INTERFACIAL SUPER-ROUGHENING BY LINEAR GROWTH EQUATIONS

NING-NING PANG* and WEN-JER TZENG†

* *Department of Physics, National Taiwan University, Taipei,
Taiwan, Republic of China*

† *Department of Physics, Tamkang University, Tamsui,
Taipei, Taiwan, Republic of China*

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We give an extensive analytical study of a class of linear growth equations in 1+1 dimensions which describe certain interfacial super-roughening processes. With our calculation, we give a first rigorous analytical affirmation on the applicability of the anomalous dynamic scaling ansatz, which has been proposed to describe the dynamics of super-rough interfaces in finite systems. In addition, we explicitly evaluate not only the leading order but also all the sub-leading orders which dominate over the ordinary dynamic scaling term. Finally, we briefly discuss the influence of the macroscopic background formation on the interfacial anomalous roughening in super-rough growth processes.

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The kinetic interfacial roughening phenomenon has drawn much attention for its generic behaviors widespread in Nature's morphology.^{1–3} One of the most important physical quantities related to the interfacial roughening phenomena is the global interfacial width $w(L, t)$, defined as

$$w^2(L, t) \equiv \overline{\langle (h(x, t) - \langle h(x, t) \rangle_L)^2 \rangle}_L, \quad (1)$$

with $h(x, t)$ denoting the interface height from a flat substrate at position x and time t , $\langle \cdots \rangle_L$ denoting the lateral spatial average over the whole system of lateral size L , and the overbar denoting the statistical average, throughout the paper. It describes the interface height fluctuation relative to the average interface height over the whole system of lateral size L . Extensive numerical works have observed that the global interfacial widths $w(L, t)$ of the kinetically roughened interfaces obey the ordinary dynamic scaling behavior⁴:

$$w(L, t) \sim \begin{cases} t^{\chi/z} & \text{for } t^{1/z} \ll L, \\ L^\chi & \text{for } t^{1/z} \gg L. \end{cases} \quad (2)$$

Here, the two independent exponents χ and z are known as the *global roughness exponent* and the *dynamic exponent*, respectively. In addition to the global interfacial

width $w(L, t)$, the equal-time height difference correlation function $G(r, t)$ and the local interfacial width $w(l, t)$ are also very important and informative. Here, $G(r, t)$ is defined as

$$G(r, t) \equiv \overline{\langle (h(x, t) - h(x + r, t))^2 \rangle_L} \quad (3)$$

and $w(l, t)$ is defined as

$$w^2(l, t) \equiv \overline{\langle (\langle h(x, t) \rangle_l - \langle h(x, t) \rangle_l)^2 \rangle_l} \quad (4)$$

with $\langle \cdots \rangle_l$ denoting, throughout the paper, the lateral spatial average calculated within a local window of lateral size l . In contrast to the global interfacial width $w(L, t)$, the local interfacial width $w(l, t)$ describes the interface height fluctuation relative to the average interface height within the local window of lateral size l ($\ll L$). From the point of the experimental measurements, the equal-time height difference correlation functions $G(r, t)$ (or the local interfacial widths $w(l, t)$) are much more accessible than the global interfacial widths $w(L, t)$ for both time and economic concerns. Since it has been generally believed that the kinetically roughened interfaces are self-affine (i.e. $w(l, t)$ and $G(r, t)$ have the same scaling behavior as $w(L, t)$), most people only measure $w(l, t)$ or $G(r, t)$ in the experiments.

