Sea-quark distributions in the pion

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Using Sullivan processes with $\rho\pi\pi$, $K^{*+}\bar{K}^0\pi$, and $\bar{K}^{*0}K^+\pi$ vertices, we describe how the sea-quark distributions of a pion may be generated in a quantitative manner. The input valence-quark distributions are obtained using the leading Fock component of the light-cone wave function, which is in accord with results obtained from the QCD sum rules. The sample numerical results appear to be reasonable as far as the existing Drell-Yan production data are concerned, although the distributions as a function of x differs slightly from those obtained by imposing counting rules for $x \rightarrow 0$ and $x \rightarrow 1$. Our results lend additional support toward the conjecture of Hwang, Speth, and Brown that the sea distributions of a hadron, at low and moderate Q^2 (at least up to a few GeV^2), may be attributed primarily to generalized Sullivan processes.

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I. INTRODUCTION

In 1972, Sullivan [1] pointed out that in deep-inelastic scattering (DIS) of a nucleon by leptons the process in which the virtual photon strikes the pion emitted by the nucleon and smashes the pion into debris will scale like the original process where the virtual photon strikes and smashes the nucleon itself. In other words, the process will contribute by a finite amount to cross sections in the Bjorken limit: $Q^2 \rightarrow \infty$ and $\nu \equiv E_l - E'_l \rightarrow \infty$, with $x \equiv Q^2/(2m_N\nu)$ fixed. Being intrigued by the fact that Sullivan processes with πNN and $\pi N\Delta$ vertices will contribute to the $\bar{u}$ and $\bar{d}$ sea distributions of a nucleon, Thomas [2] in 1983 and Frankfurt, Mankiewicz, and Strikman [3] in 1989, as well as, very recently, Kumano [4], considered the possibility of using the difference $\frac{1}{2}(\bar{u} + \bar{d}) - \bar{s}$ to constrain the size of the nucleon or, equivalently, the πNN form factor. They obtained a large size or, equivalently, a very soft πNN form factor for the nucleon. As pointed out by Hwang, Speth, and Brown [5], however, the validity of such argument is spoiled by inclusion of Sullivan processes with $KN\Lambda$, $KN\Sigma$, $K^*N\Lambda$, and $K^*N\Sigma$ vertices. Instead, they showed that a reasonably hard πNN form factor, as widely used in nuclear physics, can be reconciled with the DIS data, but then it is expected that the sea distributions of the nucleon are already saturated by contributions from the generalized Sullivan processes. By considering that the sea distributions in a nucleon at low and moderate Q^2 are essentially nonperturbative (in the jargon of QCD), they used generalized Sullivan processes to generate the sea distributions of a nucleon, which turn out to compare very well with what extracted from the DIS data.

The significance of the conjecture of attributing the sea-quark distributions of a hadron at low and moderate Q^2 to its associated meson cloud as generated by strong-

interaction processes at the hadron level is that we are now able to determine the sea-quark distributions of a hadron from the knowledge of the valence distributions of the various hadrons, which presumably are calculable from quark models. The QCD evolution equations then take us from low or moderate Q^2 to large Q^2 . The previously very "fuzzy" gap between low- Q^2 (nuclear) physics and large- Q^2 (particle) physics is now linked nicely together, should the conjecture be substantiated by future experiments.

The primary objective of the present work is to take up the conjecture of Hwang, Speth, and Brown in the case of pions. Using Sullivan processes with $\rho\pi\pi$, $K^{*+}\bar{K}^0\pi$, and $\bar{K}^{*0}K^+\pi$ vertices, we describe how the sea-quark distributions of a pion at low and moderate Q^2 may be related to its valence distributions in a quantitative manner. To be specific, we consider the structure functions of a positively charged pion π^+ . The input for this calculation is as follows: The valence distribution is taken to be the one that is calculated from a light-cone pion wave function (which satisfies the QCD sum rules), with the $\rho\pi\pi$, $K^*\bar{K}\pi$, and $\bar{K}^*K\pi$ form factors adjusted to reproduce the valence number sum rules. As the resultant $\rho\pi\pi$, $K^*\bar{K}\pi$, and $\bar{K}^*K\pi$ form factors are very similar to those [6] which describe very well the extracted phase shifts in $\pi\pi$ and πK scatterings, we also consider the case in which the form factors are fixed by the scattering data, but the input valence distribution is rescaled slightly to observe the overall valence number sum rule. Both *Ansätze* yield similar results on valence- and sea-quark distributions. Using the resultant valence- and sea-quark distributions, we then find that the existing Drell-Yan (DY) experiment using high-energy $\pi^\pm$ beams can be understood in quantitative terms. We are thus encouraged by the important prospect that the sea- and valence-quark distributions for any meson may now be treated on the same footing as for the pion.

