Large- Q^2 behavior of the Higgs mechanism as derived from one-loop diagrams and the renormalization-group equation

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The renormalization-group (RG) approach generalized by Lee and Weisberger to the case of gauge theories with spontaneous symmetry breaking is adopted in order to study the large- Q^2 behavior of the Higgs mechanism. The approach amounts to a generalization of the well-known concept of "running coupling constants" to all the couplings involved in the theory. To simplify our consideration, we have considered scalar electrodynamics (QED) in which a $U(1)$ gauge field interacts with both fermions and scalar fields (the latter responsible for spontaneous symmetry breaking). We first carry out a perturbative calculation including all one-loop diagrams with the explicit results on the various renormalization constants obtained by separating out pole parts ($\propto 1/\epsilon$) of the various diagrams regulated in the dimensional-regularization scheme. Using the explicit results on the renormalization constants, we extract the β and γ functions which govern the behavior of the theory through the RG equation as $Q^2 \rightarrow \infty$. We then solve explicitly the coupled RG equations, obtaining the result that the vacuum expectation value $v^2(Q^2)$ vanishes as $Q^2 \rightarrow \infty$ (i.e., the broken symmetry is restored) but the gauge sector (consisting of the gauge field and fermions) does not decouple from the strongly self-interacting Higgs sector. In the case of non-Abelian gauge theories, we suggest looking for theories which allow for an eventual decoupling between the strongly interacting Higgs sector and the asymptotically free gauge sector.

I. INTRODUCTION

In the 1960s, Weinberg and Salam proposed [1] a unified gauge theory of electromagnetic and weak interactions making use of the Higgs mechanism [2]. Weinberg speculated [1] that spontaneous symmetry breaking (SSB) does not destroy the renormalizability of the theory. It was later proven by 't Hooft [3] that renormalizability of the theory is indeed not affected by the presence of spontaneous symmetry breaking. In his proof of renormalizability, 't Hooft adopted the "dimensional-regularization" method [4], which preserves explicitly gauge symmetry and Lorentz invariance. Lee and Justin [5] also had discussed extensively the renormalization and renormalizability of a spontaneously broken gauge theory.

In early 1970s, Weinberg considered [6] the one-loop contributions to the fermion and pseudo Goldstone-boson masses in the context of a non-Abelian gauge theory. He showed that, when the masses are subject to some zeroth-order symmetry relations for all values of the parameters in the Lagrangian, the divergences arising from one-loop corrections to these symmetry relations cancel exactly. In a classic paper, Appelquist *et al.* [7] investigated gauge independence and the renormalization prescription, up to one-loop order, of scalar quantum electrodynamics (QED), which describes a charged-scalar particle interacting with the electromagnetic field.

The renormalization-group (RG) equation may be used to investigate the Q^2 dependence of the Higgs mechanism. For example, Lee and Weisberger [8] discussed the asymptotic behavior of symmetry breaking. They ob-

tained that, for a non-Abelian gauge field theory with the Higgs mechanism, spontaneously broken symmetry will be restored at some large Q^2 if the theory is asymptotically free.

On the other hand there are formal treatments [9–11] concerning the interplay between the asymptotically free property of a gauge theory and the role played by the Higgs mechanism. For example, Coleman and Gross [9] have shown that no renormalizable field theory can be asymptotically free without non-Abelian gauge fields. Gross and Wilczek [10] examined the problem of incorporating scalar fields in asymptotically free theories and pointed out the difficulty associated with the quartic coupling of the scalar field which is inherently unstable. Cheng, Eichten, and Li [11] went on to show in a rather systematic manner that, for a large class of non-Abelian gauge theories, introduction of scalar fields will be unstable in the Higgs sector if the gauge symmetry is broken completely.

Our motivation for attempting to understand the large- Q^2 behavior of the Higgs mechanism does not have much to do with whether renormalization works for a gauge theory with Higgs mechanism, a question central to the studies [3–8] during early 1970s. With experimental results pouring in from SLAC Linear Collider (SLC) and CERN e^+e^- collider LEP beginning from the year of 1989 and all demonstrating that the Glashow-Salam-Weinberg (GSW) electroweak theory works beautifully at energies near the Z^0 mass, it has become imperative to know with high precision how the theory works as we prepare to dive into much higher energies, such as in the TeV range. Use of the Higgs mechanism is central to the

GSW theory and, yet, the Higgs sector of the theory may have become strongly interacting in the TeV range. We believe it is timely to pick up again the efforts [3–11], as well as the accomplishments, of the early 1970s and to continue looking into how a theory such as the GSW electroweak theory behaves at large Q^2 . The primary objective of our work is to use the coupled RG equations to determine how the various parameters (couplings), including both the dimensionless and dimensional ones, behave as $Q^2 \rightarrow \infty$. The focus in the past [10,11] has been primarily with the dimensionless couplings, except in the work of Lee and Weisberger [8] where the vacuum expectation value $v(Q^2)$ was discussed in the context of the RG equation. Although we choose to work out the problem only for the simplest prototype of spontaneously broken gauge field theories, i.e., scalar QED, our result may suggest why a similar problem should also be seriously addressed for the GSW electroweak theory and its likes, especially when one anticipates pathological behavior of some sort. On an optimistic note, it is perfectly conceivable that, even though the GSW electroweak theory may not be well behaved as $Q^2 \rightarrow \infty$, a suitable extension may result in clear separation, or decoupling, of the bad component (which grows with Q^2) from the good part (in which all couplings approach zero as $Q^2 \rightarrow \infty$). In other words, instead of looking for a theory that behaves perfectly well as $Q^2 \rightarrow \infty$, we suggest to look into the structure of the theory in order to construct a theory in which, as $Q^2 \rightarrow \infty$, the strongly interacting part of the theory eventually decouples from its well-behaved part. We do not think that this is a trivial question, as it requires an explicit solution to the coupled RG equations. It is this specific aspect that has motivated our current investigations.

On a different but related frontier, we may also mention that there are much discussed subjects such as the possible dominance [12] of the instanton-induced interactions in this new frontier or utilization of a possible non-trivial QED phase [13] in understanding the observed positron or electron peaks [14] in very-low-energy heavy-ion collisions. It is again of great interest to investigate the large- Q^2 behavior of the Higgs mechanism, making use of the knowledge acquired from previous studies [3–11]. As in the case of Appelquist *et al.* [7], we find it is useful to consider scalar QED with spontaneous symmetry breaking. The lessons which we learn can then be generalized to non-Abelian gauge theories such as the GSW $SU(2) \times U(1)$ electroweak theory. Unlike what has been done nicely in the paper of Cheng, Eichten, and Li [11], who examine only the RG property of the dimensionless couplings, we wish to know how each coupling, not just those dimensionless ones, behaves as $Q^2 \rightarrow \infty$. This makes it more practical to first look into a simplified model such as scalar QED with spontaneous symmetry breaking.

Specifically, we first extend the one-loop calculation of Appelquist *et al.* [7] by taking into account all possible two-, three-, and four-point vertex functions. The procedure is a fairly standard one, which consists of obtaining the explicit results on the various renormalization constants by separating out pole parts ($\propto 1/\epsilon$) of the

various diagrams regulated in the dimensional-regularization scheme [4]. For pedagogical purposes, this step is described in some detail in Secs. II and III, which the expert in the field may choose to skip except by noting that the explicit results on the various renormalization constants, as defined by Eqs. (18)–(20), are all we need for Sec. IV. Such explicit results, which unfortunately were not touched upon at all in the paper of Appelquist *et al.* [7], in fact make it very easy to understand how the renormalization program described in the paper of Appelquist *et al.* [7] works in detail. In particular, one sees how introduction of a small number of renormalization constants indeed renormalizes the theory in a consistent manner. As in the paper of Appelquist *et al.* [7], our paper has also been prepared in a pedagogic style so that it may supplement the formal treatments in the literature [3,5,6,8–11].

Among the results which we have obtained, we wish to first mention [15], by comparing term-by-term one-loop results in scalar QED with spontaneous symmetry breaking (SSB) [$\mu^2 < 0$ in Eq. (3c)] with those in scalar QED without SSB ($\mu^2 > 0$), that the constants which renormalize the theory are independent of whether or not the symmetry is spontaneously broken.

The next step consists of using the RG equations to investigate the large- Q^2 behavior of the theory. Using the explicit results on the renormalization constants, we extract the β and γ functions which govern the behavior of the theory through the RG equation as $Q^2 \rightarrow \infty$. We find that the coupled RG equations can be solved explicitly, yielding the well-known result that the coupling constant λ in the $\lambda\phi^4$ term may diverge at some Q^2 well before the Landau ghost problem becomes relevant. Nevertheless, the vacuum expectation value $v^2(Q^2)$ vanishes as $Q^2 \rightarrow \infty$ (i.e., the broken symmetry is restored) but the gauge sector (consisting of the gauge field and fermions) does not decouple from the strongly self-interacting Higgs sector.

