Can the conductance step of a single-mode ballistic constriction be lower than $2e^2/h$?

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Abstract

We have performed four-terminal measurements on a split-gate device with three overlaying finger gates. In our system, a one-dimensional (1D) ballistic constriction can be formed by applying a negative voltage on the split-gate. By changing the finger gate voltage, we are able to modulate the potential profile within the 1D constriction. In such a 1D ballistic structure we have observed that the conductance steps show a gradual decrease from $2e^2/h$ to $0.97 \times 2e^2/h$ with increasing negative finger gate voltage. We suggest this phenomenon is due to different shifts of 1D subbands with changing split-gate voltage. Both simple analytical estimate for an adiabatic constriction and realistic modeling of the device give the same magnitude of the conductance decrease as observed in our experiments.

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It is known that the conductance of a one-dimensional (1D) ballistic wire is quantised in units of $2e^2/h$. This was discovered experimentally [1] in split-gate induced constrictions in a high-mobility two-dimensional electron gas (2DEG). Both experimental [2] and theoretical results [3] show that the accuracy of the quantisation is sensitive to the detailed shape of the confining potential and the presence of impurities. Observation of a high number of exact quantised plateaux [4] without resonant features indicates that adiabatic transport in the constriction

[5,6] is attainable. However, regular deviations of plateaux from quantised values $n \times 2e^2/h$ that have been observed in long 1D wires [7–10] were associated with either non-adiabaticity of the constriction or electron–electron interactions in the 1D system. In the seminal work of Tarucha et al. [7] a few percent deviations were observed in long (5–10 μm) weakly disordered quantum wires [7]. The deviation increases with increasing wire length and is explained in terms of electron–electron interactions in a “dirty Luttinger liquid” [11]. The reduction of conductance plateaux up to 25% observed in 5–6 μm quantum wires was ascribed to backscattering of electrons due to impurities [10]. Other structures with complicated geometry (T-shaped cleaved-edge-overgrowth [8]

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and V-groove quantum wires [9]) show significant decrease of conductance steps which was attributed to backscattering of electrons from the abrupt interface between the 2D contacts and the 1D wire.

In this paper, we show that a deviation from the quantised value $2e^2/h$ is possible even in a clean short constriction in which residual scattering and effects of non-adiabaticity are negligible. The observed deviation could not be explained by conductance renormalisation due to electron–electron interactions in the 1D wire either, because it is well established that two-terminal conductance of a clean ballistic 1D system is determined by non-interacting source and drain contacts [12].

We have studied constrictions electrostatically shaped in the 2DEG by a pair of split gates and three finger gates lying across the channel. This versatile device permits control of the electrostatic potential across and along the channel without changing series resistance R_s . We measured conductance G as a function of split gate voltage V_g at different centre finger gate voltages V_F with outer finger gates grounded to the 2DEG. With increasing negative voltage on the centre finger gate we observed in a series of $G(V_g)$ curves the transition from pronounced steps to smeared conductance steps. At the same time the height of the first conductance step gradually decreased from $2e^2/h$ to $0.97 \times 2e^2/h$. The effect was reproduced in two- and four-terminal conductance measurements, whereas the series resistance $R_s = R(V_g = 0)$ is varied by an order of magnitude. We ascribe this deviation from the conductance quantum $2e^2/h$ to the interplay between reflection in the first transmission mode and tunnelling via the second mode in the adiabatic constriction. Calculation of electrostatics shows that the tops of the first and second subbands move with different velocities in response to the change of the split-gate voltage. This leads to a slower decrease of the reflection as compared to simultaneous increase in the tunnelling, and therefore onset of the next conductance step occurs at a lower value than $G = 2e^2/h$. The effect disappears when the smeared conductance steps turn into pronounced ones.