II. FORMULATION

In Fig. 1 we show the Sullivan processes which are used in this work for generating the sea-quark distributions (out of the valence-quark distributions) for π^+ . We begin by describing briefly the derivation of the formulas for taking into account contributions from these processes.

Consider Fig. 1(a) as an illustrative example. The $\rho\pi\pi$ vertex may be specified by the effective Lagrangian

$$\mathcal{L}_{\rho\pi\pi} = f_{\rho\pi\pi} \rho_\mu \cdot (\pi \times \partial_\mu \pi). \quad (1)$$

The amplitude which characterizes Fig. 1(a) is then given by

$$\mathcal{T}_\mu = f_{\rho\pi\pi} \frac{\bar{\xi}_\alpha}{\sqrt{2p'_0}} \frac{1}{\sqrt{2p_0}} (k_\alpha + p_\alpha) \frac{1}{-t + m_\pi^2 - i\epsilon} \times \langle X | j_\mu(0) | \pi^+ \rangle, \quad (2)$$

where $t \equiv -k^2 \equiv k_0^2 - \mathbf{k}^2$ and p, p', k , and q are the four-momenta carried by the initial pion, recoiling ρ meson, t -channel pion, and struck photon, respectively. ξ_α is the polarization four-vector for the vector meson (ρ) with $\xi \cdot p' \equiv \xi \cdot p' - \xi_0 p'_0 = 0$ (the Lorentz condition which ensures that there are only three physical polarizations). The following relations [1] are useful:

$$d\sigma = \frac{d^3p'}{(2\pi)^3} \sum_X (2\pi)^4 \delta^4(p' + p_X - p - q) \sum_{\text{pol}} \mathcal{T}_\mu^* \mathcal{T}_\nu, \quad (3a)$$

FIG. 1. Generalized Sullivan processes with $\rho\pi\pi$, $K^{*+}\bar{K}^0\pi$, and $\bar{K}^{*0}K^+\pi$ vertices.

$$W_{\mu\nu}^\pi \equiv \sum_X (2\pi)^4 \delta^4(p' + p_X - p - q) \langle \pi^+ | j_\mu^*(0) | X \rangle \times \langle X | j_\nu(0) | \pi^+ \rangle, \quad (3b)$$

$$\frac{d\delta W_{\mu\nu}^\pi}{dt d\omega'} \equiv 2p_0 \frac{d\sigma}{dt d\omega'} = 2p_0 \frac{\pi p'_0}{\omega} \frac{d\sigma}{d^3p'}, \quad (3c)$$

$$\omega \equiv \frac{1}{x}, \quad \omega' \equiv \frac{1}{x'}. \quad (3d)$$

Here $W_{\mu\nu}^\pi$ and $\delta W_{\mu\nu}^\pi$ are, respectively, the structure function for the pion and the correction to the pion structure function due to the Sullivan process [Fig. 1(a)]. x and x' are the Bjorken scaling variables for the entire process and subprocess Eq. (3b), respectively. Equations (2) and (3) yield

$$\frac{d\delta W_{\mu\nu}^\pi}{dt d\omega'} = \frac{f_{\rho\pi\pi}^2}{4\pi} \frac{1}{4\pi\omega} \frac{1}{(-t + m_\pi^2)^2} 4 \sum_{\text{pol}} |\xi \cdot k|^2 W_{\mu\nu}^\pi. \quad (4)$$

To carry out the summation over the three possible polarizations in a simple and Lorentz-covariant manner, we adopt the relation

$$\sum_{\text{pol}} \bar{\xi}_\mu \xi_\nu = \delta_{\mu\nu} + \frac{p'_\mu p'_\nu}{m_\rho^2}, \quad (5)$$

which vanishes identically upon contraction by p'_μ or p'_ν (Lorentz condition). We wish to caution that the off-shellness of the hadrons involved does present some problem in the formulation of the Sullivan processes. We nevertheless hope that such off-shellness represents nonperturbative physics of even higher twists (than the process in question) and may then be neglected by comparison. In this way we obtain

$$\sum_{\text{pol}} |\xi \cdot k|^2 = \left[-t + \frac{1}{4m_\rho^2} (t - m_i^2 + m_f^2)^2 \right]. \quad (6)$$