We proceed to generalize our considerations to non-Abelian gauge theories. Our focus is to discuss the validity of an assertion made by Lee and Weisberger [8] that, for a non-Abelian gauge field theory with Higgs mechanism which is asymptotically free, the spontaneously broken symmetry is restored at large Q^2 . Upon looking into their proof closely, we believe that, although the gauge coupling and the vacuum expectation value may both approach zero as $Q^2 \rightarrow \infty$, the quartic coupling of the scalar fields diverges as already pointed out by Gross and Wilczek [10] and by Cheng, Eichten, and Li [11].

The RG approach generalized by Lee and Weisberger [8] to the case of gauge theories with spontaneous symmetry breaking is what we adopt here to study the large- Q^2 behavior of the theory. The approach amounts to generalization of the well-known concept of “running coupling constants” to all the couplings involved in the theory. When the program is carried out order by order (in, e.g., the number of loops), the result is by definition (of a renormalizable field theory) rigorous. In practice, the task to go beyond the one-loop order is already horrendous so that the effective potential approach developed by many people [16], applied to the Higgs potential by Coleman and Weinberg [17], and reviewed ex-

tensively by Sher [18], has often been employed for investigating the pathological behavior associated with the Higgs sector. The connection between the standard approach which we adopt here and the effective potential method is not yet completely clear to us although it is obvious that the renormalization program and summation over all one-particle-irreducible loop diagrams in the effective potential method are not quite compatible to each other in their present forms. Finally, we should remark that, as only the pole parts ($\propto 1/\epsilon$) of the various diagrams are needed in our investigations, the question concerning which renormalization scheme is to be adopted, or the finite part of a specific diagram, is not relevant for our purpose. (See, e.g., the work of Marciano and Sirlin [19] or of Aoki *et al.* [20].)

II. SCALAR QED WITH SPONTANEOUS SYMMETRY BREAKING

The Lagrangian for scalar quantum electrodynamics (QED) is specified by [21]

$$L_{\text{eff}} = L_{\text{gauge}} + L_F + L_S, \quad (1)$$

$$L_{\text{gauge}} = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu}, \quad (2a)$$

$$L_F = -\bar{\Psi}\gamma_\mu D_\mu \Psi - m\bar{\Psi}\Psi, \quad (2b)$$

$$L_S = -[D_\mu \phi]^\dagger [D_\mu \phi] - V(\phi), \quad (2c)$$

where the gauge-invariant derivative D_μ and the potential $V(\phi)$ are given by

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu, \quad (3a)$$

$$D_\mu \equiv (\partial_\mu - igA_\mu), \quad (3b)$$

$$V(\phi) = \frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4. \quad (3c)$$

Here the coupling strength is denoted by g , which is to be differentiated from e in genuine QED as one should not confuse the U(1) gauge field treated here with the photon field.

For $\mu^2 > 0$, the ground state, at the tree level, is at $\phi = 0$ and is gauge invariant. Thus, gauge symmetry is exact and there is no “spontaneous symmetry breaking” (SSB). For $\mu^2 < 0$, gauge symmetry is spontaneously broken and the ϕ field has a nonvanishing vacuum expectation value (VEV) v ($=\sqrt{-\mu^2/\lambda}$). In the rest of this section, we shall discuss the general case of scalar QED with SSB. However, the results apply equally well to the case of scalar QED without SSB once we set the VEV to identically zero.

To facilitate a systematic treatment of loop calculations, we find it is useful to denote the complex scalar field $\phi = \phi_1 + i\phi_2$ by a vector notation $\phi \equiv (\phi_1, \phi_2)$ and its vacuum expectation value (VEV) by $\mathbf{v}$. The generalization to n -component scalar fields is then obvious. Now, quantum fields are to be interpreted as fluctuations around the VEV, so that we may expand the Lagrangian around $\mathbf{v}$. In particular, the potential $V(\phi)$ becomes

$$V(\phi) = V(\mathbf{v}) + \frac{1}{2!}\phi_i M_{ij}^2 \phi_j + \frac{1}{3!}f_{ijk}\phi_i \phi_j \phi_k + \frac{1}{4!}f_{ijmn}\phi_i \phi_j \phi_m \phi_n, \quad (4)$$

where

$$M_{ij}^2 = \left. \frac{\partial^2 V(\phi)}{\partial \phi_i \partial \phi_j} \right|_{\phi=\mathbf{v}}, \quad (5a)$$

$$f_{ijk} = \left. \frac{\partial^3 V(\phi)}{\partial \phi_i \partial \phi_j \partial \phi_k} \right|_{\phi=\mathbf{v}}, \quad (5b)$$

$$f_{ijmn} = \left. \frac{\partial^4 V(\phi)}{\partial \phi_i \partial \phi_j \partial \phi_m \partial \phi_n} \right|_{\phi=\mathbf{v}}. \quad (5c)$$

The values of M_{ij}^2 , f_{ijk} , and f_{ijmn} are given by

$$M_{ij}^2 = \mu^2 \delta_{ij} + \lambda v^2 \delta_{ij} + 2\lambda v_i v_j = 2\lambda v_i v_j, \quad (6a)$$

$$f_{111} = 6\lambda v_1, \quad f_{112} = 2\lambda v_2, \quad (6b)$$

$$f_{122} = 2\lambda v_1, \quad f_{222} = 6\lambda v_2, \quad f_{1111} = 6\lambda, \quad f_{1112} = 0, \quad f_{1122} = 2\lambda, \quad f_{1222} = 0, \quad f_{2222} = 6\lambda. \quad (6c)$$

Note that f_{ijk} and f_{ijmn} are totally symmetric in the indices.

It is useful to introduce

$$\tau \equiv \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad v_{1i} \equiv \tau_{ij} v_j, \quad (7)$$

which yields

$$\tau^2 = -I, \quad \tau_{ij} v_{1j} = -v_i, \quad \mathbf{v}_1 \cdot \mathbf{v}_1 = v^2. \quad (8)$$

Note that $V(\phi)$ has the rotation symmetry; i.e., $\delta V(\phi)$ vanishes upon a rotation in the (12) plane:

$$\delta \phi_1 = \epsilon \phi_2, \quad \delta \phi_2 = -\epsilon \phi_1. \quad (9)$$

This implies that the various constants v_i , M_{ij}^2 , f_{ijk} , and f_{ijmn} satisfy certain algebraic relations which realize the symmetry of the scalar potential. For instance, we have

$$f_{ijrs} f_{mnrs} + f_{imrs} f_{jnrs} + f_{inrs} f_{mjrs} = 20\lambda f_{ijmn}, \quad (10a)$$

$$f_{ijmn} f_{kmn} + f_{kimn} f_{jmn} + f_{jkmn} f_{imn} = 20\lambda f_{ijk}. \quad (10b)$$

Relations of such kind are very useful for reducing sums of one-loop results to simple forms. Additional relations, some of which also involve τ and v_{1j} introduced in Eq. (7), are listed in Appendix A.

Upon expanding $-[D_\mu \phi]^\dagger [D_\mu \phi]$, we find a term which describes the $\phi_1 \rightarrow A$ conversion and which can be eliminated by choosing the gauge-fixing term as

$$L_{\text{gf}} = -\frac{1}{2\xi}[\partial \cdot A - \xi g(\mathbf{v}_1 \cdot \phi)]^2 = -\frac{1}{2\xi}(\partial \cdot A)^2 + g(\mathbf{v}_1 \cdot \phi) \partial \cdot A - \frac{1}{2} \xi g^2 (\mathbf{v}_1 \cdot \phi)^2. \quad (11)$$

This gives rise to the renormalizable R_ξ gauge [22].

We may then carry out quantization of the gauge theory making use of the prescription of Faddeev and Popov [23] in the path integral formulation. It is easy to see that the gauge condition $F = \partial \cdot A - \xi g(\mathbf{v}_\perp \cdot \boldsymbol{\phi}) = 0$ yields the Faddeev-Popov ghost Lagrangian as given by

$$L_{\text{FP}} = -\bar{c}(x)[-(\partial^2 - \xi g^2 v^2) + \xi g^2 \mathbf{v}_\perp \cdot \boldsymbol{\phi}]c(x). \quad (12)$$

Combining all the things together, we obtain

$$L_{\text{eff}} = L_0 + L_I + L'_I + L_{\text{gf}} + L_{\text{FP}}, \quad (13)$$

where

$$L_0 = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} - \bar{\Psi}(\gamma_\mu \partial_\mu + m)\Psi - \frac{1}{2}(g^2 v^2)A^2 - \frac{1}{2}\phi_i M_{ij}^2 \phi_j - \frac{1}{2}\partial_\mu \boldsymbol{\phi} \cdot \partial_\mu \boldsymbol{\phi}, \quad (14a)$$

$$L_I = +ig\bar{\Psi}\gamma_\mu A_\mu \Psi - \frac{1}{3!}f_{ijk}\phi_i \phi_j \phi_k - \frac{1}{4!}f_{ijmn}\phi_i \phi_j \phi_m \phi_n + gA_\mu(\boldsymbol{\phi}_\perp \cdot \partial_\mu \boldsymbol{\phi}) - \frac{1}{2}g^2 A^2 \boldsymbol{\phi}^2 - g^2 A^2(\mathbf{v} \cdot \boldsymbol{\phi}), \quad (14b)$$

$$L'_I = gA_\mu(\mathbf{v}_\perp \cdot \partial_\mu \boldsymbol{\phi}). \quad (14c)$$

It is clear that L'_I can be combined with the second term of Eq. (11) to give a total derivative which can then be dropped completely.