Recently, this assumption about kinetically roughened interfaces being self-affine is challenged by both the numerical^{5–10} and experimental^{11–13} observations of the peculiar interfacial features of the growth processes with the global roughness exponent $\chi > 1$. This class of growth processes has been coined the name “super-rough”,⁶ since their relative global interfacial widths diverge with L ; i.e., $[w(L, t \rightarrow \infty)/L] \rightarrow \infty$, as $L \rightarrow \infty$. People have proposed that the equal-time height difference correlation functions $G(r, t)$ and the local interfacial widths $w(l, t)$ of the super-rough growth processes in finite systems of lateral size L obey the *anomalous dynamic scaling ansatz*⁶:

$$G(r, t) \sim \begin{cases} t^{2\chi/z} & \text{for } t^{1/z} \ll r, \\ r^{2(\chi-\kappa)} t^{2\kappa/z} & \text{for } r \ll t^{1/z} \ll L, \\ r^{2(\chi-\kappa)} L^{2\kappa} & \text{for } L \ll t^{1/z}, \end{cases} \quad (5)$$

and

$$w^2(l, t) \sim \begin{cases} t^{2\chi/z} & \text{for } t^{1/z} \ll l, \\ l^{2(\chi-\kappa)} t^{2\kappa/z} & \text{for } l \ll t^{1/z} \ll L, \\ l^{2(\chi-\kappa)} L^{2\kappa} & \text{for } L \ll t^{1/z}. \end{cases} \quad (6)$$

To compare them with the ordinary dynamic scaling behavior, we see that the spatial scaling of $G(r, t)$ versus r is described by the *local roughness exponent* $\chi' (\equiv \chi - \kappa)$ instead of the global roughness exponent χ . Thus, the appearance of the third independent nonzero exponent κ is the signature of anomalous dynamic scaling behaviors. A Flory-type scaling argument was proposed in Ref. 14 to estimate the value of the exponent κ .

Although many numerical works have been done, rigorous analytical affirmations of the anomalous dynamic scaling ansatz in finite systems are still lacking. Moreover, there is still limited understanding about the origin which causes anomalously roughened interfaces. Naively, people generally conjecture that the local interfacial orientational instability is the sole mechanism responsible for the occurrence of anomalous roughening in super-rough growth processes. However, we have numerically shown that, for some super-rough growth models, this conjecture fails.¹⁵ The origin which causes interfacial anomalous roughening in super-rough growth processes is actually much more subtle and intriguing.

Since most super-rough growth models are nonlinear and it is thus difficult, if not impossible, to get any analytical results without approximation, we are motivated to take an extensive analytical study of the super-rough growth processes described by the following class of *linear* growth equations in 1+1 dimensions⁹:

$$\partial_t h(x, t) = (-1)^{m+1} \nu \partial_x^{2m} h(x, t) + \eta(x, t) \quad \text{with integer } m \geq 2, \quad (7)$$

where $h(x, t)$ denotes the interface height at position x and time t and $\eta(x, t)$ represents white noise with zero mean and correlation

$$\overline{\eta(x, t)\eta(x', t')} = D\delta(x - x')\delta(t - t'). \quad (8)$$

Recall that, for $m = 1$ and 2 , the above equation denotes respectively the well-known Edwards–Wilkinson Equation,¹⁶ mimicking the particle sedimentation under gravity, and the Mullins–Wolf–Villain Equation,¹⁷ mimicking the surface-diffusion-driven growth mechanism in molecular-beam-epitaxy growth processes. By using the simple scaling analysis, it is straightforward to obtain the values of the global roughness exponent $\chi = (2m - 1)/2$ and the dynamic exponent $z = 2m$. Note that, for $m \geq 2$, the global roughness exponent $\chi > 1$. Thus, the interfacial growth processes described by the above class of linear growth equations with $m \geq 2$ display super-roughening phenomena. Since the linear growth equations are exactly solvable, the systematic rigorous study of the above class of linear growth equations can help us to understand and tackle some unsettled issues in interfacial super-roughening phenomena. We first want to take an extensive analytical study of the equal-time height difference correlation functions $G(r, t)$ for systems described by Eq. (7) with finite lateral size of L and rigorously evaluate their asymptotic forms in different time regimes. By employing the obtained results of $G(r, t)$, we then also obtain the asymptotics of the local interfacial width $w(l, t)$ in different time regimes and analytically verify the validity of the anomalous dynamic scaling ansatz in finite systems. Finally, we will discuss the influence of the macroscopic background formation on the interfacial anomalous roughening.