The multi-layered gated 1D structure was lithographically defined 158 nm above the 2DEG as shown in Fig. 1a. There is a 30-nm-thick layer of polymethylmethacrylate (PMMA) which has been highly

Fig. 1. (a) Schematic view of device. (b) Four-terminal measurement setup. (c) Measured dependences of resistance R on the split-gate voltage V_g , with all finger gates grounded to the 2DEG (the solid line). Horizontal dashed lines show values $R_n = h/2ne^2$ for $n = 1, 2, 3$. For comparison two-terminal measurement of $R(V_g)$ is shown by the dotted line.

irradiated by an electron beam, to act as a dielectric between the split-gate and three overlaying finger gates. The 2DEG has a carrier density of $2.5 \times 10^{15} \text{ cm}^{-2}$ with a mobility of $3.0 \times 10^6 \text{ cm}^2/\text{V s}$ after brief illumination with a red light emitting diode. Experiments were performed in a pumped ^3He cryostat at 300 mK and the four-terminal resistance was measured using an AC driving current of 10 nA at a frequency of 77 Hz with standard phase-sensitive techniques.

In our set-up, the outer finger gates were grounded to the 2DEG, while centre finger gate voltage V_F and split gate voltage V_g were varied over a wide range. In this sample, potentiometric contacts were deliberately brought close to the constriction (four-terminal measurements) to reduce the contribution of series resistance of the 2DEG (Fig. 1b). The measured resistance R_s between the contacts in the absence of the constriction was only 115Ω , that is 0.9% of the quantum resistance $h/2e^2$. Thus, we were able to measure the conductance of the single-mode constriction precisely enough without resorting to subtraction of the series resistance (Fig. 1c). Note that the series resistance was 725Ω in the two-terminal measurements.

Fig. 2 shows the measured dependences of conductance G on the split gate voltage V_g and derivative $dV_g(G)/dG$. We consider only the first conductance step and determine its height G_* by the value of G

Fig. 2. (a) Measured four-terminal conductance $G(V_g)$ at different centre finger gate voltage V_F . From left to right: $V_F = 0$ – -4.8 V in 0.2 V steps. (b) Derivative $dV_g(G)/dG$ at different center finger gate voltage V_F . From bottom to top: $V_F = 0$ – -4.8 V in 0.2 V steps.

at a point of inflection, where the derivative dV_g/dG reaches a maximum. As the voltage on the centre finger gate V_F is made more negative the step becomes narrower and eventually transforms into a point of inflection. The reason of smearing of conductance steps lies in decrease of the ratio ω_y/ω_x of transverse and longitudinal frequencies that define the shape of saddle potential in the constriction [6]. Fig. 2b shows that $G_* = 2e^2/h$ at $V_F = 0$ and $G_* = 0.97 \times 2e^2/h$ at a large negative voltage $V_F = -1.6$ V. In the voltage range $-1.6 \text{ V} < V_F < 0$ a continuous transition between these two values is observed. Exact conductance quantisation at small negative V_F confirms the correctness of four-terminal measurements. Non-locality of resistance in this set-up can complicate measurements of the conductance of the constriction. But making the resistance of the potentiometric contacts smaller than that of the single-mode channel one can neglect the non-locality.

It is worth mentioning that the gradual decrease of the height of the first step had also been detected in two-terminal conductance measurements, where R_s subtracted. The decrease has been shown to be reproducible for different cool-downs, which indicated that it cannot be explained by residual disorder scattering. The lack of resonant features and the pronounced conductance steps demonstrate that we have a clean 1D system in which impurity scattering is

negligible. Therefore, we can exclude the possibility of so-called “dirty Luttinger liquid” behaviour in our case. Moreover, in contrast to previous observations [7–10] where the deviation from exact quantisation increased with the length of the quantum wire, in our case the deviation increases when the 1D region shortens. Therefore, we deal with the deviation of another physical origin.

Let us now see how the observed decrease of the conductance steps can be explained in the model of quasi-1D ballistic transport for an adiabatic constriction. The transverse quantisation of electrons allows us to view the scattering problem in terms of the local transverse modes. The conductance is then given by the Landauer formula: $G = 2e^2/h \sum_{kj} |t_{kj}|^2$, where $|t_{kj}|^2$ is the probability that flux input in subband j is traveling forward into subband k . The solution of the transverse Schrödinger equation at each value x along the constriction gives energies of 1D subbands $E_n(x)$. These subbands $E_n(x)$ have a shape of smooth potential barriers offset from each other by transverse quantum $y_n(x)$. If y_n changes slowly along x then intersubband scattering is suppressed and total transmission coefficient is equal to the sum of contributions of all subbands [5]: $T = \sum_n T_n$, $T_n = |t_{nn}|^2$. At $G \approx 2e^2/h$ the conductance can be expressed in the form