Using Eqs. (11)–(14c), we may derive the Feynman rules in a rather straightforward manner. We list some results here for the benefits of a beginner on the subject.

To the gauge field propagator as illustrated by Fig. 1(a), we assign the factor

$$\Delta_{\mu\nu}^A(k) = \frac{1}{i}(k^2 + g^2 v^2 - i\epsilon)^{-1} \left[\delta_{\mu\nu} + (\xi - 1) \frac{k_\mu k_\nu}{k^2 + \xi g^2 v^2} \right]. \quad (15a)$$

Analogously we assign to the propagators illustrated by Figs. 1(b) (scalar field), 1(c) (fermion), and 1(d) (Faddeev-

FIG. 1. Propagators for scalar QED with spontaneous symmetry breaking.

Popov ghost) the following factors, respectively:

$$\Delta^S(k)_{ij} = \frac{1}{i}[(k^2 + M^2 - i\epsilon)_{ij}^{-1} - \xi g^2 v_{\perp i} v_{\perp j} (k^2)^{-1} (k^2 + \xi g^2 v^2 - i\epsilon)^{-1}], \quad (15b)$$

$$\Delta^F(k) = \frac{1}{i}[(i\gamma_\mu k_\mu + m) - i\epsilon]^{-1}, \quad (15c)$$

$$\Delta^g(k) = \frac{1}{i}[k^2 + \xi g^2 v^2 - i\epsilon]^{-1}. \quad (15d)$$

To the triple couplings as illustrated by Figs. 2(a)–2(e), we assign, respectively the factors

$$\begin{aligned} (a) & -g\gamma_\mu; \quad (b) \quad g\tau_{ij}(k+p)_\mu; \\ (c) & if_{ijk}; \quad (d) \quad -2ig^2 v_i \delta_{\mu\nu}; \\ (e) & -ig^2 \xi v_i. \end{aligned} \quad (16)$$

Similarly, we assign to the quartic couplings illustrated by Figs. 3(a) and 3(b) respectively the factors

$$\begin{aligned} (a) & -if_{ijmn}; \\ (b) & -2ig^2 \delta_{ij} \delta_{\mu\nu}. \end{aligned} \quad (17)$$

An important observation can be made at this juncture that, by setting $\mathbf{v}$ to identically zero, we recover the Feynman rules needed to investigate scalar QED without spontaneous symmetry breaking (SSB). We shall come back to this point in the next section when we obtain explicit results from one-loop calculations, showing that the various counterterms, except Z_K which is absent for scalar QED without SSB, actually do not depend explicitly on $\mathbf{v}$.

FIG. 2. Triple couplings for scalar QED with spontaneous symmetry breaking.

FIG. 3. Quartic couplings for scalar QED with spontaneous symmetry breaking.

In concluding this section, we wish to remark that, for a theory with more than one dimensionless coupling constant, the choice of an expansion parameter in perturbative calculations is not obvious. Nevertheless, it has been suggested [24] that the loop numbers of the Feynman diagrams may be a natural choice. Expanding in terms of loop numbers corresponds to the expansion in the Planck constant $\hbar$. For our model, the loop expansion is equivalent to the expansion by regarding λ to be of the same order as g^2 .

In our one-loop calculations, the “dimensional-regularization method” is adopted. The basic idea underlying dimensional regularization is to separate out the divergent parts of the unrenormalized amplitudes by defining the integrals in the complex degree of dimension and making the analytic continuation. We take the minimal subtraction scheme [4] to define the finite parts, which means that only pole parts of the divergences are subtracted away. Since choosing different schemes differs only in finite quantities and does not influence the asymptotic behavior of the theory, the minimal subtraction scheme is well suited for our purpose.

III. RENORMALIZATION OF SCALAR QED

The renormalization procedure of a gauge theory with Higgs mechanism, such as the one described in the preceding section, has been known since the early 1970s [3–5]. As in the σ model [25], a linear ϕ term is needed to absorb the higher-order corrections to the vacuum expectation value. We note that, although there is no linear

term at the tree level, there are higher-order corrections which, in fact, are infinite. Accordingly, the linear ϕ counterterm is needed and may be set by requiring that the one-point Green’s function of the scalar field still vanishes in higher orders. In our case, the term may be chosen to be $K\mu^2(\mathbf{v}\cdot\phi)$, which does not break the gauge symmetry, nor the reflection symmetry $\phi \rightarrow -\phi$ of the original Lagrangian.

Following closely the work of Appelquist *et al.* [7], we renormalize the theory with the Lagrangian specified by Eqs. (13) and (14a)–(14c) making use of the reparametrization

$$\phi \rightarrow \phi_0 = Z^{1/2}\phi, \quad v \rightarrow v_0 = Z^{1/2}v, \quad (18a)$$

$$\lambda \rightarrow \lambda_0 = Z_\lambda Z^{-2}\lambda, \quad \mu^2 \rightarrow \mu_0^2 = Z_\mu Z^{-1}\mu^2; \quad (18b)$$

$$K \rightarrow K_0 = Z_K Z_\mu^{-1} Z^{-1}K, \quad (18c)$$

$$\Psi \rightarrow \Psi_0 = Z_1^{1/2}\Psi, \quad m \rightarrow m_0 = Z_m Z_1^{-1}m; \quad (18d)$$

$$A_\mu \rightarrow A_{0\mu} = Z_3^{1/2}A_\mu, \quad g \rightarrow g_0 = Z_3^{-1/2}g; \quad (18e)$$

$$\xi \rightarrow \xi_0 = Z_\xi \xi. \quad (18f)$$

The renormalized Lagrangian is then given by

$$L_{\text{ren}} = L_{0,\text{ren}} + L_{I,\text{ren}} + L'_I + L_{\text{gf}} + L_{\text{FP}}, \quad (19)$$

where

$$\begin{aligned} L_{0,\text{ren}} = & -\frac{1}{4}Z_3 F_{\mu\nu}F_{\mu\nu} - Z_1 \bar{\Psi}\gamma_\mu \partial_\mu \Psi - Z_m m \bar{\Psi}\Psi \\ & -\frac{1}{2}Z(g^2 v^2)A^2 - \frac{1}{2}Z_\mu \phi_i M_{ij}^2 \phi_j \\ & -\frac{1}{2}Z \partial_\mu \phi \cdot \partial_\mu \phi + Z_K K \mu^2(\mathbf{v}\cdot\phi), \end{aligned} \quad (20a)$$

$$\begin{aligned} L_{I,\text{ren}} = & +iZ_1 g \bar{\Psi}\gamma_\mu \Psi A_\mu - \frac{1}{3!}Z_\lambda f_{ijk} \phi_i \phi_j \phi_k \\ & -\frac{1}{4!}Z_\lambda f_{ijmn} \phi_i \phi_j \phi_m \phi_n + Zg A_\mu (\phi_1 \cdot \partial_\mu \phi) \\ & -\frac{1}{2}Zg^2 A^2 \phi^2 - Zg^2 A^2(\mathbf{v}\cdot\phi). \end{aligned} \quad (20b)$$

Here we have used relations among the renormalization constants, as derived from gauge symmetry, in obtaining Eqs. (18)–(20). It will become clear that both the gauge-fixing and Faddeev-Popov ghost terms are not renormalized. Consequently, the two $A_\mu \mathbf{v}_\perp \cdot \phi$ terms no longer cancel each other in higher orders, although they cancel at the tree level.

In what follows, we describe separately, but briefly, our one-loop results on the one-, two-, three-, and four-point vertex functions.

FIG. 4. Tadpole diagrams.

A. One-point vertex function: Tadpole diagrams

To one-loop order, the one-point vertex function is determined by three different sources: (1) the tadpole diagrams which are illustrated by Fig. 4, (2) the counterterms obtained by shifting the scalar fields, $\phi_i \rightarrow \phi_i + v_i$, associated with the ϕ^2 and ϕ^4 terms [as can easily be seen

from Eqs. (3c) and (18a)], and (3) the counterterm in Z_K [i.e., the last term in Eq. (20a) which is needed for reasons explained earlier in this section].