Accordingly, we obtain the formulas for determining the contribution from Fig. 1(a):

$$\delta F_{2\pi}^\pi(x, Q^2) = \int_x^1 dy f_\pi(y) F_{2\pi} \left[\frac{x}{y}, Q^2 \right], \quad (7a)$$

$$f_\pi(y) = \frac{\bar{f}_{\rho\pi\pi}^2}{4\pi} \frac{1}{4\pi y} \int_{-\infty}^t dt \frac{|F_\pi(t)|^2}{(-t + m_\pi^2)^2} \times \left[-4t + \frac{1}{m_\rho^2} (t - m_i^2 + m_f^2)^2 \right], \quad (7b)$$

where $t^m = m_i^2 y - m_f^2 y / (1 - y)$. $F_{2\pi}(x)$ is the pion structure function as would be measured in deep-inelastic electron (or muon) scattering. $\delta F_{2\pi}(x)$ is the correction to the pion structure function due to the Sullivan process [Fig. 1(a)]. $f_\pi(y)$ is the probability of finding a cloud pion carrying the original pion momentum fraction y . The form factor $F_\pi(t)$, as defined through $f_{\rho\pi\pi} \equiv \bar{f}_{\rho\pi\pi} F(t)$, characterizes the t dependence of the $\rho\pi\pi$ vertex when π is in the t channel. Similarly, we obtain, for the Sullivan process [Fig. 1(b)],

$$\delta F_{2\pi}^{\rho}(x, Q^2) = \int_x^1 dy f_{\rho}(y) F_{2\rho} \left[\frac{x}{y}, Q^2 \right], \quad (8a)$$

$$f_{\rho}(y) = \frac{\bar{f}_{\rho\pi\pi}^2}{4\pi} \frac{1}{4\pi} y \int_{-\infty}^m dt \frac{|F_{\rho}(t)|^2}{(-t + m_{\rho}^2)^2} \times \left[-4m_f^2 + \frac{1}{m_{\rho}^2} (t - m_i^2 + m_f^2)^2 \right]. \quad (8b)$$

We note that the difference between Figs. 1(a) and 1(b) in the way how different particles are off shell spoils the equality between $f_{\rho}(y)$ and $f_{\pi}(y)$, an equality expected when all particles involved in the process are on shell. In our calculation we choose the $F_{\pi}(t)$ and $F_{\rho}(t)$ such that $N_{\rho} \equiv \int f_{\rho}(y) dy$ is the same as $N_{\pi} \equiv \int f_{\pi}(y) dy$, thereby reducing the number of independent cutoff parameters (or form factors).

It is then straightforward to generalize the consideration slightly to take into account the Sullivan processes with $K^{*+} \bar{K}^0 \pi$ and $\bar{K}^{*0} K^+ \pi$ vertices. We shall use flavor-SU(3) symmetry and adopt $f_{K^{*+} K \pi} = -\frac{1}{2} f_{\rho\pi\pi}$. As a specific attempt, we consider only the case of parameterizing the various form factors in a monopole form [6]:

$$F(t) = \frac{\Lambda_{\alpha}^2 - m_{\alpha}^2}{\Lambda_{\alpha}^2 - t}. \quad (9)$$

We wish to pause a little to consider the derivation of valence-quark distributions from light-cone wave functions, as we shall try to generate sea distributions via Sullivan processes using valence distributions as the input. This question has recently been considered by one [7] of us. In short, we adopt the “light-cone perturbation theory,” which, as suggested by Lepage and Brodsky [8], may be used for obtaining the hard-scattering amplitudes for high-energy exclusive processes. In conjunction with the suggestion, it was proposed that the hadron wave function may be represented as an infinite series of Fock components. In particular, the pion π^+ may be described

in the light-cone language as

$$|\pi^+(P)\rangle = C_0 |u\bar{d}\rangle + C_g |u\bar{d}g\rangle + C_q |u\bar{d}q\bar{q}\rangle + \cdots, \quad (10)$$

where g is a gluon and $q\bar{q}$ are quark-antiquark pairs. Specifically, it was shown [8] that, for exclusive processes at sufficiently large Q^2 , the contribution from the leading Fock component $|u\bar{d}\rangle$ dominates over all the others. Considerable progress was made by Chernyak and Zhitnitsky [9] who were able to improve upon the hadron wave functions making use of the results from QCD sum-rule studies [10] of properties of low-lying hadrons. To describe a specific component in the wave function, we adopt, using light-cone variables,

$$x_i \equiv \frac{p_i^+}{P^+} = \frac{p_{i0} + p_{i3}}{P_0 + P_3}, \quad \mathbf{p}_{i\perp} \equiv (p_{i1}, p_{i2}), \quad (11)$$