$$G = \frac{2e^2}{h} (1 - R_1 + T_2), \quad (1)$$

where R_1 is the reflection coefficient of the first subband and T_2 —transmission coefficient through the second subband. When the plateaux of $G(V_g)$ are smeared out, their height is determined by interplay of two basic phenomena of quantum mechanics—above-barrier reflection and tunnelling. If $R_1(V_g)$ reduces slower than $T_2(V_g)$ increases, then at the point of inflection of curve $G(V_g)$ the height of the steps G_* can be less than the conductance quantum $2e^2/h$.

The effect is most easily seen in an approximation of a saddle point potential $U(x, y) = V_0 + \frac{1}{2} m(\omega_y^2 y^2 - \omega_x^2 (x - x_{\max})^2)$, for which the transmission coefficient via the n th subband $E_n(x) = V_0 + y(n - \frac{1}{2}) - \frac{1}{2} m\omega_x^2 (x - x_{\max})^2$ has a simple analytical expression [6]: $T_n = (1 + e^{-\pi \varepsilon_n})^{-1}$, where $\varepsilon_n = 2(E - y(n - \frac{1}{2}) - V_0)/\omega_x$, $n = 1, 2, \dots$. With increasing negative voltage V_g on the split gate the saddle becomes narrower. For small changes in V_g one can assume that only ω_y and V_0 are changed whilst ω_x remains constant. Then the second

subband moves more rapidly than the first one

$$E'_1 = \frac{dE_1}{dV_g} = \frac{dV_0}{dV_g} + \frac{1}{2} \frac{\hbar d\omega_y}{dV_g},$$

$$E'_2 = \frac{dE_2}{dV_g} = \frac{dV_0}{dV_g} + \frac{3}{2} \frac{\hbar d\omega_y}{dV_g}. \quad (2)$$

Hence, at the minimum of dT/dV_g it holds that $R_1 > T_2$. Indeed, at $T \approx 1$ the coefficients can be approximated as

$$R_1 = 1 - T_1 \approx \exp\left(-2\pi \frac{E - E_1(x_{\max})}{x}\right),$$

$$T_2 \approx \exp\left(-2\pi \frac{E_2(x_{\max}) - E}{x}\right). \quad (3)$$

At the point of inflection, $d^2T/dV_g^2 = 0$, we have $T_2 \approx R_1(E'_1/E'_2)^2$, $E = V_0 + y(1 + \ln(E'_1/E'_2)/2\pi)$, and

$$T_* \approx 1 - e^{-\pi\omega_y/\omega_x} \left(\frac{E'_2}{E'_1} - \frac{E'_1}{E'_2} \right). \quad (4)$$

For the steps on the plot $T(V_g)$ be observable it needs that $\omega_y/\omega_x > 1.25$ [6]. Thus the exponent in Eq. (4) does not exceed 2%; on the other hand from computation of electrostatics (see below) $dV_0/dV_g \sim \frac{1}{2} \hbar d\omega_y/dV_g$ and $E'_2/E'_1 \approx 2$. Therefore, the step of conductance can decrease by 3%. When the negative voltage on the centre finger gate approaches zero, ω_x decreases, and the effect vanishes exponentially. Thus, the simple equation (4) predicts the behaviour that qualitatively agrees with experiment (Fig. 2).

In conclusion, we have shown that a deviation from exact conductance quantisation in a short adiabatic constriction is possible. With increasing negative finger gate voltage we observed gradual decrease of the height of the first conductance step from $2e^2/h$ to $0.97 \times 2e^2/h$. We explain the effect by interplay

between reflection in the first mode and tunnelling via the second mode. We have found that the top of the first subband moves two times slower than the second one in response to the change of the split-gate voltage. This leads to a slower decrease of the reflection as compared to simultaneous increase in the tunnelling, and therefore when the conductance step smears out its height is also reduced. Realistic numerical modelling of the electrostatics of the structure and transmission through the constriction gives the results similar to our experimental data.

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