Evaluation of the three tadpole diagrams illustrated in Fig. 4 using dimensional regularization [4] is straightforward. The expressions which we need to work out are given by

$$\begin{aligned} T^{(1)} &= \frac{1}{2!} (-i) f_{ijn} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{i} [v_j v_n (k^2 + 2\lambda v^2)^{-1} + v_{\perp j} v_{\perp n} (k^2 + \xi g^2 v^2)^{-1}] , \\ T^{(2)} &= \frac{1}{2!} (-2ig^2 v_i \delta_{\mu\nu}) \int \frac{d^4 k}{(2\pi)^4} \left[\frac{1}{i} (k^2 + g^2 v^2)^{-1} \left[\delta_{\mu\nu} + (\xi - 1) \frac{k_\mu k_\nu}{k^2 + \xi g^2 v^2} \right] \right] , \\ T^{(3)} &= (-1) (-i\xi g^2 v_i) \int \frac{d^4 k}{(2\pi)^4} \left[\frac{1}{i} (k^2 + \xi g^2 v^2)^{-1} \right] . \end{aligned} \quad (21)$$

Here $\epsilon = \frac{1}{2}(4-n)$ with n the dimension of space-time for defining the integral. The sum of their pole parts is, as evaluated in the Euclidean space with an extra factor i after Wick's rotation,

$$\sum_{i=1}^3 T^{(i)} = \frac{1}{16\pi^2} \lambda v^2 v_i \left[6\lambda + 3 \frac{g^4}{\lambda} + \xi g^2 \right] \frac{1}{\epsilon} . \quad (22)$$

Note that this result is gauge dependent, an aspect which was emphasized previously by Appelquist *et al.* [7]. Gauge dependence of the tadpole contribution serves to cancel out other gauge-dependent terms in order to produce gauge-independent S -matrix elements for physical observables. This has little to do with whether we shift the vacuum in light of higher-order corrections.

The counterterms obtained by shifting the scalar fields associated with the ϕ^2 and ϕ^4 terms can be combined to yield $-\lambda v^2 (Z_\lambda - Z_\mu)(\mathbf{v} \cdot \boldsymbol{\phi})$, which are yet to be determined. Once we calculate Z_λ and Z_μ to one-loop order, we can combine it with Eq. (22) in order to fix Z_K such that one-loop corrections to the vacuum expectation value of the scalar fields get canceled out completely. We shall come back to this point later in this section.

B. Two-point vertex functions

In this subsection, we consider evaluation of the following two-point vertex functions: (i) the self-energy of the gauge field as illustrated by Fig. 5, (ii) the self-energy of the scalar fields as illustrated by Fig. 6, (iii) the self-energy diagrams of the Faddeev-Popov ghost (Fig. 7) which cancel among themselves, and (iv) the conversion between the gauge field and the scalar field as shown by Fig. 8. Note that the tadpole contribution, which has just been evaluated in Eqs. (21) and (22) above, appears as one of the terms for each of Figs. 5–8.

First, we consider the self-energy of the gauge field as illustrated by Fig. 5. Here diagram (e) refers to the tadpole contribution while diagram (f) is the contribution

from the counterterm, $-\lambda v^2 (Z_\lambda - Z_\mu)(\mathbf{v} \cdot \boldsymbol{\phi})$, which is yet to be determined. Pole parts of diagrams (a)–(f) in Fig. 5 are given, respectively, by

$$\begin{aligned} \Gamma_{\mu\nu}^{(1)} &= \frac{1}{16\pi^2 \epsilon} (2\lambda + \xi g^2) g^2 v^2 \delta_{\mu\nu} , \\ \Gamma_{\mu\nu}^{(2)} &= \frac{1}{16\pi^2 \epsilon} \left[-\frac{1}{3} g^2 (k^2 \delta_{\mu\nu} - k_\mu k_\nu) - 2g^2 \lambda v^2 \delta_{\mu\nu} - \xi g^4 v^2 \delta_{\mu\nu} \right] , \\ \Gamma_{\mu\nu}^{(3)} &= \frac{1}{16\pi^2 \epsilon} (3 + \xi) g^4 v^2 \delta_{\mu\nu} , \\ \Gamma_{\mu\nu}^{(4)} &= \frac{1}{16\pi^2 \epsilon} \left(-\frac{4}{3} N_f \right) g^2 (k^2 \delta_{\mu\nu} - k_\mu k_\nu) , \\ \Gamma_{\mu\nu}^{(5)} &= \frac{1}{16\pi^2 \epsilon} \left[-6\lambda - 3 \frac{g^4}{\lambda} - \xi g^2 \right] g^2 v^2 \delta_{\mu\nu} , \\ \Gamma_{\mu\nu}^{(6)} &= g^2 v^2 (Z_\lambda - Z_\mu) \delta_{\mu\nu} . \end{aligned} \quad (23)$$

Here N_f is the number of fermion flavors which couple to

FIG. 5. Self-energy diagrams for the gauge field.

FIG. 6. Self-energy diagrams for the scalar fields.

the gauge field. Accordingly, we have

$$\sum_{i=1}^6 \Gamma_{\mu\nu}^{(i)} = \frac{1}{16\pi^2} \frac{1}{\epsilon} \left[-\frac{g^2}{3} (4N_f + 1) (k^2 \delta_{\mu\nu} - k_\mu k_\nu) + \left[-6\lambda - 3\frac{g^4}{\lambda} + 3g^2 \right] g^2 v^2 \delta_{\mu\nu} \right] + (Z_\lambda - Z_\mu) g^2 v^2 \delta_{\mu\nu}. \quad (24)$$

Comparing Eq. (24) with the counterterms obtained from Eqs. (14) and (20),

$$-\frac{1}{4}(Z_3 - 1)F_{\mu\nu}F_{\mu\nu} - \frac{1}{2}(Z - 1)g^2 v^2 A_\mu A_\mu - \frac{1}{2\xi}(Z_\xi^{-1}Z_3 - 1)(\partial \cdot A_\mu)^2,$$

we find

$$Z_3 = 1 - \frac{1}{16\pi^2} \left[\frac{g^2}{3} \right] (4N_f + 1) \frac{1}{\epsilon}, \quad (25)$$

$$Z - 1 = (Z_\lambda - Z_\mu) + \frac{1}{16\pi^2} \left[-6\lambda - 3\frac{g^4}{\lambda} + 3g^2 \right] \frac{1}{\epsilon}, \quad (26)$$

$$Z_\xi = Z_3. \quad (27)$$

Equation (27) indicates that the longitudinal component of the gauge field, which would be absent without spontaneous symmetry breaking, is not renormalized to one-loop order.

Next, we proceed to consider the self-energy of the scalar fields as illustrated by Fig. 6. The sum of pole parts of

FIG. 7. Self-energy diagrams for the Faddeev-Popov ghost.

FIG. 8. Self-energy diagrams for the A - ϕ coupling.

diagrams (a)–(f) in Fig. 6 is given by

$$\frac{1}{16\pi^2} \left[(3 - \xi) g^2 k^2 \delta_{ij} + (6\lambda^2 + 3g^4 - \xi g^2 \lambda) v^2 \delta_{ij} + \left[20\lambda^2 - 4\xi g^2 + 6\frac{g^4}{\lambda} \right] v_i v_j \right] \frac{1}{\epsilon},$$

which are to be compensated by the counterterms:

$$-\frac{1}{2}(Z - 1)\partial_\mu \phi \partial_\mu \phi - \frac{1}{2}\lambda v^2 (Z_\lambda - Z_\mu) \phi^2 - (Z_\lambda - 1)\lambda (\mathbf{v} \cdot \phi)^2.$$

The term-by-term comparison gives rise to

$$Z = 1 + \frac{1}{16\pi^2} (3 - \xi) g^2 \frac{1}{\epsilon}, \quad (28)$$

$$Z_\lambda = 1 + \frac{1}{16\pi^2} \left[10\lambda + 3\frac{g^4}{\lambda} - 2\xi g^2 \right] \frac{1}{\epsilon}, \quad (29)$$

$$Z_\mu = 1 + \frac{1}{16\pi^2} (4\lambda - \xi g^2) \frac{1}{\epsilon}. \quad (30)$$

As a consistency check, we see that Eq. (26) is indeed satisfied by the results given by Eqs. (28)–(30). As the tadpole terms are always followed by the counterterm $(Z_\mu - Z_\lambda)\lambda v^2 (\mathbf{v} \cdot \phi)$, we may use Eqs. (22), (29), and (30) to combine them together:

$$\sum_{i=1}^3 T^{(i)} + \Gamma_{(Z_\lambda - Z_\mu)} = \frac{1}{16\pi^2} 2\xi g^2 \lambda v^2 v_i \frac{1}{\epsilon}. \quad (31)$$

We may now turn our attention to the self-energy of the Faddeev-Popov ghost as illustrated by Fig. 7. The contribution from diagram (a) is specified by

$$\Gamma_{ij}^{(1)}(k) = (-i\xi g^2)^2 v_i v_j \int \frac{d^4 q}{(2\pi)^4} \Delta_{ij}^{(S)}(q) \Delta^{(W)}(k + q). \quad (32)$$

Its pole part is

$$\frac{1}{16\pi^2} (\xi^2 g^4 v^2) \frac{1}{\epsilon},$$

which is canceled exactly by the contributions from diagrams (b) and (c).

The last two-point vertex function which we consider here is the A - ϕ vertex as illustrated by Fig. 8. Note that, at the tree level, we choose the gauge-fixing term such that the A - ϕ term is absent in the Lagrangian. The pole contribution from diagrams (a) and (b) is given by

$$\frac{1}{16\pi^2} 3ig^3 v_{\perp i} k_{\mu} \frac{1}{\epsilon},$$

while diagrams (c) and (d) yield

$$\frac{1}{16\pi^2} (-i\xi g^3) v_{\perp i} k_{\mu} \frac{1}{\epsilon}.$$

Thus, the sum as given by

$$\frac{1}{16\pi^2} (3 - \xi) ig^3 v_{\perp i} k_{\mu} \frac{1}{\epsilon},$$

is canceled by the counterterm $(Z - 1)g v_{\perp i} \partial_{\mu} \phi$, which comes from the renormalization of $[D_{\mu} \phi]^{\dagger} [D_{\mu} \phi]$ [i.e., the difference between the fourth terms of Eqs. (20b) and (14b)]. Accordingly, the gauge-fixing term is not renormalized, just like the self-energy of the Faddeev-Popov ghost.

