ELSEVIER

Available online at www.sciencedirect.com

ScienceDirect

Nuclear Physics B (Proc. Suppl.) 168 (2007) 350–352

**NUCLEAR PHYSICS B
PROCEEDINGS
SUPPLEMENTS**

www.elsevierphysics.com

A_4 Group and Tri-bimaximal Neutrino Mixing – A Renormalizable Model

Xiao-Gang He^a

^aDepartment of Physics and Center for Theoretical Sciences
National Taiwan University, Taipei, Taiwan

The tetrahedron A_4 group has been widely used in studying neutrino mixing matrix. It provides a natural framework of model building for the tri-bimaximal mixing matrix. In this class of models, it is necessary to have two Higgs fields, χ and χ' , transforming under A_4 as 3 with one of them having vacuum expectation values for the three components to be equal and another having only one of the components to be non-zero. These specific vev structures require separating χ and χ' from communicating with each other. The clash of the different vev structures for χ and χ' is the so called sequestering problem. In this work, I show that it is possible to construct renormalizable supersymmetric models producing the tri-bimaximal neutrino mixing with no sequestering problem.

The current data from neutrino oscillation experiments[1] can be described by three neutrino mixing. The mixing matrix V can be well fitted by the tri-bimaximal mixing of the form[2]

$$V_{tri-bi} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} P. \quad (1)$$

Here $P = \text{Diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3})$ is a Majorana phase matrix. Since an overall phase does not play a role in any physical process, only two of the $\alpha_{1,2,3}$ are physically independent.

The tri-bimaximal form for neutrino mixing was first proposed by Harrison, Perkins and Scott[2], and further studied by Xing[2]. Also independently proposed by He and Zee[2]. Many theoretical efforts have been made to produce such a mixing pattern. Among them theories based on A_4 symmetry provide some interesting examples[3–5]. Most of the attempts made in the literature assumed certain vacuum expectation value (vev) structures for Higgs fields without specific renormalizable models to realize them. Attempts to build renormalizable models have been made in Refs.[4,5]. Here I construct a realistic renormalizable model with supersymmetry (SUSY) which produces the tri-bimaximal mixing.

In addition to the standard $SU(3)_C \times SU(2)_L \times$

$U(1)_Y$ gauge symmetry, this model has additional global symmetries $A_4 \times Z_4 \times Z_3 \times Z_2$ acting on various fields. Under the global symmetries in order of A_4 , Z_4 , Z_3 , and Z_2 , the relevant lepton and Higgs fields transform as: $L(3, 1, 0, 1)$, $e^c(1 + 1' + 1'', 0, 2, 1)$, $N^c(3, 0, 1, 1)$, $E(3, 0, 1, 1)$, $E^c(3, 2, 2, 1)$, $H_u(1, 3, 2, 0)$, $H_d(1, 1, 1, 0)$, $\chi(3, 0, 0, 0)$, $\chi'(3, 0, 1, 0)$, $S(1, 0, 0, 0)$, $S'(1, 0, 1, 0)$, $S''(1, 2, 0, 0)$.

The A_4 group is the tetrahedron group. It has 12 elements with 4 inequivalent representations 1, $1'$, $1''$ and 3. The multiplication rules of these representations are $1 \times 1 = 1$, $1 \times 1' = 1'$, $1 \times 1'' = 1''$, $1' \times 1'' = 1$, $1' \times 1' = 1''$, $1'' \times 1'' = 1'$, $3 \times 3 = 1 + 1' + 1'' + 3 + 3$. The 1, $1'$, $1''$ and the two 3's formed from two 3's $a = (a_1, a_2, a_3)$ and $b = (b_1, b_2, b_3)$ are given by

$$\begin{aligned} 1 &: a_1 b_1 + a_2 b_2 + a_3 b_3, \\ 1' &: a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3, \\ 1'' &: a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3, \\ 3_s &: (a_2 b_3 + a_3 b_2, a_3 b_1 + a_1 b_3, a_1 b_2 + a_2 b_1), \\ 3_a &: (a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1). \end{aligned}$$

The Z_n charge N in the above are defined as $\exp[i2N\pi/n]$.