Let us consider a one-dimensional interface $h(x, t)$ defined on a linear substrate, from $x = 0$ to $x = L$, with periodic boundary conditions. By Fourier transforming Eq. (7) into k -space, i.e.,

$$\tilde{f}(k_n, t) \equiv \frac{1}{L} \int_0^L dx e^{-ik_n x} f(x, t) \quad (9)$$

in which $f(x, t)$ denotes $h(x, t)$ or $\eta(x, t)$ and $k_n \equiv n(2\pi/L)$ with $n = 0, \pm 1, \pm 2, \dots$, we obtain

$$\partial_t \tilde{h}(k_n, t) = -\nu k_n^{2m} \tilde{h}(k_n, t) + \tilde{\eta}(k_n, t). \quad (10)$$

Under the assumption of the flat initial conditions (i.e., $h(x, t=0) = 0$ for all x), the solution is easily obtained:

$$\tilde{h}(k_n, t) = e^{-\nu k_n^{2m} t} \int_0^t d\tau e^{\nu k_n^{2m} \tau} \tilde{\eta}(k_n, \tau). \quad (11)$$

Consequently, by Fourier transforming this result back to x -space, i.e.

$$f(x, t) = \sum_{n=-\infty}^{\infty} e^{ik_n x} \tilde{f}(k_n, t) \quad (12)$$

with $\tilde{f}(k_n, t)$ denoting $\tilde{h}(k_n, t)$ or $\tilde{\eta}(k_n, t)$, we then obtain the interface height $h(x, t)$ in the (x, t) -space

$$h(x, t) = \frac{1}{L} \int_0^L dr \int_0^t d\tau \eta(x+r, t-\tau) \sum_{n=-\infty}^{\infty} e^{-ik_n r} e^{-\nu k_n^{2m} \tau}. \quad (13)$$

In the following, we would like to employ the above result to study the equal-time height difference correlation function $G(r, t)$ in detail. By definition,

$$\begin{aligned} G(r, t) &= \overline{\langle h(x, t) - h(x+r, t) \rangle_L} \\ &= \overline{\langle h^2(x, t) \rangle_L} + \overline{\langle h^2(x+r, t) \rangle_L} - 2\overline{\langle h(x, t)h(x+r, t) \rangle_L}. \end{aligned} \quad (14)$$

By employing Eqs. (8) and (13), it is straightforward to obtain

$$\overline{\langle h(x, t)h(x+r, t) \rangle_L} = \frac{D}{2L} \sum_{n=-\infty}^{\infty} e^{ik_n r} \frac{1 - e^{-2\nu k_n^{2m} t}}{\nu k_n^{2m}}. \quad (15)$$

After some simple calculation, the equal-time height difference correlation function $G(r, t)$ is then obtained as follows

$$G(r, t) = \frac{2D}{L} \sum_{n=1}^{\infty} [1 - \cos(k_n r)] \frac{1 - e^{-2\nu k_n^{2m} t}}{\nu k_n^{2m}}. \quad (16)$$

It is easily seen from Eq. (16) that there exists a characteristic wave-vector k_c ($\equiv (2\nu t)^{-1/2m}$). It separates the time evolution of $G(r, t)$ into three regimes: $k_c \gg 1/r$, $1/r \gg k_c \gg 1/L$, and $k_c \ll 1/L$, which correspond to $t \ll r^z/\nu$ (the early time regime), $r^z/\nu \ll t \ll L^z/\nu$ (the intermediate time regime), and $t \gg L^z/\nu$ (the late time regime), respectively. (Recall that the dynamic exponent $z = 2m$.) Subsequently, we would like to study the asymptotics of $G(r, t)$ in these three successive time regimes. We will pay special attention to the intermediate and late time regimes and undertake extensive studies of the asymptotics of $G(r, t)$ in these two regimes, because the anomalous dynamic scaling behaviors of super-rough interfaces appear in these two regimes.