In concluding our discussions on the two-point vertex functions, we note that treatment of the fermion propagator in spontaneously broken scalar QED ($\mu^2 < 0$) is identical to that in a nonbroken theory ($\mu^2 > 0$). A straightforward calculation yields

$$Z_1 = 1 - \frac{1}{16\pi^2} \xi g^2 \frac{1}{\epsilon}, \quad (33)$$

$$Z_m = 1 - \frac{1}{16\pi^2} (3 + \xi) g^2 \frac{1}{\epsilon}. \quad (34)$$

So far, we have already determined all the renormalization constants as required by Eqs. (20a) and (20b), except the coefficient Z_K which we introduced in connection with the one-point function. With Z_{λ} and Z_{μ} already given by Eqs. (29) and (30), we may now determine Z_K . As indicated earlier, we may combine the term $-\lambda v^2 (Z_{\lambda} - Z_{\mu})(\mathbf{v} \cdot \boldsymbol{\phi})$ and the tadpole contributions [Eq. (22)] with the Z_K term such that one-loop corrections to the vacuum expectation value of the scalar fields get canceled out completely. As the counterterm from $-\lambda v^2 (Z_{\lambda} - Z_{\mu})(\mathbf{v} \cdot \boldsymbol{\phi})$ contributes

$$-\frac{1}{16\pi^2} \lambda v^2 v_i \left[6\lambda + 3 \frac{g^4}{\lambda} - \xi g^2 \right] \frac{1}{\epsilon}, \quad (35)$$

we obtain, using also Eq. (22),

$$Z_K = 1 + \frac{1}{16\pi^2} \frac{2\xi g^2}{K} \frac{1}{\epsilon}. \quad (36)$$

As all renormalization constants introduced in Eqs. (18a) and (18b) have been determined by evaluation of the one-point and two-point vertex functions, an explicit calculation of a three- or four-point vertex function provides a consistent check on whether introduction of a small number of renormalization constants indeed renormalizes the theory consistently. Although we offer in Secs. III C and III D examples to show that this is indeed the case, we urge an interested reader to study the paper of Appelquist *et al.* [7] where the consistency question was addressed in a somewhat formal manner (primarily because the explicit results on the various renormalization constants were yet to be separated out).

C. Three-point vertex functions

As an illustrative example, we consider the ϕ^3 vertex as described by Fig. 9. Diagram (a) yields

$$\Gamma_{ijk}^{(a)}(k) = \frac{1}{2!} f_{ijmn} f_{krs} \int \frac{d^4 q}{(2\pi)^4} \Delta_{mr}^{(S)}(q) \Delta_{ns}^{(S)}(k+q), \quad (37)$$

with the pole part readily determined to be

$$\frac{1}{16\pi^2} \frac{1}{2!} f_{ijmn} f_{kmn} \frac{1}{\epsilon}. \quad (38)$$

Thus, we have, from cyclic permutations,

$$\sum \Gamma_{ijk}^{(a)} = \frac{1}{16\pi^2} \frac{1}{2!} (f_{ijmn} f_{kmn} + f_{kimn} f_{jmn} + f_{jkmn} f_{imn}) \frac{1}{\epsilon}, \quad (39)$$

which, upon using Eq. (10b), becomes

$$\frac{1}{16\pi^2} 10\lambda f_{ijk} \frac{1}{\epsilon}. \quad (40)$$

Analogously, contributions from diagrams (b), (c), and (d) and their cyclic permutations are given by

$$\begin{aligned} \sum \Gamma_{ijk}^{(b)} &= \frac{1}{16\pi^2} 8g^4 (\delta_{ij} v_k + \delta_{ki} v_j + \delta_{jk} v_i) \frac{1}{\epsilon} \\ &= \frac{1}{16\pi^2} \left[\frac{4g^4}{\lambda} \right] f_{ijk} \frac{1}{\epsilon}, \end{aligned} \quad (41)$$

$$\begin{aligned} \sum \Gamma_{ijk}^{(c)} &= \frac{1}{16\pi^2} (-\xi g^2) (f_{kmn} \tau_{mi} \tau_{nj} + f_{jmn} \tau_{mk} \tau_{ni} \\ &\quad + f_{imn} \tau_{mj} \tau_{nk}) \frac{1}{\epsilon} \\ &= \frac{1}{16\pi^2} (-\xi g^2) f_{ijk} \frac{1}{\epsilon}, \end{aligned} \quad (42)$$

$$\begin{aligned} \sum \Gamma_{ijk}^{(d)} &= \frac{1}{16\pi^2} (-2g^4) (\delta_{ij} v_k + \delta_{ki} v_j + \delta_{jk} v_i) \frac{1}{\epsilon} \\ &= \frac{1}{16\pi^2} \left[-\frac{g^4}{\lambda} \right] f_{ijk} \frac{1}{\epsilon}. \end{aligned} \quad (43)$$

FIG. 9. One-loop contributions to the ϕ^3 vertex function.

Diagrams (e) and (f) yield

$$\begin{aligned}\sum_{e,f} \Gamma_{ijk} &= \frac{1}{16\pi^2} (-\xi g^2) (v_m f_{ijkm}) \frac{1}{\epsilon} \\ &= \frac{1}{16\pi^2} (-\xi g^2) f_{ijk} \frac{1}{\epsilon} .\end{aligned}\quad (44)$$

The sum of Eqs. (40)–(44) is then given by

$$\sum_a^f \Gamma_{ijk}^{(l)} = \frac{1}{16\pi^2} \left[10\lambda + 3\frac{g^4}{\lambda} - 2\xi g^2 \right] f_{ijk} \frac{1}{\epsilon} , \quad (45)$$

which is canceled exactly by the counterterm, $-(1/3!)(Z_\lambda - 1)f_{ijk}\phi_i\phi_j\phi_k$, as represented pictorially by diagram (g) in Fig. 9. Alternatively, we may use the ϕ^3

three-point function to determine Z_λ , which then must satisfy certain relations when we calculate two- and four-point vertex functions.

D. Remarks on four-point vertex functions

Let us for the moment consider scalar QED *without* spontaneous symmetry breaking. Evaluation of the renormalization constants is straightforward as we have learned from Sec. II that Feynman rules can easily be obtained by setting the vacuum expectation value of the scalar fields to identically zero. As all our results obtained from Secs. III A and III B are already independent of v , it readily follows that, for scalar QED without SSB, we have

$$\begin{aligned}\Phi \rightarrow \Phi_0 &= Z^{1/2} \Phi, \quad Z = 1 + \frac{1}{16\pi^2} (3 - \xi) g^2 \frac{1}{\epsilon} ; \\ \Psi \rightarrow \Psi_0 &= Z_1^{1/2} \Psi, \quad Z_1 = 1 - \frac{1}{16\pi^2} \xi g^2 \frac{1}{\epsilon} ; \\ A_\mu \rightarrow A_{0\mu} &= (Z_3)^{1/2} A_\mu, \quad Z_3 = 1 - \frac{1}{16\pi^2} \left[\frac{1}{3} (4N_f + 1) g^2 \right] \frac{1}{\epsilon} ; \\ g \rightarrow g_0 &= Z_3^{-1/2} g, \\ \lambda \rightarrow \lambda_0 &= Z_\lambda Z^{-2} \lambda, \quad Z_\lambda = 1 + \frac{1}{16\pi^2} \left[10\lambda + 3\frac{g^4}{\lambda} - 2\xi g^2 \right] \frac{1}{\epsilon} ; \\ \mu^2 \rightarrow \mu_0^2 &= Z_\mu Z^{-1} \mu^2, \quad Z_\mu = 1 + \frac{1}{16\pi^2} (4\lambda - \xi g^2) \frac{1}{\epsilon} ; \\ m \rightarrow m_0 &= Z_m Z_1^{-1} m, \quad Z_m = 1 - \frac{1}{16\pi^2} (3 + \xi) g^2 \frac{1}{\epsilon} ; \\ \xi \rightarrow \xi_0 &= Z_\xi \xi, \quad Z_\xi = Z_3 ;\end{aligned}\quad (46)$$

Without complications from nonzero v , it is relatively simple to consider other vertices such as three- or four-point functions in order to demonstrate that the reparametrization introduced in Eq. (46) together with the explicit one-loop results indeed renormalizes the theory in a consistent manner.