which are invariant under Lorentz boosts in the z direction. In light-cone language, moreover, Lorentz boosts in the z direction are purely kinematical—that is, no particles are created or destroyed. Thus there is an *invariant* description of the complicated hadron wave function such as Eq. (1) in all frames which are related by Lorentz boosts in the z direction. In this way we may eventually go over to the infinite-momentum limit ($P_3 \rightarrow \infty$) to study Bjorken scaling and its violations. For these reasons we expect that, if the quark distributions in the parton model can ever be described in terms of wave functions of any sort, the hadron wave functions written in light-cone language appear to be the best candidate for such a description. To work in light-cone language, we need to depart, temporarily until Eq. (19) below, from the notation (and metric) used so far and adopt the notation of Kogut and Soper [11]. The need for a change in the notations arises as the $\mp$ components of a four-vector are now regarded as the zeroth and third components, respectively. As a result, we note, for example, that $g_{\mu\nu}$ is no longer diagonal, γ matrices are suitable linear combinations of the old ones, and the Dirac spinors u and v differ from the old ones. Nevertheless, the cross sections and physical observables are understood in the (ordinary) instant form. In the light-cone language, we write [11]

$$\psi(x) = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2}} \int d^2 p_{\perp} \int \frac{d\eta}{\eta} \sum_{\lambda} [u(p, \lambda) e^{-i p \cdot x} b(p, \lambda) + v(p, \lambda) e^{i p \cdot x} d^{\dagger}(p, \lambda)], \quad (12)$$

$$\{b(p, \lambda), b^{\dagger}(p', \lambda')\} = \delta_{\lambda\lambda'} (2\pi)^3 2\eta \delta(\eta - \eta') \delta^2(p_{\perp} - p'_{\perp}), \quad (13a)$$

$$\{d(p, \lambda), d^{\dagger}(p', \lambda')\} = \delta_{\lambda\lambda'} (2\pi)^3 2\eta \delta(\eta - \eta') \delta^2(p_{\perp} - p'_{\perp}), \quad (13b)$$

$$\sum_{\lambda} u(p, \lambda) \bar{u}(p, \lambda) = \gamma^{\mu} p_{\mu} + m, \quad \sum_{\lambda} v(p, \lambda) \bar{v}(p, \lambda) = \gamma^{\mu} p_{\mu} - m. \quad (13c)$$

The electromagnetic current $J_{\mu}(x)$ is specified by

$$J_{\mu}(x) = :\bar{\psi}(x) \gamma_{\mu} \psi(x):, \quad (14)$$

where $:AB:$ denotes the normal-ordered product of A and B . (Note that the electric charge Q_i may be inserted at the end of manipulations.) For the commutator $[:AB::CD:]$, we may apply Wick's theorem separately to the products $:AB::CD:$ and $:CD::AB:$ and then take the difference. In this way we obtain

$$[J_\mu(x), J_\nu(y)] = \gamma_{\mu ab} \gamma_{\nu cd} [\langle \bar{\psi}_a(x) \psi_d(y) \rangle_0 + : \bar{\psi}_a(x) \psi_d(y) :] \{ \psi_b(x), \bar{\psi}_c(y) \} \\ - \gamma_{\mu ab} \gamma_{\nu cd} \{ \bar{\psi}_a(x), \psi_d(y) \} [\langle \bar{\psi}_c(y) \psi_b(x) \rangle_0 + : \bar{\psi}_c(y) \psi_b(x) :] . \quad (15)$$

The leading Fock component of π^+ may be described as follows, with the $\mathbf{p}_\perp$ dependence explicitly taken into account:

$$|P\rangle \equiv \int \frac{d\eta_1 d^2 p_{1\perp}}{(2\pi)^3 2\eta_1} \int \frac{d\eta_2 d^2 p_{2\perp}}{(2\pi)^3 2\eta_2} (2\pi)^3 2\eta \delta(\eta_1 + \eta_2 - \eta) \delta^2(\mathbf{p}_{1\perp} + \mathbf{p}_{2\perp} - \mathbf{P}_\perp) \phi_\pi(p_1, \lambda_1; p_2, \lambda_2; P) d^\dagger(p_2, \lambda_2) b^\dagger(p_1, \lambda_1) |0\rangle , \quad (16a)$$

$$\phi_\pi(p_1, \lambda_1; p_2, \lambda_2; \mathbf{P}_\perp = 0) = \Psi(x, k_\perp^2) \bar{u}(p_1, \lambda_1) (\gamma \cdot P + M) \gamma_5 v(p_2, \lambda_2) , \quad (16b)$$

$$\Psi(x, k_\perp^2) = A_\pi \left[\frac{1-x}{x} \right]^{1/2} \exp \left[- \frac{k_\perp^2 + m^2}{8\beta^2 x(1-x)} \right] , \quad (16c)$$

where $\mathbf{k}_\perp \equiv \mathbf{k}_{1\perp} \equiv \mathbf{p}_{1\perp} - x_1 \mathbf{P}_\perp$ and $x \equiv x_1 \equiv \eta_1/\eta$, etc. Note [12] that the form for ϕ_π as given by Eq. (16b) is related to the rest-frame wave function $\delta_{\lambda_1, -\lambda_2}$ by a Melosh transformation if the so-called “mock hadron” approximation is adopted. As the wave function adopted by Chernyak and Zhitnitsky [9] or by Dziembowski [12] reproduces the QCD sum rules only approximately, we set out to choose an explicit form for our $\psi(x, k_\perp^2)$, that may better reproduce the sum rules at $\mu_0^2 = (500 \text{ MeV})^2$, with $\xi \equiv x_1 - x_2 = 2x - 1$:

$$\langle \xi^2 \rangle = 0.46, \quad \langle \xi^4 \rangle = 0.30, \quad \langle \xi^6 \rangle = 0.21 . \quad (17)$$