The superpotential relevant to lepton masses are given by: $W_Y = M_E E_i E_i^c S'' + f_e L_i E_i^c H_d + h_j^e E_i E_j^c \chi_k + \frac{1}{2} f_{S'} N_i^c N_i^c S' + \frac{1}{2} f_{ijk} N_i^c N_j^c \chi_k' +$

$f_\nu L_i N_i^c H_u$. The Z_2 is needed to prevent $N^c \chi'^2$ term which induces mixing between N^c and χ' with a non-zero vev for χ' and causes problem in obtaining the tri-bimaximal mixing.

The vev's of $\langle H_u \rangle = v_u$, $\langle H_d \rangle = v_d$, $\langle S' \rangle = v_{s'}$, $\langle S'' \rangle = v_{s''}$, and $\langle \chi(\chi') \rangle$ break the gauge symmetry and also the global symmetries. If $\langle \chi_i \rangle = x_i$ and $\langle \chi'_i \rangle = x'_i$ have the following form,

$$x_1 = x_2 = x_3 = v_\chi, \quad x'_1 = x'_3 = 0, \quad x'_2 = v_{\chi'}, \quad (2)$$

the mass matrices M_{eE} and $M_{\nu N}$ in the Lagrangian $L = -(e, E)M_{eE}(e^c, E^c)^T - (\nu^c, N)M_{\nu N}(\nu, N^c)^T$ are given by

$$\begin{aligned} M_{eE} &= \begin{pmatrix} 0 & M_{eE^c} \\ M_{e^c E} & M_{EE^c} \end{pmatrix}, \\ M_{\nu N} &= \begin{pmatrix} 0 & M_{\nu N^c} \\ M_{\nu^c N} & M_{NN^c} \end{pmatrix}, \end{aligned} \quad (3)$$

with

$$\begin{aligned} M_{eE^c} &= \begin{pmatrix} f_e v_d & 0 & 0 \\ 0 & f_e v_d & 0 \\ 0 & 0 & f_e v_d \end{pmatrix}, \\ M_{e^c E} &= \begin{pmatrix} h_1^e v_\chi & h_2^e v_\chi & h_3^e v_\chi \\ h_1^e v_\chi & h_2^e \omega v_\chi & h_3^e \omega^2 v_\chi \\ h_1^e v_\chi & h_2^e \omega^2 v_\chi & h_3^e \omega v_\chi \end{pmatrix}, \\ M_{EE^c} &= \begin{pmatrix} f_E v_{s''} & 0 & 0 \\ 0 & f_E v_{s''} & 0 \\ 0 & 0 & f_E v_{s''} \end{pmatrix}, \\ M_{\nu N^c} = M_{\nu^c N} &= \begin{pmatrix} f_\nu v_u & 0 & 0 \\ 0 & f_\nu v_u & 0 \\ 0 & 0 & f_\nu v_u \end{pmatrix}, \\ M_{NN^c} &= \begin{pmatrix} f_{s'} v_{s'} & 0 & f_{\chi'} v_{\chi'} \\ 0 & f_{s'} v_{s'} & 0 \\ f_{\chi'} v_{\chi'} & 0 & f_{s'} v_{s'} \end{pmatrix}. \end{aligned} \quad (4)$$

The above results in the following form for the light lepton mass matrices,

$$\begin{aligned} M_e &= U_L \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \\ M_\nu^{light} &= m_0 \begin{pmatrix} 1 & 0 & x \\ 0 & 1 - x^2 & 0 \\ x & 0 & 1 \end{pmatrix}, \\ U_L &= \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \end{aligned} \quad (5)$$

where the charged lepton masses $m_{e,\mu,\tau}$ are give by

$$m_i = \sqrt{3} \left(\frac{f_e v_d v_\chi / f_E v_{s''} h_i^e (1 + (h_i^e v_\chi)^2)}{(f_E v_{s''})^2} \right)^{-1/2},$$

$$\text{and } m_0 = f_\nu^2 v_u^2 f_{s'} v_{s'} / (f_{s'}^2 v_{s'}^2 - f_{\chi'}^2 v_{\chi'}^2), \quad x = -f_{\chi'} v_{\chi'} / f_{s'} v_{s'} = |x| e^{i\psi}.$$

Diagonalizing the lepton mass matrices, we obtain the neutrino mixing matrix given by eq.(1). The Majorana phase matrix P is given by: $P = \text{Diag}(e^{-i\phi_1/2}, e^{-i(\phi_1+\phi_2)/2}, e^{-i(\phi_2+\pi)/2})$ with $\phi_1 = \arg(1+x)$, $\phi_2 = \arg(1-x)$. The eigen-masses are given by $m_1 = |m_0||1+x|$, $m_2 = |m_0||1-x^2|$ and $m_3 = |m_0||1-x|$. Both normal and inverted neutrino mass hierarchies are allowed[4].