With $k_c \gg 1/L$ in both the early and intermediate time regimes, we can approximate Eq. (16) in the two regimes by taking the limit $L \rightarrow \infty$ and, thus,

$$G(r, t)|_{t \ll L^z/\nu} \simeq \frac{D}{\pi\nu} \int_0^\infty dk [1 - \cos(kr)] \frac{1 - e^{-2\nu k^{2m}t}}{k^{2m}}. \quad (17)$$

With simple change of variables ($k \equiv k_c x$), Eq. (17) can then be rewritten as

$$G(r, t)|_{t \ll L^z/\nu} \simeq \frac{D}{\pi\nu} k_c^{1-2m} \int_0^\infty dx [1 - \cos(k_c r x)] \frac{1 - e^{-x^{2m}}}{x^{2m}}. \quad (18)$$

We then derive the early and intermediate time asymptotics of $G(r, t)$ from Eq. (18). First, for the early time regime $t \ll r^z/\nu$, we have $k_c \gg 1/r \gg 1/L$. Thus, by employing this property, the early time asymptotics of $G(r, t)$ is readily obtained to the leading order

$$G(r, t)|_{t \ll r^z/\nu} \sim O\left(\frac{D}{\nu} k_c^{1-2m}\right). \quad (19)$$

After substituting $k_c = (2\nu t)^{-1/2m}$, the global roughness exponent $\chi = (2m - 1)/2$, and the dynamic exponent $z = 2m$ into Eq. (19), the early time asymptotics of $G(r, t)$ can also be expressed as

$$G(r, t)|_{t \ll r^z/\nu} \sim O\left(\frac{D}{\nu^{1/z}} t^{2\chi/z}\right). \quad (20)$$

It is just as expected that the leading order of the early time asymptotics of $G(r, t)$ is independent of r , since the spatial correlation between interface heights has not yet developed at this stage.

Subsequently, for the intermediate time regime $r^z/\nu \ll t \ll L^z/\nu$, we now have $1/L \ll k_c \ll 1/r$. Naively, people would be tempted to directly expand $\cos(k_c r x)$ in Eq. (18) by its power series expansion $\sum_{p=0}^\infty [(-1)^p/(2p)!] (k_c r x)^{2p}$ and perform the integration term by term. However, a closer look at Eq. (18) tells us that this way of calculation won't work, since it will involve some diverging terms like $\int_0^\infty x^p (1 - e^{-x^{2m}}) dx$ with $p \geq -1$. Thus, we need to find some other way out. We first systematically perform integration by parts and thus rewrite Eq. (18) as follows:

$$\begin{aligned} G(r, t)|_{t \ll L^z/\nu} \simeq & \frac{D}{\pi\nu} k_c^{1-2m} \left\{ \frac{2m}{2m-1} \int_0^\infty dx [1 - \cos(k_c r x)] e^{-x^{2m}} - \frac{2m}{(2m-1)!} \right. \\ & \times \sum_{p=1}^{m-1} \left[(-1)^p (k_c r)^{2p-1} \left((2m-2p-1)! \int_0^\infty dx x^{2p-1} \right. \right. \\ & \times \sin(k_c r x) e^{-x^{2m}} + k_c r (2m-2p-2)! \int_0^\infty dx x^{2p} \cos(k_c r x) e^{-x^{2m}} \Big) \Big] \\ & \left. - (-1)^m \frac{(k_c r)^{2m-1}}{(2m-1)!} \left[\frac{\pi}{2} - \int_0^\infty dx x^{-1} \sin(k_c r x) e^{-x^{2m}} \right] \right\}. \quad (21) \end{aligned}$$

Since $\int_0^\infty dx x^p e^{-x^{2m}}$ is finite for all $p \geq 0$, it is now legitimate to substitute $\cos(k_c r x)$ and $\sin(k_c r x)$ in Eq. (21) by their power series expansions, $\sum_{n=0}^\infty \frac{(-1)^n}{(2n)!} (k_c r x)^{2n}$ and $\sum_{n=0}^\infty \frac{(-1)^n}{(2n+1)!} (k_c r x)^{2n+1}$, respectively, and perform the integration term by term. Thus, we can rewrite $\{\dots\}$ at the r.h.s. of Eq. (21) as a polynomial of $(k_c r)^2$ in addition to a term of $(k_c r)^{2m-1}$. Consequently, after some tedious calculation, the intermediate time asymptotics of $G(r, t)$ is obtained as follows:

