Hadronic decays involving heavy pentaquarks

Xiao-Gang He*

Department of Physics, Peking University, Beijing, China

Xue-Qian Li†

Department of Physics, Nankai University, Tianjin, China

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Recently several experiments have reported evidence for pentaquark Θ^+ . H1 experiment at HERA-B has also reported evidence for Θ_c . Θ^+ is interpreted as a bound state of an $\bar{s}$ with four other light quarks $udud$ which is a member of the antidecuplet of flavor $SU(3)_f$, while Θ_c is a heavy state obtained with the $\bar{s}$ in Θ^+ being replaced by a $\bar{c}$ which is a member of an antisixtet. A heavy pentaquark triplet can also be formed with a $\bar{c}$ and four light quarks. Similarly heavy pentaquark antisixtet and triplet can be formed with the $\bar{c}$ being replaced by a $\bar{b}$. We study decay processes involving at least one heavy pentaquark using $SU(3)_f$ and estimate the decay widths for some decay modes. We find several relations for heavy pentaquark decays into another heavy pentaquark and a $B(B^*)$ or a $D(D^*)$ which can be tested in the future. The B can also decay through weak interaction to charmed heavy pentaquarks. We also study some B decay modes with a heavy pentaquark in the final states. Experiments at B factories will provide important information about the heavy pentaquark properties.

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I. INTRODUCTION

Recently several experiments have reported evidence for pentaquarks Θ^+ and other states [1], although there are also experiments reporting null results [2]. The $\Theta^+(1540)$ pentaquark has strangeness $S=+1$ and has quark content $udud\bar{s}$. This particle is an isosinglet which is a member of the antidecuplet [3] in flavor $SU(3)_f$ symmetry. At present there is very limited experimental information on the detailed properties such as the decay width, spin, and parity. Several models have been proposed to accommodate these antidecuplet states [3–13].

Replacing the $\bar{s}$ in Θ^+ by a heavy quark such as $\bar{c}$ or $\bar{b}$, it is also possible to form bound heavy pentaquark states [5,6,8,10–12,14–18] Θ_c or Θ_b . When implementing them into $SU(3)_f$, in the model of Jaffe and Wilczek [5], where pentaquark Θ^+ is composed of two (ud) diquarks with spin-0 and an $\bar{s}$ quark, heavy pentaquarks form a fundamental representation of $SU(3)_f$ triplet $R_{c,b}$ (the subindices c and b indicate whether the pentaquark is formed with a $\bar{c}$ or a $\bar{b}$), and an antisixtet $S_{c,b}$ [10]. Discovery of these states and a study of their properties can provide important information about the inner structure of matter. H1 Collaboration has recently reported observation of a narrow resonance in Ref. [19] $D^{*-}p$ and $D^{*+}\bar{p}$ in inelastic eP collisions at center-of-mass energies of 300 and 320 GeV at HERA. This resonance has a mass of $3099 \pm 3(\text{stat}) \pm 5(\text{syst})$ with a Gaussian width of 12 ± 3 MeV and can be interpreted as evidence for Θ_c . If this observation is confirmed, other related

states should exist. In this paper we study some properties of decay processes involving at least one heavy pentaquark using $SU(3)_f$ flavor symmetry.

The triplet $R_{c,b}$ transforms under $SU(3)_f$ in a similar way as the light quark (u,d,s) triplet. To indicate this fact and also to distinguish the pentaquark triplet from the quark one, we use U, D, S to indicate the elements in $R_{c,b}$. For the antisixtet $S_{c,b}$, we use $T_{c,b}$, $N_{c,b}$, and $\Theta_{c,b}$ to indicate the isospin triplet, doublet, and singlet members, respectively. We have