In order to obtain the tri-bimaximal mixing it is crucial to have the χ and χ' representation to have the specific vev structure in eq.(2). One needs to make sure that this vev structure is obtainable in a given model. In the following we demonstrate that the model proposed here can have the desired vev structure.

Non-zero vev's of the Higgs break A_4 , but left some residual symmetries. The vev of χ with equal value for all three components breaks A_4 down to a Z_3 generated by $\{I, c, a\}$, and the vev of χ' with x'_2 non-zero breaks A_4 down to a Z_2 generated by $\{I, r_2\}$. Here a, c, r_2 are A_4 group elements defined in Ref.[3]. Acting on 3, these group elements are represented by

$$\begin{aligned} a &= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad c = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \\ r_2 &= \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \end{aligned} \quad (6)$$

If in the Higgs potential there are terms directly involve both χ and χ' , it is not possible to have the desired vev structure. This is the so called “sequestering” problem. To separate χ from communicating with χ' requires additional constraints. This is one of the crucial roles played by SUSY in this model. Without SUSY there is no way to forbid terms of the form $\chi^\dagger \chi \chi'^\dagger \chi'$ and therefore destroys the desired vev structure in

four dimensional renormalizable theories. With SUSY, potentials are derived from F-terms in the superpotential and D-terms involving gauge interactions. Terms of the type $\chi^\dagger \chi \chi'^\dagger \chi'$ are forbidden if one only allows renormalizable terms in the model.

In the model discussed here the relevant terms in the superpotential consistent with the global symmetry imposed is given by

$$\begin{aligned} W_V = & \lambda_{\chi s} \chi^2 S + \lambda_{\chi} \chi^3 + \lambda_{\chi' s'} \chi'^2 S' \\ & + \lambda_{\chi'} \chi'^3 + \lambda_s S^3 + \mu_s S^2 + \delta_s S + \lambda_{s'} S'^3 \\ & + \mu_{s''} S''^2 + \lambda_{s''} S''^2 S + \mu_{ud} H_u H_d. \end{aligned} \quad (7)$$

As is well known that soft SUSY breaking terms are need to construct phenomenologically consistent model, one needs to have these terms here too. Adding all terms which softly break SUSY but keep $A_4 \times Z_4 \times Z_3 \times Z_2$ symmetries, we have

$$\begin{aligned} V_{soft} = & b_1 \chi^\dagger \chi + b_2 \chi'^\dagger \chi' + b_3 S^\dagger S + b_4 S'^\dagger S' \\ & + b_5 S''^\dagger S'' + \mu_u^2 H_u^\dagger H_u + \mu_d^2 H_d^\dagger H_d \\ & + c_1 \chi^2 S + c_2 \chi^3 + c_3 \chi'^2 S' + c_4 \chi'^3 \\ & + c_5 S^3 + c_6 S^2 + c_7 S + c_8 S'^3 \\ & + c_9 S''^2 + c_{10} S''^2 S + c_{11} H_u H_d. \end{aligned} \quad (8)$$

This model differs the model in Ref.[4] in that the global symmetries are not broken by soft SUSY breaking terms.