$$G(r, t)|_{r^z/\nu \ll t \ll L^z/\nu} \simeq \frac{D}{\pi\nu} k_c^{1-2m} \left[\sum_{p=1}^\infty C_{2p} (k_c r)^{2p} + C_{2m-1} (k_c r)^{2m-1} \right] \quad (22)$$

with the coefficients

$$C_{2p} = \begin{cases} \frac{(-1)^{p+1}}{(2m-1)!} \Gamma\left(\frac{p+\frac{1}{2}}{m}\right) \sum_{q=1}^{2m-1} \frac{(2m-q-1)!}{(2p-q+1)!} & \text{for } p < m \\ \frac{(-1)^{p+1}}{(2m-1)!} \Gamma\left(\frac{p+\frac{1}{2}}{m}\right) \sum_{q=1}^{2m-1} \frac{(2m-q-1)!}{(2p-q+1)!} + \frac{(-1)^p}{(2m)!(2p-2m-1)!} & \text{for } p \geq m \end{cases} \quad (23)$$

and

$$C_{2m-1} = \frac{\pi}{2} \cdot \frac{(-1)^{m+1}}{(2m-1)!}. \quad (24)$$

After substituting $k_c = (2\nu t)^{-1/2m}$, the global roughness exponent $\chi = (2m-1)/2$, and the dynamic exponent $z = 2m$ into Eq. (22), the intermediate time asymptotics of $G(r, t)$ can also be expressed as

$$G(r, t)|_{r^z/\nu \ll t \ll L^z/\nu} \simeq \sum_{p=1}^\infty C'_{2p} t^{2(\chi-p)/z} r^{2p} + C'_{2\chi} r^{2\chi} \quad (25)$$

with the coefficients $C'_{2p} = C_{2p} \cdot [D(2\nu)^{2(\chi-p)/z}/(\pi\nu)]$ and $C'_{2\chi} = C_{2m-1} \cdot [D/(\pi\nu)]$. Note that, in the r.h.s. of Eq. (25), the terms $C'_{2p} t^{2(\chi-p)/z} r^{2p}$ with $p < \chi (= (2m-1)/2)$ are all dominant over the term $C'_{2\chi} r^{2\chi}$ in the limit of $t \gg r^z/\nu$. Recall that, for the ordinary dynamic scaling behaviors, the intermediate and late time asymptotics of $G(r, t)$ scale as $r^{2\chi}$; i.e., $G(r, t \gg r^z/\nu) \sim O(Dr^{2\chi}/\nu)$. Thus, the growth processes described by Eq. (7) all display the anomalous dynamic scaling behaviors with the leading anomalous term ($C'_2 t^{2(\chi-1)/z} r^2$) and the sub-leading anomalous terms ($C'_4 t^{2(\chi-2)/z} r^4, \dots, C'_{2m-2} t^{2(\chi-m+1)/z} r^{2m-2}$) dominant over the ordinary dynamic scaling term $C'_{2\chi} r^{2\chi}$.

Subsequently, for the late time regime $t \gg L^z/\nu$, we now have $k_c \ll 1/L$. Thus, Eq. (18) is no more applicable and we need to go back to Eq. (16). The property of $k_c \ll 1/L$ in this regime implies that $2\nu k_n^{2m} t \gg 1$ for all positive integer n in Eq. (16). (Recall that $k_n \equiv n(2\pi/L)$.) Thus,

$$\begin{aligned}
 G(r, t)|_{t \gg L^z/\nu} &\simeq \frac{2D}{\nu L} \sum_{n=1}^{\infty} \left[\frac{1 - \cos(k_n r)}{k_n^{2m}} \right] \\
 &= \frac{2D}{\nu L} \left(\frac{L}{2\pi} \right)^{2m} \sum_{n=1}^{\infty} \left[\frac{1 - \cos\left(\frac{2\pi n}{L} r\right)}{n^{2m}} \right] \\
 &= \frac{DL^{2m-1}(-1)^{m-1}}{\nu(2m)!} \left[B_{2m}(0) - B_{2m}\left(\frac{r}{L}\right) \right] \quad (26)
 \end{aligned}$$