$$\begin{aligned} R_c &= (R_{c,i}) = (U_c^0, D_c^-, S_c^-), \\ S_c &= (S_c^{ij}) = \begin{pmatrix} T_c^{--} & T_c^-/\sqrt{2} & N_c^-/\sqrt{2} \\ T_c^-/\sqrt{2} & T_c^0 & N_c^0/\sqrt{2} \\ N_c^-/\sqrt{2} & N_c^0/\sqrt{2} & \Theta_c^0 \end{pmatrix}, \\ R_b &= (R_{b,i}) = (U_b^+, D_b^0, S_b^0), \\ S_b &= (S_b^{ij}) = \begin{pmatrix} T_b^- & T_b^0/\sqrt{2} & N_b^0/\sqrt{2} \\ T_b^0/\sqrt{2} & T_b^+ & N_b^+/\sqrt{2} \\ N_b^0/\sqrt{2} & N_b^+/\sqrt{2} & \Theta_b^+ \end{pmatrix}. \end{aligned} \quad (1)$$

The quark contents of these particles are

$$U_{c,b}^{0,+} = (udus)(\bar{c}, \bar{b}), \quad D_{c,b}^{-,0} = (udds)(\bar{c}, \bar{b}),$$

$$S_{c,b}^{-,0} = (dsus)(\bar{c}, \bar{b}),$$

$$T_{c,b}^{--,-} = (dsds)(\bar{c}, \bar{b}), \quad T_{c,b}^{-,0} = (dsus)(\bar{c}, \bar{b}),$$

$$T_{c,b}^{0,+} = (usus)(\bar{c}, \bar{b}),$$

*On leave of absence from Physics Department, National Taiwan University; Electronic address: hexg@phys.ntu.edu.tw

†Electronic address: lixq@nankai.edu.cn

$$\begin{aligned}
N_{c,b}^{-,0} &= (udds)(\bar{c}, \bar{b}), \quad N_{c,b}^{0,+} = (udus)(\bar{c}, \bar{b}), \\
\Theta_{c,b}^{0,+} &= (udud)(\bar{c}, \bar{b}).
\end{aligned} \tag{2}$$

The $U_{c,b}$, $D_{c,b}$, and $N_{c,b}$ particles have $S = -1$, $S_{c,b}$, and $T_{c,b}$ particles have $S = -2$, and $\Theta_{c,b}$ particles have $S = 0$.

In the diquark model of Ref. [5], $S_{c,b}$ have positive parity, whereas $R_{c,b}$ have negative parity since there is no P -wave excitation between the diquarks. In our study we emphasize on the flavor $SU(3)_f$ properties, the conclusions can be applied to both parity situations “+” or “−” for both $R_{c,b}$ and $S_{c,b}$.

II. HEAVY PENTAQUARK STRONG DECAY COUPLINGS

Whether heavy pentaquarks can have strong decay modes depends on their masses. With $SU(3)_f$ symmetry, particles in each multiplet are supposed to have the same mass. Quark model estimates for the masses of heavy pentaquarks have been carried out by several groups. In the diquark model, the $\Theta_{c,b}$ masses are estimated to be [5] 2710 and 6050 MeV, respectively, which are below the strong pD and nB decay threshold. A lattice calculation in Ref. [8], gives a m_{Θ_c} of about 3.5 GeV.

Removing the P -wave excitation energy, which is estimated by using the mass difference of Λ_c and its excitation Λ'_c , $U_{P\text{-wave}} \approx m_{\Lambda'_c} - m_{\Lambda_c} = 310$ MeV, from Θ_c , and adding a constituent strange quark contribution $\Delta_s = m_{\Xi'_c} - m_{\Lambda_c} \approx 184$ MeV, Ref. [15] obtained 2580 MeV for U_c, D_c masses. Assuming the same $U_{P\text{-wave}}$ and Δ_s for beauty heavy pentaquarks, U_b, D_b masses are estimated to be 5920 MeV [15].