Coming back to the case of scalar QED with SSB, we may now note that evaluation of the four-point vertex functions with zero superficial degrees of divergence, $D=0$, such as Γ_{ϕ^4} , $\Gamma_{\phi^2 A^2}$, and Γ_{A^4} has nothing to do with the sign of μ^2 . For a spontaneously broken theory ($\mu^2 < 0$), loop corrections cause a displacement in the vacuum expectation value (and thus in the scalar fields) but such displacement does not affect the pole parts of the four-point vertex functions with $D=0$. the renormalization constants listed in Eq. (46) for scalar QED without SSB determine all the four-point vertex functions in that theory, which are then identical to the theory with SSB. We thus have an overall consistency check on the renormalization program for the theory *with* spontaneous symmetry breaking.

As a matter of fact, the above argument applies equally well to the vertex $\Gamma_{\phi A^2}$ which has $D=1$ since, upon factoring out the external momentum k in view of the derivative coupling, such vertex has effectively $D=0$, i.e., not affected by a shift in the vacuum expectation value. Thus the renormalization constant for the vertex is the same for both with SSB and without SSB. Finally, we may also note that vertex functions with *only* an odd number of external gauge fields vanish identically.

In concluding this section, we wish to reiterate the result that, to one-loop order, all the renormalization constants as in Eq. (46) for scalar QED *without* spontaneous symmetry breaking are identical to a spontaneously broken theory, except that, in the latter case, there is an additional constant Z_K .

IV. LARGE- Q^2 BEHAVIOR OF SCALAR QED

The RG approach generalized by Lee and Weisberger [8] to the case of gauge theories with spontaneous symmetry breaking is what we adopt here to study the large-

Q^2 behavior of the theory. The approach amounts to generalization of the well-known concept of “running coupling constants” to all the couplings involved in the theory. When the program is carried out order by order (in, e.g., the number of loops), the result is by definition (of a renormalizable field theory) rigorous.

The primary objective of our work is to use the coupled RG equations to determine how the various parameters (couplings), including both the dimensionless and dimensional ones, behave as $Q^2 \rightarrow \infty$. The focus in the past [10,11] has been primarily with the dimensionless couplings, except in the work of Lee and Weisberger [8] where the vacuum expectation value $v(Q^2)$ was discussed

in the context of the RG equation. [Thus, the result on the running of the coupling constant $\alpha(t)$ as well as on the growth of the quartic coupling $\lambda(t)$ is well known.] Although we choose to work out the problem only for the simplest prototype of spontaneously broken gauge field theories, i.e., scalar QED, our result may suggest why a similar problem should also be seriously addressed for the GSW electroweak theory and its likes, especially when one anticipates pathological behavior of some sort.

We may begin by considering a theory without gauge symmetry nor spontaneous symmetry breaking (SSB) and write down the RG equation [26] for the vertex function $\Gamma(-p_j^2/\mu^2, m^2/\mu^2, g)$ with external momenta p_j :

$$\left[\mu^2 \frac{\partial}{\partial \mu^2} + \beta \left[\frac{m^2}{\mu^2}, g \right] \frac{\partial}{\partial g} + \gamma_m \left[\frac{m^2}{\mu^2}, g \right] m^2 \frac{\partial}{\partial m^2} - \gamma_\Gamma \left[\frac{m^2}{\mu^2}, g \right] \right] \Gamma \left[-\frac{p_j^2}{\mu^2}, \frac{m^2}{\mu^2}, g \right] = 0, \quad (47)$$

where

$$\beta \left[\frac{m^2}{\mu^2}, g \right] \equiv \bar{\mu}^2 \frac{\partial}{\partial \bar{\mu}^2} \bar{g} \left[\frac{\bar{\mu}^2}{\mu^2}, \frac{m^2}{\mu^2}, g \right] \bigg|_{\bar{\mu}^2 = \mu^2}, \quad (48a)$$

$$\gamma_m \left[\frac{m^2}{\mu^2}, g \right] \equiv \bar{\mu}^2 \frac{\partial}{\partial \bar{\mu}^2} \ln \bar{m}^2 \left[\frac{\bar{\mu}^2}{\mu^2}, \frac{m^2}{\mu^2}, g \right] \bigg|_{\bar{\mu}^2 = \mu^2}, \quad (48b)$$

$$\gamma_\Gamma \left[\frac{m^2}{\mu^2}, g \right] \equiv \bar{\mu}^2 \frac{\partial}{\partial \bar{\mu}^2} \ln Z_\Gamma \left[\frac{\bar{\mu}^2}{\mu^2}, \frac{m^2}{\mu^2}, g \right] \bigg|_{\bar{\mu}^2 = \mu^2}. \quad (48c)$$

Here $Z_\Gamma(\bar{\mu}^2/\mu^2, m^2/\mu^2, g)$ enters the description of the scaling property of the vertex as we vary the renormalization scale μ^2 or the probing distance:

$$\Gamma \left[-\frac{p_j^2}{\bar{\mu}^2}, \frac{\bar{m}^2}{\bar{\mu}^2}, \bar{g} \right] = Z_\Gamma \left[\frac{\bar{\mu}^2}{\mu^2}, \frac{m^2}{\mu^2}, g \right] \Gamma \left[-\frac{p_j^2}{\mu^2}, \frac{m^2}{\mu^2}, g \right], \quad (49)$$

which yields the RG equation (47). We note that, in Eqs. (47)–(49), the quantity μ^2 denotes the renormalization scale which should be distinguished from that adopted earlier for Eq. (3c) and used throughout until this section. To avoid further confusion, we introduce

$$t \equiv \ln \frac{\mu^2}{\mu_0^2}, \quad (50)$$

with μ_0^2 some initial renormalization point.

Lee and Weisberger [8] have considered a generalization of the RG equation to the case of gauge theories with spontaneous symmetry breaking. In short, one treats both the gauge-fixing parameter ξ and the vacuum expectation value v_i as “additional” coupling constants. Or, we temporarily ignore the fact that v may be tied to the other couplings entering in the potential Eq. (3)—a result already spoiled by quantum or loop corrections. In this way, Lee and Weisberger [8] obtained

$$\left[-\frac{\partial}{\partial t} + \beta_i(g) \frac{\partial}{\partial g_i} - 2\gamma_\nu \xi \frac{\partial}{\partial \xi} - (1 + \gamma_S)x \frac{\partial}{\partial x} - d_\Gamma \right] \times \Gamma(e' p_j; g_i, \xi, x) = 0. \quad (51)$$

Here $x \equiv v^2/\mu^2$ with μ^2 the renormalization scale.

What interests us is the large- Q^2 behavior of the theory. To this end, we consider the special cases of Eq. (51). We have, with $\alpha \equiv g^2/4\pi$,

$$\frac{d\alpha}{dt} = \beta(\alpha) = \frac{1}{3\pi} (N_f + \frac{1}{4}) \alpha^2 \geq 0, \quad (52a)$$

$$\frac{d\lambda}{dt} = \beta_\lambda = \frac{1}{16\pi^2} (10\lambda^2 - 6\lambda g^2 + 3g^4) \geq 0. \quad (52b)$$

Of course, Eq. (52a) or (52b) which characterizes a running coupling constant may also be derived by considering that the renormalization from a scale μ_0^2 to another scale μ^2 can either be done *directly* or carried out by going through an intermediate scale μ_1^2 . We shall come back shortly to the question of how the explicit forms for $\beta(\alpha)$ and β_λ are derived.

Analogously, the VEV parameter x and the gauge-fixing parameter ξ may also be treated as “running coupling constants:”

$$\frac{\partial \bar{x}}{\partial t} = -\bar{x} [1 + \gamma_S(\bar{g}, \bar{\lambda})], \quad \bar{x}(x, g, \lambda, t=0) = x, \quad (52c)$$

$$\frac{\partial \bar{\xi}}{\partial t} = -2\bar{\xi} \gamma_\nu(\bar{g}, \bar{\lambda}, \bar{\xi}), \quad \bar{\xi}(g, \lambda, \xi, t=0) = \xi. \quad (52d)$$

Now, let us come back to the question of extracting the β and γ functions from our results in Sec. III. Taking $\beta(\alpha)$ as our illustrative example, we reinterpret the reparametrization Eq. (18e) as

$$\bar{g}(\bar{\mu}^2, \bar{g}) = Z_3^{1/2} \left[\frac{\bar{\mu}^2}{\mu^2}, g \right] \bar{g}(\mu^2, g). \quad (53)$$

Or, we have

$$\bar{\alpha}(\bar{\mu}^2, \bar{\alpha}) = Z_3 \left[\frac{\bar{\mu}^2}{\mu^2}, \alpha \right] \bar{\alpha}(\mu^2, g). \quad (54)$$

Since all the explicit results on the renormalization constants are obtained in $(4-2\epsilon)$ dimensions, we introduce the renormalization scale by replacing the dimensionless coupling g by $g\mu^{-\epsilon}$. This standard practice enables us to calculate from Eqs. (48a)–(48c) and β and γ functions in a trivial manner. The result for $\beta(\alpha)$ and β_λ is already recorded in Eqs. (52a) and (52b). In the same vein, we find, to one-loop order,

$$\gamma_V = \frac{2}{16\pi^2} \frac{1}{3} (N_f + \frac{1}{4}) g^2 \geq 0, \quad (55a)$$

$$\gamma_S = -\frac{1}{16\pi^2} (3 - \xi) g^2. \quad (55b)$$

It is seen that Eqs. (52a)–(52d) are in fact the coupled RG equations which dictate the large- Q^2 behavior of the theory. As we shall see immediately below, the case for scalar QED is relatively simple as these coupled equations can be solved explicitly, yielding the prediction on how the couplings $\alpha(t)$, $v(t)$, $\xi(t)$, $\lambda(t)$, and $m(t)$ run with t ($\equiv \ln Q^2/Q_0^2$). The situation for the GSW theory is considerably more complicated [28] although we shall remark on a proof of Lee and Weisberger [7].