These moments are evaluated [12] as the expectation values using the absolute square of the wave function [Eqs. (16)] integrated over $k_\perp^2$ up to μ_0^2 . For a choice of $m = 330 \text{ MeV}$ (constituent quark mass), $M = 600 \text{ MeV}$ (mock pion mass, which is given approximately as the mass before the ρ - π mass splitting as caused by spin-spin interactions is taken into account), and $\beta = 500 \text{ MeV}$ (which characterizes the size of pion), we obtain exactly the values listed in Eq. (17). Note that these input parameters are very similar to what was used by Dziembowski [12], although the detailed form for $\Psi(x, k_\perp^2)$ is different. Note that [9] the normalization is fixed by the condition

$$\langle 0 | \bar{d}(0) \gamma_\mu \gamma_5 u(0) | \pi^+(P) \rangle = \sqrt{2} f_\pi P_\mu , \quad (18)$$

with f_π the pion decay constant (94 MeV).

Using Eqs. (15) and (16) with the aid of Eqs. (12) and (13), it is straightforward to evaluate the matrix element $\langle P | : [J_\mu(x), J_\nu(0)] : | P \rangle$, which is related to the $W_{\mu\nu}$ of Eq. (3) by a simple Fourier transform. The result for $W_{\mu\nu}$ can be split into a contribution from the u quark and another one from the $\bar{d}$ quark, with the two contributions equal to each other apart from the charge factors Q_i^2 . In this way the valence u -quark distribution may be identified unambiguously [7]:

$$u(x) = \frac{8\pi}{x(1-x)} \int dk_\perp^2 |\Psi(x, k_\perp^2)|^2 \\ \times \left[\frac{k_\perp^2 + m^2}{x} + xM^2 + 2Mm \right] \\ \times \left[\frac{k_\perp^2 + m^2}{1-x} + (1-x)M^2 + 2Mm \right] , \quad (19)$$

with m and M the quark and hadron masses, respectively. Using the wave function Eq. (16) that is constrained by the QCD sum rules [Eq. (17)], we find that the resultant $u(x)$ gives rise to $\int dx u(x) = 0.40$ (the number of the valence u quarks associated with the leading Fock component). Note that the shape of the resultant $u(x)$ is similar to the valence distribution extracted from the experiment [13]. (Of course, it is possible to improve on the form of $\psi(x, k_\perp^2)$ [Eq. (12)] so that the valence distribution as a function of x is close to that obtained by imposing counting rules. As counting rules are presumably applicable only at $Q^2 \rightarrow \infty$, we do not worry about this aspect in this paper.)

The valence distributions obtained from the leading Fock component may be used as the primary input. Using flavor-SU(3) rotations, we obtain valence distributions for ρ , K , and K^* , which are also needed for the input. In light of the currently crude information on the sea distributions of the pion, we do not worry about the fine details such as flavor-SU(3)-breaking effects.

Qualitatively speaking, the valence-quark distributions obtained from the leading Fock component, i.e., the first term on the right-hand side (RHS) of Eq. (10), represent the valence contribution from the “core” of the hadron. The idea of using generalized Sullivan processes to generate sea-quark distributions and additional valence-quark distributions is based upon the belief that the remaining Fock components as appearing on the RHS of Eq. (10) should organize themselves naturally into the various hadrons—as known to be true at low and moderate Q^2 . In the nucleon case, we may adopt [5], as a

phenomenological approach, the observed valence distributions for the nucleon and parton distributions for completely dressed mesons as the input for generalized Sullivan processes so that sea-quark distributions of the nucleon are obtained. In the present case for the pion, we attempt to use the valence distributions obtained from the leading Fock component as the input, and in some sense this represents a step closer to first principles.

It is expected that the gluon distribution in the pion comes from the Fock components, which contain explicit gluonic degrees of freedom. At large Q^2 where perturbative QCD is applicable, the next-to-leading Fock component, i.e., the second term in Eq. (10), may be calculated, yielding a prediction on the gluon distribution. The gluon distributions at different but large Q^2 are then related by the QCD evolution equation. At low Q^2 where nonperturbative aspects of QCD dominate, it is not clear how the gluon distribution can be calculated using the meson-baryon picture. However, once the gluon distributions in the various core hadrons are known, generalized Sullivan processes provide certain relations among these gluon distributions. In this paper we do not intend to pursue any further the issue related to the question of how the gluon distribution in a hadron can be calculated.