The mass degeneracy for the particles in a multiplet is lifted by quark mass differences m_u , m_d , and m_s . The mass terms, up to linear corrections in light quark masses, are given by

$$\begin{aligned}
L &= m_0^{R_{c,b}} \text{Tr}(\bar{R}_{c,b} R_{c,b}) + \alpha_m^{R_{c,b}} \text{Tr}[\bar{R}_{c,b} (M + M^\dagger) R_{c,b}] \\
&\quad + m_0^{S_{c,b}} \text{Tr}(\bar{S}_{c,b} S_{c,b}) + \alpha_m^{S_{c,b}} \text{Tr}[\bar{S}_{c,b} (M + M^\dagger) S_{c,b}].
\end{aligned} \tag{3}$$

We have neglected terms of the form $\text{Tr}[\bar{R}(\bar{S})R(S)]\text{Tr}(M)$ which only rescale m_0 . M is the quark mass matrix and is given by

$$M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}. \tag{4}$$

Neglecting small $m_{u,d}$ masses, we obtain

$$\begin{aligned}
m_{U_{c,b}} &= m_{D_{c,b}} = m_0^{R_{c,b}}, \quad m_{S_{c,b}} = m_0^{R_{c,b}} + 2\alpha_m^{R_{c,b}} m_s, \\
m_{T_{c,b}} &= m_0^{S_{c,b}}, \quad m_{N_{c,b}} = m_0^{S_{c,b}} + \alpha_m^{S_{c,b}} m_s, \\
m_{\Theta_{c,b}} &= m_0^{S_{c,b}} + 2\alpha_m^{S_{c,b}} m_s.
\end{aligned} \tag{5}$$

Taking into account the $SU(3)_f$ breaking effects via differences in light quark masses from $\Delta_s = m_{\Xi'_c} - m_{\Lambda_c}$ for constituent strange quarks, the masses of S_c^- and S_b^0 were estimated to be 2770 and 6100 MeV, respectively in Ref. [15]. Making the same assumption we obtain the masses of N_c , N_b , T_c , and T_b to be 2894, 6236, 3078, and 6420 MeV, respectively. These values are similar to the estimates obtained in Ref. [16].

There are other model estimates for heavy pentaquark masses which give larger masses. For example, in the model of Karliner and Lipkin, where the pentaquarks are formed from a triquark and a diquark [6], the masses are estimated to be 2985 and 6398 MeV, respectively, which are above the strong pD and nB decay threshold. And the masses of N_c , N_b , T_c , and T_b are estimated to be 3165, 6570, 3340, and 6740 MeV, respectively [16]. Removing the P -wave excitation energy [6] $\delta E^{P\text{-wave}} \approx 207$ MeV between the diquark and triquark from $\Theta_{c,b}$ and adding the mass difference due to the replacement of a light u or d quark by an s quark, one obtains the masses of U_c^0, D_c^- and U_b^+, D_b^0 to be 2858 and 6533 MeV, respectively. S_c^- and S_b^0 are approximately 3028 and 6708 MeV. Clearly the above estimates for the masses are rather rough and should not be expected to hold with an accuracy better than 50 or even 100 MeV.

If the H1 narrow resonance of mass 3099 MeV is indeed the Θ_c particle, both the diquark and triquark-diquark model predictions for the mass are slightly lower than the data. There is also the possibility that the narrow resonant state observed at H1 is a chiral partner of Θ_c . At present the uncertainties involved in the estimates are large so it is too early to make a decisive conclusion. With a mass 3099 MeV for Θ_c , it is possible for it to decay into $D^{*-}p$ and $D^+\bar{p}$. Similar situation may happen for beauty pentaquarks. We therefore will consider processes involving both D , B and D^* , B^* .