with $B_{2m}(x)$ denoting the Bernoulli polynomial.¹⁸ By employing the property $B_n(x) = \sum_{q=0}^n \frac{n!}{q!(n-q)!} B_{n-q} x^q$ with B_{n-q} denoting the Bernoulli number,¹⁸ we can rewrite Eq. (26) as

$$G(r, t)|_{t \gg L^z/\nu} \simeq \frac{DL^{2m-1}}{\nu} \sum_{q=1}^{2m} \left[\frac{(-1)^m B_{2m-q}}{q!(2m-q)!} \left(\frac{r}{L}\right)^q \right]. \quad (27)$$

Note that the Bernoulli number consists of a very special property:

$$\begin{cases} B_n = 0, & \text{for } n = 3, 5, 7, \dots; \\ B_n \neq 0, & \text{otherwise.} \end{cases}$$

Consequently, by employing the above property and substituting the exponent $\chi = (2m - 1)/2$ into Eq. (27), the asymptotics of $G(r, t)$ in the late time regime is obtained as follows:

$$G(r, t)|_{t \gg L^z/\nu} \simeq \sum_{p=1}^m B'_{2p} L^{2(\chi-p)} r^{2p} + B'_{2\chi} r^{2\chi} \quad (28)$$

with the coefficients $B'_{2p} = (D/\nu)[(-1)^m B_{2m-2p}]/[(2p)!(2m-2p)!]$ and $B'_{2\chi} = (D/\nu)[(-1)^m B_1]/[(2m-1)!]$. It is just as expected that, at the late time regime, the range of the spatial correlation between the interface heights has reached the system size L . At this stage, $G(r, t)$ has reached its saturated value and, thus, no more depends on time. Furthermore, in the r.h.s. of Eq. (28), the terms $B'_{2p} L^{2(\chi-p)} r^{2p}$ with $p < \chi (= (2m - 1)/2)$ are all dominant over the ordinary scaling term $B'_{2\chi} r^{2\chi}$, since $L \gg r$. To compare them with Eq. (25), it is just like the time t in the intermediate time asymptotics of $G(r, t)$ being substituted by $O(L^z/\nu)$.

With our rigorous calculation, we have analytically affirmed the validity of anomalous dynamic scaling ansatz⁶ to describe the correlation function of the super-rough growth processes governed by Eq. (7) with finite lateral system size of L and explicitly obtained not only the leading order but also all the sub-leading orders which dominate over the ordinary dynamic scaling term. Furthermore, the extensive studies about the asymptotics of $G(r, t)$ enable us to obtain the asymptotics of the local interfacial width $w(l, t)$ easily. First, from Eqs. (3) and (4), the relation between the local interfacial width $w(l, t)$ and the equal-time height difference

correlation function $G(r, t)$ is readily obtained as follows:

$$w^2(l, t) = \frac{1}{l^2} \int_0^l (l-r) G(r, t) dr. \quad (29)$$

Then, by substituting Eqs. (20), (25), and (28) into Eq. (29), we obtain the asymptotics of the local interfacial width $w(l, t)$ as follows.

(i) For $t \ll l^z/\nu$,

$$w^2(l, t) \sim O\left(\frac{D}{\nu^{1/z}} t^{2\chi/z}\right). \quad (30)$$

(ii) For $l^z/\nu \ll t \ll L^z/\nu$,

$$w^2(l, t) \simeq \sum_{p=1}^{\infty} C''_{2p} t^{2(\chi-p)/z} l^{2p} + C''_{2\chi} l^{2\chi} \quad (31)$$

with the coefficients

$$C''_{2p} = C'_{2p}/[(2p+1)(2p+2)] \quad \text{and} \quad C''_{2\chi} = C'_{2\chi}/[(2\chi+1)(2\chi+2)].$$