Coming back to the question of how to solve Eqs. (51a)–(52d), we first integrate out Eq. (52) and recover the well-known formula for a “running coupling:”

$$\alpha(t) = \frac{\alpha_0}{1 - \alpha_0 \frac{1}{3\pi} (N_f + \frac{1}{4}) t}, \quad (56)$$

with $\alpha_0 = \alpha(t=0)$ (i.e., at μ_0^2). Equation (52d) together with Eq. (55a) can be combined with Eq. (52a) to yield

$$\frac{\partial \alpha}{\partial \xi} = -\frac{\alpha}{\xi},$$

which can be solved trivially:

$$\alpha(t)\xi(t) = \alpha_0\xi_0. \quad (57)$$

Using Eqs. (56) and (57) to eliminate g^2 and ξ in favor of an explicit t dependence, we may now integrate out Eq. (52c):

$$x(t) = x_0 \exp \left[- \left[1 + \frac{\alpha_0 \xi_0}{4\pi} \right] t \right] \times \left[1 - \alpha_0 \frac{1}{3\pi} (N_f + \frac{1}{4}) t \right]^{-9/4(N_f + 1/4)} \quad (58)$$

As $\alpha_0 \xi_0 > 0$, we see that the VEV, or x , rapidly approaches zero before the coupling $\alpha(t)$ gets strong as $t \rightarrow \infty$. This is a useful result since we are assured that spontaneous symmetry breaking indeed disappears as $Q^2 \rightarrow \infty$. On the other hand, if the gauge coupling $\alpha(Q^2)$ vanishes at large Q^2 [which is, however, not true in the $U(1)$ case], then the gauge sector decouples from the

Higgs sector. We will then have a simple theory although the Higgs sector gets strongly self-interacting.

The Higgs sector indeed becomes strongly interacting even in the $U(1)$ case. Note that Eq. (52b) describes a “running” $\lambda(t)$. Instead of solving Eq. (52b) as it is, we may write

$$\frac{d\lambda}{dt} = \frac{1}{16\pi^2} [7\lambda^2 + 3(\lambda - g^2)^2] \geq \frac{7\lambda^2}{16\pi^2}.$$

We may integrate the “inequality” equation from t_0 (say $t_0=0$ or $\mu^2=\mu_0^2$) to a larger value t to obtain an inequality relation

$$\lambda(t) \geq \frac{\lambda_0}{1 - \lambda_0 \frac{7}{16\pi^2} t}. \quad (59)$$

Accordingly, we see an important and well-known result that $\lambda \rightarrow \infty$ before $(7/16\pi^2)\lambda_0 t \rightarrow 1$. This happens well ahead of $\alpha(t) \rightarrow \infty$ for theories with $\lambda_0 \geq (16\pi/21)(N_f + \frac{1}{4})\alpha_0$. This may in fact be good news since it saves the theory from running into the “Landau ghost” problem i.e., $\alpha(t) \rightarrow \infty$. However, the theory becomes strongly interacting (and is also nonlinear) in the Higgs sector and it ceases to be justified to work with perturbation expansions.

It may be useful to know how the other couplings behave as $Q^2 \rightarrow \infty$. For the fermion mass $m(t)$, we find

$$\alpha(t)m^d(t) = \alpha_0 m_0^d, \quad (60)$$

with $d = \frac{4}{9}(N_f + \frac{1}{4})$. Thus, $m(t)$ behaves in a way similar to $\xi(t)$ as $t \rightarrow \infty$. On the other hand, the strength of the ϕ^2 term [in Eq. (3c)] is characterized by $\mu_\phi^2(t)$ (here we add a subscript ϕ to distinguish it from the renormalization scale):

$$\frac{d\mu_\phi^2}{dt} = \beta(\mu_\phi^2) = \frac{1}{16\pi^2} (4\lambda - 3g^2)\mu_\phi^2. \quad (61)$$

Here we see clearly that there is a competition between λ and g^2 (or α). For $\lambda > 3\pi\alpha$, μ_ϕ^2 is increasing as t increases. This should happen eventually starting at some t_c for theories with $\lambda_0 \geq (16\pi/21)(N_f + \frac{1}{4})\alpha_0$. Equation (58) indicates that $v^2(t)$ rapidly approaches zero but never reaches zero until the Landau ghost point. This implies that $\mu_\phi^2(t)$ first becomes more and more negative, turns around at $t=t_c$, but also never reaches zero until either $\lambda(t)$ or $\alpha(t)$ diverges.

V. REMARKS ON THE NON-ABELIAN GAUGE THEORY

In this section, we wish to generalize our considerations to non-Abelian gauge theories. Our focus is to discuss the validity of an assertion made by Lee and Weisberger [8] that, for a non-Abelian gauge field theory with Higgs mechanism which is asymptotically free, the spontaneously broken symmetry is restored at large Q^2 .

First, we recall from the paper of Lee and Weisberger [8] that they have considered the asymptotic symmetry

restoration of a non-Abelian gauge theory as described by the Lagrangian

$$L = \frac{1}{2} |\partial_\mu \phi - ig T^\alpha \phi A_\mu^\alpha|^2 - \frac{1}{2} m_0^2 |\phi|^2 - \frac{1}{4} \lambda_0 |\phi|^4 + L_{\text{gauge}}, \quad (62)$$

where $(T^\alpha)_{ab}$ is a real representation of the gauge group G , $(T^\alpha)^t = -T^\alpha$. They apparently had adopted a different regularization scheme and obtained, to one-loop order,

$$Z_S = 1 + \frac{g^2}{16\pi^2} \sum_\beta (-T^\beta T^\beta) (1 + \xi) \ln(m^2/\Lambda^2) + O(g^4), \quad (63)$$

with Λ some ultraviolet cutoff. The γ functions are calculated from the formula

$$\gamma_S = -\partial \ln Z_S / \partial \ln \Lambda^2, \quad \text{etc.} \quad (64)$$

They obtained

$$\gamma_S = \frac{g^2}{16\pi^2} \sum_\beta (-T^\beta T^\beta) (1 + \xi) + O(g^4), \quad (65)$$

and

$$\gamma_V = -\frac{g^2}{16\pi^2} \left(\frac{13}{3} - \xi \right) C_2(G) + \frac{g^2}{16\pi^2} \frac{1}{3} \text{Tr}(-T^\beta T^\beta)_{aa}, \quad (66)$$

where $C_2(G)$ is the quadratic Casimir operator of the adjoint representation.

In the last part of their paper, they tried to base on Eqs. (65) and (66) to prove, but incorrectly, an important assertion that, for a non-Abelian gauge theory with Higgs mechanism which is asymptotically free, the spontaneously broken symmetry is restored at large Q^2 . One important error comes from the determination of the fixed point for ξ (which they call α):

$$\xi_1 = \frac{g^2}{16\pi^2} [13C_2(G) + \text{Tr}(T^\beta T^\beta)], \quad (67')$$

which is determined from $\gamma_V = 0$ (implying $d\xi/dt = 0$). This equation was later used in their argument in the sense that $\xi \rightarrow 0$ as $g^2 \rightarrow 0$ in the limit of $t \rightarrow \infty$ for an asymptotic free theory.

In fact, it is easy to see from Eq. (66) that Eq. (67') is incorrect. It should read

$$\xi_1 = [13C_2(G) + \text{Tr}(T^\beta T^\beta)] / 3C_2(G), \quad (67)$$

which is positive for an asymptotic free theory. As the condition they wish to prove is given by

$$\lim_{t \rightarrow \infty} \gamma_S > -1, \quad (68)$$

the proof would have been complete if we could use their result on γ_S , Eq. (65). Here we note that $\text{Tr}(-T^\beta T^\beta) > 0$.

However, there are good reasons to cast doubt on the correctness of Eqs. (65) and (66). For example, the dependence of their γ_S on the gauge-fixing parameter ξ differs from that in the literature [27] (as well as from ours). If the dependence is the standard $(3 - \xi)$ instead of

their $(1 + \xi)$, the conclusion has to be different from theirs. In addition, the function $\xi(t)$ is tied to the running coupling $\alpha(t)$ through a formula such as Eq. (57), which is true even for non-Abelian theories. [In other words, the ξ -dependence in Eq. (66) also appears to differ from that in the literature.] Thus, Eq. (68) cannot hold if we use the factor $(3 - \xi)$ instead of $(1 + \xi)$. This seems to invalidate their proof of the statement that $v^2(t) \rightarrow 0$ as $t \rightarrow \infty$ (i.e., the broken symmetry is restored).