We now write down the strong decay amplitudes to the leading order using the $SU(3)_f$ symmetry for heavy pentaquark decays with a B or D in the final states. We have

$$\begin{aligned}
L_{R_b NB} &= c_{R_b NB} \bar{R}_b^i N_j^i \bar{B}_i + \text{H.c.}, \\
L_{S_b NB} &= c_{S_b NB} \bar{S}_b^i N_j^i \bar{B}_i \epsilon^{ikl} + \text{H.c.}, \\
L_{R_c ND} &= c_{R_c ND} \bar{R}_c^i N_j^i \bar{D}_i + \text{H.c.}, \\
L_{S_c ND} &= c_{S_c ND} \bar{S}_c^i N_j^i \bar{D}_i \epsilon^{ikl} + \text{H.c.}
\end{aligned} \tag{6}$$

In the above N is the ordinary baryon octet, D^i and B^i are the charm and beauty mesons. They are given by

TABLE I. Couplings for $B\bar{N}R_b(D\bar{N}R_c)$ in unit $c_{RNB}(c_{RND})$. The Lorentz structure for the bi-spinor product is of the form $\bar{N}\Gamma_p R$ with $\Gamma_p = +1$ and γ_5 for negative and positive parity for R , respectively. For $B^*\bar{N}R_b$, the Lorentz structure for the bispinor product should be changed to $\bar{N}\gamma_\mu\gamma_5\Gamma_p R_b$.

B_u	$\left(\frac{1}{\sqrt{6}}\bar{\Lambda} + \frac{1}{2}\bar{\Sigma}^0\right)U_b^+ + \bar{\Sigma}^-D_b^0 + \bar{\Xi}^-S_b^0$	D_u	$\left(\frac{1}{\sqrt{6}}\bar{\Lambda} + \frac{1}{2}\bar{\Sigma}^0\right)U_c^0 + \bar{\Sigma}^-D_c^- + \bar{\Xi}^-S_c^-$
B_d	$\bar{\Sigma}^+U_b^+ + \left(\frac{1}{\sqrt{6}}\bar{\Lambda} - \frac{1}{\sqrt{2}}\bar{\Sigma}^0\right)D_b^0 + \bar{\Xi}^0S_b^0$	D_d	$\bar{\Sigma}^+U_c^0 + \left(\frac{1}{\sqrt{6}}\bar{\Lambda} - \frac{1}{\sqrt{2}}\bar{\Sigma}^0\right)D_c^- + \bar{\Xi}^0S_c^-$
B_s	$\bar{p}U_b^+ + \bar{n}D_b^0 - \sqrt{\frac{2}{3}}\bar{\Lambda}S_b^0$	D_s	$\bar{p}U_c^0 + \bar{n}D_c^- - \sqrt{\frac{2}{3}}\bar{\Lambda}S_c^-$

$$N = (N_i^j) = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}.$$

$$D = (D^i) = (D_u, D_d, D_s) = (c\bar{u}, c\bar{d}, c\bar{s}),$$

$$B = (B^i) = (B_u, B_d, B_s) = (b\bar{u}, b\bar{d}, b\bar{s}). \quad (7)$$

In Tables I and II, we list the couplings for $\bar{N}R_b(S_b)B$. One can obtain the couplings for $\bar{N}R_c(S_c)D$ by replacing B_i by D_i and the subindex b to c . Throughout the paper, in all the equations for pentaquark couplings only group indices are properly labeled and all fields in the Lagrangian are going outwards. The Lorentz structures are suppressed in the equations, while the proper ones are given in the tables. Also we will assume that heavy pentaquarks are spin-1/2 particles. To obtain results for spin-3/2 heavy pentaquarks, one can just use at an appropriate place the Rarita-Schwinger vector spinor for the relevant fields.

We would like to point out that the above equations can be equally applied to processes with B and D replaced by D^*

TABLE II. Couplings for $B\bar{N}S_b(D\bar{N}S_c)$ in unit $c_{SNB}(c_{SND})$. The Lorentz structure is the same as in Table I.