III. NUMERICAL RESULTS AND DISCUSSION

As mentioned earlier, the idea of using generalized Sullivan processes to generate sea-quark distributions is based upon the belief that the various Fock components, as appearing on the RHS of Eq. (10), should organize themselves naturally into the various hadrons—as known to be true at low and moderate Q^2 . For instance, the first term represents the core pion, the third term a combination of two-meson states, etc. An important aspect is that the nonleading Fock components also contribute to valence distributions. The situation for the pion is somewhat amusing as it is difficult to tell the difference between the meson in the “meson cloud” and the meson in the core, but it is clear that the valence distributions must obey the valence number sum rules—e.g., adding up to one u quark and one d antiquark in π^+ .

Specifically, using the cutoff (or form factors) obtained [6] from meson-meson scatterings, one immediately recognizes that the valence distribution contributed by Sullivan processes (Fig. 1) is comparable in amount to that from the core. Therefore we may choose the form factors such that the number of the valence u quarks (or $\bar{d}$ quarks) from the cloud (via Sullivan processes) and that from the core add up to unity. The valence number sum rules thus serve as an important guideline when one takes into account contributions from generalized Sullivan processes.

Numerically, we may try out the following two *Ansätze* and compare the predictions in the two scenarios.

(1) The valence distribution is taken to be the one that is calculated from the light-cone pion wave function (which satisfies the QCD sum rules), with the $\rho\pi\pi$, $K^*\bar{K}\pi$, and $\bar{K}^*K\pi$ form factors adjusted to reproduce the valence number sum rules. The $\rho\pi\pi$, $K^*\bar{K}\pi$, and $\bar{K}^*K\pi$ couplings are taken from Ref. [6]. The Q^2 is tak-

en to be 3 GeV^2 . Note that the resultant $\rho\pi\pi$ form factor (with ρ in the t channel) is 2450 MeV , which is not very far from that [6] obtained from fitting the extracted phase shifts in $\pi\pi$ and πK scatterings ($\Lambda_\rho = 1600 \text{ MeV}$). As there is little clue on the amount of the strangeness in a pion, we choose $\Lambda_{K^*} = 4000 \text{ MeV}$, which represents a similar increase over that [6] used in the study of meson-meson scatterings. Note that Λ_K and Λ_π are adjusted to ensure that quarks and antiquarks are produced in pairs. This yields $\Lambda_\pi = 1882 \text{ MeV}$ and $\Lambda_K = 3480 \text{ MeV}$. The results from this calculation are denoted as solid curves in Figs. 2–4.

The integrated numbers of mesons in the cloud may be determined by

$$\int f_\pi(y) dy = \int f_\rho(y) dy = 0.477$$

and

$$\int f_{K^*}(y) dy = \int f_{\bar{K}}(y) dy = 0.271.$$

[See Eqs. (7b) and (8b).] The momentum fractions carried by the various partons (in π^+) are $\langle x \rangle_u = \langle x \rangle_{\bar{d}} = 21.5\%$, $\langle x \rangle_{\bar{u}} = \langle x \rangle_d = 2.5\%$, $\langle x \rangle_s = \langle x \rangle_{\bar{s}} = 3.5\%$ and $\langle x \rangle_g = 45\%$. (Note that the momentum fraction carried by gluons is a derived quantity once the momentum fractions carried by quarks and antiquarks are determined.) All these results appear to be rather reasonable.

(2) In light of the results obtained in the previous case, we may also consider the case in which the form factors are fixed by the scattering data, but the input valence distribution is rescaled slightly to observe the overall valence number sum rule. In order to do so, we rescale

FIG. 2. Valence momentum distributions obtained from the two calculations described in the text are shown as solid and dashed curves, respectively. For comparison, the valence momentum distribution of Owens and Reya [14], also at $Q^2 = 3 \text{ GeV}^2$, is shown as a dotted curve. In addition, the valence momentum distribution (at $Q^2 \approx 25 \text{ GeV}^2$) obtained by the NA3 collaboration [13] in fitting to their Drell-Yan data using $\pi^\pm$ beams is shown as a dot-dashed curve. Note that the thick solid curve describes the result of evolving the solid curve to 25 GeV^2 via the Altarelli-Parisi equation.