(iii) For $t \gg L^z/\nu$,

$$w^2(l, t) \simeq \sum_{p=1}^m B''_{2p} L^{2(\chi-p)} l^{2p} + B''_{2\chi} l^{2\chi} \quad (32)$$

with the coefficients

$$B''_{2p} = B'_{2p}/[(2p+1)(2p+2)] \quad \text{and} \quad B''_{2\chi} = B'_{2\chi}/[(2\chi+1)(2\chi+2)].$$

It is just as expected that the local interfacial widths $w(l, t)$ have the same scaling behaviors as the equal-time height difference correlation functions $G(r, t)$ with the variable r substituted by the variable l . Namely, for the intermediate time regime $l^z/\nu \ll t \ll L^z/\nu$, the leading anomalous term ($C''_2 t^{2(\chi-1)/z} l^2$) and the sub-leading anomalous terms ($C''_4 t^{2(\chi-2)/z} l^4, \dots, C''_{2m-2} t^{2(\chi-m+1)/z} l^{2m-2}$) dominate over the ordinary dynamic scaling term $C''_{2\chi} l^{2\chi}$ and, for the late time regime $t \gg L^z/\nu$, the leading anomalous term ($B''_2 L^{2(\chi-1)} l^2$) and the sub-leading anomalous terms ($B''_4 L^{2(\chi-2)} l^4, \dots, B''_{2m-2} L^{2(\chi-m+1)} l^{2m-2}$) dominate over the ordinary dynamic scaling term $B''_{2\chi} l^{2\chi}$.

Note that, after comparing the obtained leading anomalous terms in the asymptotics of $G(r, t)$ and $w(l, t)$ with the anomalous dynamic scaling ansatz in Eqs. (5) and (6), we see that the local roughness exponent $\chi' = 1$, independent of m , for all the growth processes described by Eq. (7), while the exponent $\kappa = \chi - 1 = m - 3/2$. From a geometrical argument,¹⁹ it has been shown that the local roughness exponent χ' must be less than or equal to 1 for any interfacial roughening processes. Since $G(r, t)$ measures the interfacial height difference with the lateral separation r , the growth process with $\chi' = 1$ implies that the interface, starting from the flat initial condition, gradually develops a macroscopic background with the lateral size comparable to L . Besides, the restriction of the periodic boundary condition

requires that the macroscopic background must be in the form of global mountains or valleys. For the growth process described by Eq. (7) with larger value of m , there is less restriction on the local interfacial slope variation and, consequently, the macroscopic background becomes rougher. Thus, as the value of m increases, the value of the exponent κ becomes larger and more anomalous terms are dominant over the ordinary dynamic scaling term in the asymptotics of $G(r, t)$ and $w(l, t)$. From Eqs. (31) and (32), we see that these leading anomalous terms in the intermediate and late time asymptotics of $w^2(l, t)$ are simply the polynomials of l with even power from l^2 up to l^{2m-2} . Thus, it implies that these leading anomalous terms in the asymptotics of $w^2(t)$ can be suppressed by the subtraction of a macroscopic background $\tilde{h}_l(x, t) (= \sum_{p=0}^{m-1} S_p(l, t)x^p$ with the appropriate choices of the coefficients $S_p(l, t)$ obtained from the least squares fit to the interface configuration) from the original interface height function $h(x, t)$ within a local window of size l and, consequently, the resulting local interfacial width will retrieve the ordinary dynamic scaling behavior. Namely, the *residual* local interfacial width

$$\tilde{w}^2(l, t) \equiv \overline{\langle (h(x, t) - \tilde{h}_l(x, t))^2 \rangle_l}, \quad (33)$$

which measures the interface height fluctuation relative to the macroscopic background within a local window of size $l (\ll L)$, does retain the ordinary dynamic scaling behaviors. This concept of the residual local interfacial width can be generalized and applied to other super-rough growth processes if they are governed by the growth equations containing the dominant terms like $\nabla^{2m}h$. However, we must note that, for some peculiar super-rough growth processes such as the Das Sarma–Tamborenea model,²⁰ the interface contains very narrow and sharp local grooves or spikes^{7,8,15} and, thus, the concept of the residual local interfacial width is not applicable in those circumstances. For the future work, we plan to take a rigorous and extensive analysis on this issue.