B_u	$\frac{1}{2}[\sqrt{2}\bar{\Xi}^-T_b^0 + 2\bar{\Xi}^0T_b^+ - \sqrt{2}\bar{\Sigma}^-N_b^0 - \sqrt{3}\bar{\Lambda}N_b^+ + \bar{\Sigma}^0N_b^+ - 2\bar{n}\Theta_b^+]$
B_d	$\frac{1}{2}[-2\bar{\Xi}^-T_b^- - \sqrt{2}\bar{\Xi}^0T_b^0 + \sqrt{2}\bar{\Sigma}^+N_b^+ + \sqrt{3}\bar{\Lambda}N_b^0 + \bar{\Sigma}^0N_b^0 + 2\bar{p}\Theta_b^+]$
B_s	$\frac{1}{2}[2\bar{\Sigma}^-T_b^- - 2\bar{\Sigma}^0T_b^0 - 2\bar{\Sigma}^+T_b^+ - \sqrt{2}\bar{p}N_b^+ + \sqrt{2}\bar{n}N_b^0]$
D_u	$\frac{1}{2}[\sqrt{2}\bar{\Xi}^-T_c^- + 2\bar{\Xi}^0T_c^0 - \sqrt{2}\bar{\Sigma}^-N_c^- - \sqrt{3}\bar{\Lambda}N_c^0 + \bar{\Sigma}^0N_c^0 - 2\bar{n}\Theta_c^0]$
D_d	$\frac{1}{2}[-2\bar{\Xi}^-T_c^- - \sqrt{2}\bar{\Xi}^0T_c^- + \sqrt{2}\bar{\Sigma}^+N_c^0 + \sqrt{3}\bar{\Lambda}N_c^- + \bar{\Sigma}^0N_c^- + 2\bar{p}\Theta_c^0]$
D_s	$\frac{1}{2}[2\bar{\Sigma}^-T_c^- - 2\bar{\Sigma}^0T_c^- - 2\bar{\Sigma}^+T_c^0 - \sqrt{2}\bar{p}N_c^0 + \sqrt{2}\bar{n}N_c^-]$

and B^* , respectively. We will not distinguish them in the equations in later discussions unless specifically indicated.

If the diquark model for pentaquarks is the right one, we see that all the strong decay modes are forbidden due to restriction of phase space. However, if the masses are close to the triquark-diquark model predictions, strong decays are allowed. The H1 data indicate that the above strong decays are possible. One can use the allowed decay modes to determine the parameter c_{abc} and then the widths of the heavy pentaquarks. Here we give the formula for the couplings in terms of decay widths assuming the decays are allowed. For decays with B in the final states we have

$$c_{R_bNB}^2 = \frac{16\pi m_{S_b^0}\Gamma(S_b^0 \rightarrow \Xi^- \bar{B}_u)}{[(\hat{\pi}m_{S_b^0} + m_{\Xi^-})^2 - m_{B_u}^2]Ph(m_{S_b^0}, m_{\Xi^-}, m_{B_u})},$$

$$c_{S_bNB}^2 = \frac{16\pi m_{\Theta_b^+}\Gamma(\Theta_b^+ \rightarrow p \bar{B}_d)}{[(\hat{\pi}m_{\Theta_b^+} + m_p)^2 - m_{B_d}^2]Ph(m_{\Theta_b^+}, m_p, m_{B_d})}, \quad (8)$$

where $Ph(a, b, c) = \sqrt{1 - (b+c)^2/a^2}(1 - (b-c)^2/a^2)$. $\hat{\pi}$ is the eigenvalue of parity of the heavy pentaquark.

For decays with B^* in the final states, we have

$$c_{R_bNB^*}^2 = \frac{16\pi m_{S_b^0}\Gamma(S_b^0 \rightarrow \Xi^- \bar{B}_u^*)}{f(m_{S_b^0}, m_{\Xi^-}, m_{B_u^*})Ph(m_{S_b^0}, m_{\Xi^-}, m_{B_u^*})},$$

$$c_{S_bNB^*}^2 = \frac{16\pi m_{\Theta_b^+}\Gamma(\Theta_b^+ \rightarrow p \bar{B}_d^*)}{f(m_{\Theta_b^+}, m_p, m_{B_d^*})Ph(m_{\Theta_b^+}, m_p, m_{B_d^*})}, \quad (9)$$