Using the stationary conditions of the Higgs potential, we obtain

$$\begin{aligned} x_2 \frac{\partial V}{\partial x_1} - x_1 \frac{\partial V}{\partial x_2} &= -2(x_1^2 - x_2^2)(\lambda_{\chi}^2 x_1 x_2 \\ &+ 6\lambda_{\chi} \lambda_{\chi s} v_s x_3 + c_2 x_3) = 0, \\ x_2 \frac{\partial V}{\partial x_3} - x_3 \frac{\partial V}{\partial x_2} &= -2(x_3^2 - x_2^2)(\lambda_{\chi}^2 x_3 x_2 \\ &+ 6\lambda_{\chi} \lambda_{\chi s} v_s x_1 + c_2 x_1) = 0, \\ x_2 \frac{\partial V}{\partial x_1} - x_1 \frac{\partial V}{\partial x_2} &= -2(x_1^2 - x_2^2)(\lambda_{\chi}^2 x_1 x_3 \\ &+ 6\lambda_{\chi} \lambda_{\chi s} v_s x_2 + c_2 x_2) = 0, \\ x'_2 \frac{\partial V}{\partial x'_1} - x'_1 \frac{\partial V}{\partial x'_2} &= -2(x'^2_1 - x'^2_2)(\lambda_{\chi'}^2 x'_1 x'_2 \\ &+ 6\lambda_{\chi'} \lambda_{\chi' s} v_{s'} x'_3 + c_4 x'_3) = 0, \\ x'_2 \frac{\partial V}{\partial x'_3} - x'_3 \frac{\partial V}{\partial x'_2} &= -2(x'^2_3 - x'^2_2)(\lambda_{\chi'}^2 x'_3 x'_2 \end{aligned}$$

$$\begin{aligned} &+ 6\lambda_{\chi'} \lambda_{\chi' s} v_{s'} x'_1 + c_4 x'_1) = 0, \\ x'_3 \frac{\partial V}{\partial x'_1} - x'_1 \frac{\partial V}{\partial x'_3} &= -2(x'^2_1 - x'^2_3)(\lambda_{\chi'}^2 x'_1 x'_3 \\ &+ 6\lambda_{\chi'} \lambda_{\chi' s} v_{s'} x'_2 + c_4 x'_2) = 0. \end{aligned} \quad (9)$$

From the above one sees clearly that it is possible to have $\langle \chi \rangle$ to be of the form $x_1 = x_2 = x_3 = v_{\chi}$, and $\langle \chi' \rangle$ to be of the form $x'_1 = x'_3 = 0$, $x'_2 = v_{\chi'}$, at the minimal of the potential. The correct vev structure to produce the tri-bimaximal neutrino mixing pattern has, therefore, been obtained.

I finally comment on the quark mixing. If the quark fields are assigned under the $A_4 \times Z_4 \times A_3 \times Z_2$ as: $Q(1, 1, 0, 1)$, $U^c(1, 0, 1, 1)$, $D^c(1, 2, 2, 1)$, one would obtain a superpotential, $W_Q = Q\lambda_U H_u U^c + Q\lambda_D H_d D$. This superpotential then gives an unconstrained quark mixing. Efforts have been made in Ref.[5] to explain the small off-diagonal elements in quark mixing by requiring that they are zero at tree level, but generated at loop levels. The model constructed here has a simpler Higgs sector, although less predictions for quark mixing.

I thank Babu, Keum, Volkas and Zee for collaborations on related subjects reported here. This work is partly supported by NSC and NCTS.

REFERENCES

1. W.-M. Yao, et al., *Jou. Phys.* **G33**, 1(2006); G. Fogli et al, hep-ph/0608060.
2. P. F. Harrison, D.H. Perkins and W.G. Scott, *Phys. Lett.* **B530**, 167(2002); Z.-Z. Xing, *Phys. Lett.* **B533**, 85(2002); X.-G. He and A. Zee, *Phys. Lett.* **B560**, 87(2003).
3. E. Ma and G. Rajasekaran, *Phys. Rev.* **D64**, 113012 (2001); K. S. Babu, E. Ma, and J. W. F. Valle, *Phys. Lett.* **B552**, 207 (2003); G. Altarelli and F. Feruglio, *Nucl. Phys.* **B720**, 64(2005); E. Ma, *Phys. Rev.* **D72**, 037301 (2005); A. Zee, *Phys. Lett.* **B630**, 58(2005); X.-G. He and A. Zee, hep-ph/0607163.
4. K. S. Babu and X.-G. He, hep-ph/0507217.
5. C. I. Low and R. R. Volkas, *Phys. Rev.* **D68**, 033007 (2003); X.-G. He, Y.-Y. Keum and R. Volkas, *JHEP*, 0604:039(2006).