FIG. 3. Sea $\bar{u}$ -quark momentum distributions at $Q^2=3 \text{ GeV}^2$ obtained from the two calculations described in the text are shown as solid and dashed curves, respectively. For comparison, the corresponding distribution of Owens and Reya [14], also at $Q^2=3 \text{ GeV}^2$, is shown as dotted curve. In addition, the sea distribution used by NA3 at $Q^2=25 \text{ GeV}^2$ is shown as a dot-dashed curve.

by a multiplicative factor of about $\frac{5}{3}$ the valence distribution that is calculated from the light-cone pion wave function so that the valence u quarks from the core and cloud add up to unity with the input of the (somewhat softer) form factors [6] $\Lambda_\rho=1600 \text{ MeV}$ and $\Lambda_{K^*}=3000 \text{ MeV}$. Again, Λ_K and Λ_π are adjusted to ensure that quarks and antiquarks are produced in pairs. The results from this calculation are similar to those in the previous case and are to be denoted as dashed curves in Figs. 2–4.

We note that both *Ansätze* differ somewhat from what we have adopted in the nucleon case [5], where for the

FIG. 4. Sea strange-quark momentum distributions at $Q^2=3 \text{ GeV}^2$ obtained from the two calculations described in the text are shown as solid and dashed curves, respectively. For comparison, the corresponding distribution of Owens and Reya [14], also at $Q^2=3 \text{ GeV}^2$, is shown as a dotted curve. In addition, the sea distribution used by NA3 at $Q^2=25 \text{ GeV}^2$ is shown as dot-dashed curve.

sake of consistency the mesons in the cloud are taken as fully dressed, i.e., the physical mesons which contain both valence and sea distributions. In the present case, however, we have only mesons so that the distinction between the cloud and core is no longer there. This results in a slightly different (perturbation) scheme for the pion, although in principle the present scheme is equivalent to the one used previously [5] for the nucleon when effects to all orders are included.

In Fig. 2 we show the valence momentum distributions obtained from these two calculations. For comparison, the valence momentum distribution of Owens and Reya [14], also at $Q^2=3 \text{ GeV}^2$, is shown as a dotted curve. In addition, the valence momentum distribution (at $Q^2 \approx 25 \text{ GeV}^2$) obtained by the NA3 Collaboration [13] in fitting to their Drell-Yan data using $\pi^\pm$ beams is shown as a dot-dashed curve. Using our routine to evolve the valence distributions via Altarelli-Parisi equations from $Q^2=3$ to 25 GeV^2 (which has the primary effect of softening the distributions, i.e., shifting the weight to the smaller x region), the result is displayed as a thick solid curve. As shall be explained shortly later, our valence distribution at 25 GeV^2 , together with our sea distributions, is consistent with the NA3 data. Note that we have chosen to show the results for $x \leq 0.5$ so as to display clearly the differences among the various curves, but it may be useful to keep in mind that for $x > 0.5$ all curves approach zero very smoothly (in fact, almost linearly in the case of valence distributions).

In Figs. 3 and 4, we show, respectively, the sea $\bar{u}$ -quark and sea strange-quark momentum distributions obtained from the two calculations described above. For comparison, the corresponding distributions of Owens and Reya, also at $Q^2=3 \text{ GeV}^2$, are shown as dotted curves. In addition, the sea distributions used by NA3 at $Q^2=25 \text{ GeV}^2$ are shown as dot-dashed curves. It is clear that the sea distributions obtained via Sullivan processes are “harder” than those used by Owens and Reya (on the basis of counting rules). We see this as reasonable as QCD evolution always tends to make a distribution softer, in addition to populating sea quarks at very small x , and in addition the distributions which satisfy the counting rules are in principle those at $Q^2 = \infty$ rather than at $Q^2=3 \text{ GeV}^2$ (if the counting rules apply at all).

It is of practical interest to consider how the NA3 Drell-Yan data can be understood quantitatively in our model. To this end we focus our results from the first calculation (shown as solid curves), keeping in mind that QCD evolution from 3 to 25 GeV^2 has the effect of softening the distributions as going from the solid curve to the thick solid curve in Fig. 2. Without knowing the acceptance of their apparatus as a function of (x_1, x_2) , we make the comparison in the following manner.

First, we consider the NA3 data in relation to their “extracted” valence and sea distributions. According to their presentation [13] on the $F_2^\pi(x)$, their data can be explained entirely by their valence distribution for $x > 0.4$, and for x starting from 0.4 down to 0.2 (the minimum x which they managed to go down), the data start calling for the existence of their sea distributions—negligible at $x=0.4$, but increasing steadily to about 25% at $x=0.2$.