In conclusion, we take an extensive analytical study of certain super-rough growth processes described by a class of linear growth equations, Eq. (7), in 1+1 dimensions with finite lateral system size of L . Through our detailed analysis of the equal-time height difference correlation functions $G(r, t)$ and the local interfacial widths $w(l, t)$ in the intermediate and late time regimes, we give a first rigorous analytical affirmation on the applicability of the anomalous dynamic scaling ansatz for super-rough growth processes with finite lateral system size. Moreover, we explicitly obtain not only the leading anomalous term but also all the sub-leading anomalous terms which are dominant over the ordinary dynamic scaling term. The intermediate and late time asymptotics of $G(r, t)$ and $w(l, t)$ also tell us that the cause of the anomalous dynamic scaling behaviors in the growth processes governed by Eq. (7) is totally attributed to the formation of global mountains and valleys in the interface morphology and, thus, the residual local interfacial width $\tilde{w}(l, t)$ still retains the ordinary dynamic scaling behaviors. This concept of the residual local interfacial width can be generalized to other super-rough growth processes governed by the growth equations whose dominant terms are like $\nabla^{2m}h$ with $m \geq 2$, but it

can not be applied to the super-rough growth processes of which the interfaces form very narrow and sharp local grooves or spikes.

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References

1. J. Krug, *Adv. Phys.* **46**, 139 (1997).
2. T. Halpin-Healy and Y.-C. Zhang, *Phys. Rep.* **254**, 215 (1995).
3. P. Meakin, *Fractals, Scaling & Growth Far From Equilibrium* (Cambridge University Press, Cambridge, 1998).
4. F. Family and T. Vicsek, *J. Phys.* **A18**, L75 (1985).
5. M. Schroeder, M. Siegert, D. E. Wolf, J. D. Shore and M. Plischke, *Europhys. Lett.* **24**, 563 (1993).
6. S. Das Sarma, S. V. Ghaisas and J. M. Kim, *Phys. Rev.* **E49**, 122 (1994).
7. J. Krug, *Phys. Rev. Lett.* **72**, 2907 (1994).
8. S. Das Sarma, C. J. Lanczycki, R. Kotlyar and S. V. Ghaisas, *Phys. Rev.* **E53**, 359 (1996).
9. J. M. López and M. A. Rodríguez, *Phys. Rev.* **E54**, R2189 (1996).
10. N.-N. Pang and W.-J. Tzeng, *Phys. Rev.* **E59**, 234 (1999).
11. H.-N. Yang, Y.-P. Zhao, G.-C. Wang and T.-M. Lu, *Phys. Rev. Lett.* **76**, 3774 (1996).
12. J. H. Jeffries, J. K. Zuo and M. M. Craig, *Phys. Rev. Lett.* **76**, 4931 (1996).
13. S. Morel, J. Schmittbuhl, J. M. López, and G. Valentin, *Phys. Rev.* **E58**, 6999 (1998).
14. J. M. López, *Phys. Rev. Lett.* **83**, 4594 (1999).
15. N.-N. Pang and W.-J. Tzeng, *Phys. Rev.* **E61**, 3212 (2000).
16. S. F. Edwards and D. R. Wilkinson, *Proc. R. Soc. (London)* **A381**, 17 (1982).
17. D. E. Wolf and J. Villain, *Europhys. Lett.* **13**, 389 (1990).
18. See, for example, *An Atlas of Functions*, eds. J. Spanier and K. B. Oldham (Springer-Verlag, Berlin, 1987); *Integrals and Series*, eds. A. P. Prudnikov, Y. A. Brychkov and O. I. Marichev (Gordon and Breach, New York, 1986).
19. H. Leschhorn and L.-H. Tang, *Phys. Rev. Lett.* **70**, 2973 (1993).
20. S. Das Sarma and P. Tamborenea, *Phys. Rev. Lett.* **66**, 325 (1991).