Nevertheless, we may offer an alternative proof which makes no use of the condition (68). Once we adopt a formula such as Eq. (57), we may in fact try to integrate out the RG equation governing $x(t)$. It will automatically contain a factor $\exp[-(1 + c_0 \alpha_0 \xi_0)t]$ for $\gamma_S = c_0 \xi \alpha + c_1 \alpha$. Thus, for a theory with $c_0 \alpha_0 \xi_0 > -1$, we have $v^2(t) \rightarrow 0$ as $t \rightarrow \infty$. The condition $c_0 \alpha_0 \xi_0 > -1$, instead of Eq. (68) advocated by Lee and Weisberger [7], is a more stringent condition to guarantee $v^2(t) \rightarrow 0$ as $t \rightarrow \infty$.

Once we know that $v^2(t) \rightarrow 0$ and $\alpha(t) \rightarrow 0$ as $t \rightarrow \infty$, we are assured that the Higgs sector, which may become strongly interacting, decouples completely from the sector containing the gauge fields and fermions. So we again have a simple theory. The remaining issue is the effect of Yukawa couplings between the fermions and the scalar fields, which may prevent decoupling between the Higgs sector and the gauge sector. It is therefore highly desirable if the Yukawa couplings are introduced such that the decoupling between the gauge sector and the strongly self-interacting Higgs sector will indeed occur at some Q^2 .

In concluding this section, we note that it is rather unlikely that the strongly interacting Higgs sector can be avoided altogether, since the various contributions to β_λ are determined primarily from the gauge principle. Inclusion of the Yukawa couplings of fermions to the scalar fields may introduce some additional loops but it will be a long way [11] to have any chance to compensate effects due to loops consisting of gauge fields or scalar fields. Thus, we believe that, although the gauge coupling and the vacuum expectation value may both approach zero as $Q^2 \rightarrow \infty$, the quartic coupling of the scalar fields diverges as already pointed out by Gross and Wilczek [10] and by Cheng, Eichten, and Li [11]. Nevertheless, we wish to suggest looking for theories that allow for eventual decoupling between the Higgs sector and the gauge sector. We believe that this is an important, as well as urgent, issue for a realistic theory such as the GSW electroweak theory. As a simple exercise, we use scalar QED with SSB to illustrate our point. The value of λ is given, in the GSW theory, by

$$\lambda = \frac{\pi \alpha M_H^2}{2M_Z^2 \sin^2 \theta_W \cos^2 \theta_W}. \quad (69)$$

Using the currently "believed" value for M_H (say, > 130 GeV), we immediately come to realize, if we can borrow from scalar QED with SSB Eq. (59) with $Q_0^2 \approx 1 \text{ GeV}^2$, that the interactions in the Higgs sector are already very strong ($> +0.5$) somewhere in the TeV range.

VI. SUMMARY

The RG approach generalized by Lee and Weisberger [8] to the case of gauge theories with spontaneous symmetry breaking has been adopted in this work in order to study the large- Q^2 behavior of the theory. The approach amounts to generalization of the well-known concept of “running coupling constants” to all the couplings involved in the theory.

In particular, we have considered scalar electrodynamics (QED) in which a U(1) gauge field interacts with both fermions and scalar fields (the latter responsible for spontaneous symmetry breaking). The model was illustrated in detail in Sec. II. We have solved explicitly the coupled RG equations to determine how the various parameters (couplings), including both the dimensionless and dimensional ones, behave as $Q^2 \rightarrow \infty$. The focus in the past [10,11] has been primarily with the dimensionless couplings, except in the work of Lee and Weisberger [8] where the vacuum expectation value $v(Q^2)$ was discussed in the context of the RG equation. [Thus, the result on the running of the coupling constant $\alpha(t)$ as well as on the growth of the quartic coupling $\lambda(t)$ is well known.]

In Sec. III, we have carried out a perturbative calculation including all one-loop diagrams with the explicit results on the various renormalization constants obtained by separating out pole parts ($\propto 1/\epsilon$) of the various diagrams regulated in the dimensional-regularization scheme [4]. Such explicit results, which unfortunately were not touched upon at all in the paper of Appelquist *et al.* [7], in fact make it very easy to understand how the renormalization program described in the paper of Appelquist *et al.* [7] works in detail. In particular, one sees how introduction of a small number of renormalization constants indeed renormalizes the theory in a consistent manner. By comparing term-by-term one-loop results in scalar QED with spontaneous symmetry breaking (SSB) [$\mu^2 < 0$ in Eq. (3c)] with those in scalar QED without SSB ($\mu^2 > 0$), we also find [15] that the constants which renormalize the theory are independent of whether or not the symmetry is spontaneously broken.

In Sec. IV, we have used the explicit results on the renormalization constants to extract the β and γ_i functions which govern the behavior of the theory through the RG equation as $Q^2 \rightarrow \infty$. We find that the coupled RG equations can be solved explicitly, yielding the well-known result that the coupling constant $\lambda(Q^2)$ in the $\lambda\phi^4$ term may diverge at some Q^2 well before the Landau ghost problem becomes relevant. In addition, we find that the vacuum expectation value $v^2(Q^2)$ vanishes as $Q^2 \rightarrow \infty$ (i.e., the broken symmetry is restored) but the gauge sector (consisting of the gauge field and fermions) does not decouple from the strongly self-interacting Higgs sector.

In Sec. V, we have also generalized our considerations to non-Abelian gauge theories. Our focus was to discuss the validity of an assertion made by Lee and Weisberger [8] that, for a non-Abelian gauge field theory with Higgs mechanism which is asymptotically free, the spontaneously broken symmetry is restored at large Q^2 . Upon looking into their proof closely, we believe that, although the gauge coupling and the vacuum expectation value

may both approach zero as $Q^2 \rightarrow \infty$, the quartic coupling of the scalar fields diverges as already pointed out by Gross and Wilczek [10] and by Cheng, Eichten, and Li [11]. Nevertheless, we suggest looking for theories which allow for eventual decoupling between the strongly interacting Higgs sector and the asymptotically free gauge sector.

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APPENDIX A: RELATIONS AMONG THE COEFFICIENTS OF THE POTENTIAL $V(\phi)$

In this appendix, we list those algebraic relations among the coefficients of the potential $V(\phi)$ [Eqs. (4)–(6) in the text] which are useful for reducing sums of the loop results to simple forms. Note that τ and v_{lj} have been defined by Eq. (8) in the text. We also introduce

$$\tilde{f}_{ijm} \equiv f_{ijmn} v_{ln} . \quad (A1)$$

In addition to Eqs. (10) in the text, we have

$$f_{ijk} \delta_{jk} = 8\lambda v_i , \quad (A2)$$

$$f_{ijmn} \delta_{mn} = 8\lambda \delta_{ij} , \quad (A3)$$

$$\delta_{ij} \tau_{ij} = 0 , \quad (A4)$$

$$f_{ijk} \tau_{jk} = 0 , \quad (A5)$$

$$f_{ijmn} \tau_{mn} = 0 , \quad (A6)$$

$$f_{ijk} v_k = 2\lambda [3v_i v_j + v_{li} v_{lj}] , \quad (A7)$$

$$f_{ijmn} v_n = f_{ijm} , \quad (A8)$$

$$f_{ijk} v_{lk} = 2\lambda (v_i v_{lj} + v_{li} v_{lj}) , \quad (A9)$$

$$\tilde{f}_{ijk} \delta_{jk} = 8\lambda v_{li} , \quad (A10)$$

$$\tilde{f}_{ijk} \tau_{jk} = 0 , \quad (A11)$$

$$\tilde{f}_{ijk} v_k = 2\lambda (v_i v_{lj} + v_{li} v_{lj}) , \quad (A12)$$

$$\tilde{f}_{ijk} v_{lk} = \lambda (2v_i v_j + 6v_{li} v_{lj}) , \quad (A13)$$

$$f_{imn} f_{jmn} = 8\lambda^2 (5v_i v_j + v_{li} v_{lj}) , \quad (A14)$$

$$\delta_{ij} v_k + \delta_{ik} v_j + \delta_{jk} v_i = \frac{1}{2\lambda} f_{ijk} , \quad (A15)$$

$$\delta_{ij} \delta_{mn} + \delta_{im} \delta_{jn} + \delta_{in} \delta_{jm} = \frac{1}{2\lambda} f_{ijmn} , \quad (A16)$$

$$f_{imn} \tau_{mj} \tau_{nk} + f_{kmn} \tau_{mi} \tau_{nj} + f_{kmj} \tau_{mk} \tau_{ni} = f_{ijk} , \quad (A17)$$

$$f_{rsmn} \tau_{ri} \tau_{sj} + f_{rsmi} \tau_{rn} \tau_{sj} + f_{rsmj} \tau_{ri} \tau_{sn} \\ + f_{rsij} \tau_{rm} \tau_{sn} + f_{rsni} \tau_{rm} \tau_{sj} + f_{rsnj} \tau_{ri} \tau_{sm} = 2f_{ijmn} . \quad (A18)$$

Note that these relations with $\mathbf{v}$ set to zero hold for the theory without spontaneous symmetry breaking ($\mu^2 > 0$).

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