However, the “existence” of the sea is only marginal as the data at $x=0.2-0.4$ have large experimental errors and the valence distributions alone are lower than but still compatible with the data within the cited experimental errors. In particular, we note that, at $x \approx 0.2$, the 280-GeV data could be understood if the amount of the sea is in the range 0–60 % of the proposed NA3 valence distribution. Of course, the proposed NA3 sea distributions represent a best fit to their data if their *Ansätze*, including primarily the assumption on the nucleon parton distributions and the working hypotheses on the relative strengths and the asymptotic behaviors (see $x \rightarrow 0$ and $x \rightarrow 1$) of the various sea distributions in the pion, are adopted. The experimental situation is not satisfactory, but can be improved in the near future.

Next, we consider if our calculated parton distributions are compatible with the NA3 data. Figure 2 indicates that our valence distribution at 25 GeV² is stronger than the fitted NA3 one by about 30–40 % at $x \approx 0.2$, or about 10% at $x \approx 0.3$, crossing at $x \approx 0.35$, and becoming weaker than the NA3 one from there on. By comparing also our sea distribution with the proposed NA3 one, we find that our sea distribution (after evolution) contributes by about the same amount starting at $x \approx 0.2$, decreasing more slowly than the NA3 sea as x increases. Thus it is seen that our model prediction is compatible with the pion structure function $F_2^\pi(x)$ as extracted by the NA3 group from the Drell-Yan production cross sections in π^+p collisions (by imposing counting rules for $x \rightarrow 0$ and $x \rightarrow 1$)—it is the same as the prediction from the proposed NA3 distributions for x near 0.4, but with a slightly sharper slope, but compatible with the data. Note that the missing strength due to the weaker valence distribution at large x is compensated by the contribution due to our sea distributions, as can easily be seen by comparing the various curves at $x = 0.5$. Thus the NA3 data can be understood in a quantitative manner—there is also some contribution from the sea at smaller x , but such contribution falls off more slowly than the fitted NA3 distributions as x increases.

As in the nucleon case, we could in principle adopt as the input the valence distributions which are obtained by imposing counting rules for $x \rightarrow 0$ and $x \rightarrow 1$, such as the NA3 [13] or Owens-Reya distribution [14]. We may then expect to produce sea distributions which differ from those shown in Figs. 3 and 4. However, we believe that whether or not such theoretical bias should be imposed at Q^2 of a few GeV² should be left for testing by future experiments. It is clearly desirable that future experiments can be analyzed without resorting to such biases.

The normalization condition imposed by Eq. (18) appears to be of marginal importance in our calculations: We may increase the valence number by multiplying the input valence distribution Eq. (18) by an overall constant, but then the form factors need to be reduced accordingly, to ensure again that the total valence number is unity. On the other hand, there is no *a priori* reason why the leading Fock component in the wave function [i.e., the

first term in Eq. (10)] should be normalized according to Eq. (18) as, if it should indeed be so, it would amount to saying that the physics of pion decay is determined solely by the leading Fock component (or the core). It is therefore of critical importance to observe that the renormalization property which we have just mentioned arises in a qualitative manner. Nevertheless, the arbitrary renormalization point may eventually be fixed, should there be sufficient experimental information on the sea-quark distributions, as in the case of the proton [5]. In fact, our first calculation already yields results which are doing somewhat better than those from the second choice, as far as the NA3 data are concerned.

One should remark that in principle one may in fact *first* evolve the input valence distribution to higher Q^2 and *then* calculate the sea-quark distributions. It is clear from Eqs. (7) and (8) that the results remain the same for practical purposes. In other words, QCD evolution and convolution for Sullivan processes commute. The issue is when we should start doing QCD evolution via Altarelli-Parisi equations. In our opinion, the Q^2 must be high enough (or the resolution good enough) in order to see the substructure (or occurrence of subprocesses) at the quark-gluon level. In this paper we take it to be 3 GeV², which we believe is a reasonable guess. For Q^2 below such a value, we believe that quarks and gluons exist only by associating themselves with hadrons and thus Sullivan processes provide a natural way for organizing parton distributions at these Q^2 . We do not have to justify our statements for low Q^2 as all of us know that the baryon-meson picture works nicely there. What is intriguing is that the contribution from Sullivan processes never goes away—it scales or it does not vanish in the Bjorken limit. In other words, one could in principle tell that a quark actually comes from some parton via QCD evolution and that this parton belongs to the core of the hadron or to some hadron in the cloud.

In summary, we have described how the sea-quark distributions of a pion may be generated in a quantitative manner, using generalized Sullivan processes with $\rho\pi\pi$, $K^{*+}\bar{K}^0\pi$, and $\bar{K}^{*0}K^+\pi$ vertices. The numerical results appear to be very reasonable as far as the NA3 Drell-Yan data [13] are concerned, thus encouraging further use of the conjecture of Hwang, Speth, and Brown [5] in obtaining the sea distributions of a hadron, at low and moderate Q^2 (at least up to a few GeV²), on the basis of generalized Sullivan processes.

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