where

$$f(a, b, c) = a^2 + b^2 - c^2 - \hat{\pi}6ab + [(a^2 - b^2)^2 - c^4]/c^2.$$

Similarly one can obtain $c_{R_c(S_c)ND(D^*)}^2$ by considering $S_c^- \rightarrow \bar{D}_u^0(D_u^{*0})\Xi^-$ and $\Theta_c^0 \rightarrow p\bar{D}_d^+(\bar{D}_d^{*+})$ decays, respectively. At present there is only some information on the width of $\Gamma(\Theta_c \rightarrow p\bar{D}_d^{*+})$. Assuming that the narrow resonant state of width 12 ± 3 MeV at H1 is the Θ_c particle, we obtain

$$c_{S_cND^*}^2 \approx \begin{cases} 1.712, & \hat{\pi} = 1, \\ 0.167, & \hat{\pi} = -1. \end{cases} \quad (10)$$

Using these numbers, the decay widths for decay modes in Table II involving D^* can be predicted. These predictions can be tested.

III. WEAK HADRONIC DECAYS OF HEAVY PENTAQUARKS

Heavy pentaquark can also decay through weak interaction. If kinematically the strong decays discussed in the previous section are not allowed, weak interaction will dominate heavy pentaquark decays. These decays can be semileptonic or purely hadronic ones. Analysis on some of the heavy pentaquark properties have been carried out [11,12,15–18]. Here we will concentrate on some two body hadronic heavy pentaquark decays.

A. $R_b(S_b) \rightarrow R_c(S_c) + \Pi$ decays

In this subsection we study pentaquark decays of the type $R_b(S_b) \rightarrow R_c(S_c)\Pi$. Here Π represents a meson in the pseudoscalar octet which is given by

$$\Pi = (\Pi_i^j) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}. \quad (11)$$

The quark level effective Hamiltonian for $R_b(S_b) \rightarrow R_c(S_c)\Pi$ is given by

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} [V_{cb}^* V_{uq} (c_1 O_1 + c_2 O_2) + V_{ub} V_{cq}^* (c_1 \tilde{O}_1 + c_2 \tilde{O}_2)], \quad (12)$$

$$O_1 = \bar{b} \gamma_\mu (1 - \gamma_5) c \bar{u} \gamma^\mu (1 - \gamma_5) q, \quad (13)$$

$$O_2 = \bar{b} \gamma_\mu (1 - \gamma_5) q \bar{u} \gamma^\mu (1 - \gamma_5) c, \quad (13)$$

$$\tilde{O}_1 = \bar{b} \gamma_\mu (1 - \gamma_5) u \bar{c} \gamma^\mu (1 - \gamma_5) q,$$

$$\tilde{O}_2 = \bar{b} \gamma_\mu (1 - \gamma_5) q \bar{c} \gamma^\mu (1 - \gamma_5) u. \quad (14)$$

The two operators $O_{1,2}$ in the above can induce decays of the type $R_b(S_b) \rightarrow R_c(S_c) + \Pi$, while the operators $\tilde{O}_{1,2}$ will not cause beauty heavy pentaquark to charmed pentaquark transitions. We write it down here for later discussions.

Under the $SU(3)_f$ symmetry $O_{1,2}$ transforms as an octet H_j^i . With proper normalization the nonzero entries of H_j^i can be written as

$$q=d, \quad H_2^1=1; \quad q=s, \quad H_3^1=1. \quad (15)$$

The $SU(3)_f$ invariant decay amplitudes are

$$H(R_b \rightarrow R_c \Pi) = V_{cb}^* V_{uq} [r_{81} \bar{R}_{b,i} R_c^i \Pi_k^j H_j^k + r_{82} \bar{R}_{b,i} R_c^j \Pi_k^i H_j^k + r_{83} \bar{R}_{b,i} R_c^j \Pi_k^i H_j^k],$$

$$H(R_b \rightarrow S_c \Pi) = V_{cb}^* V_{uq} [s_{81} \bar{R}_{b,i} S_{c,jl} \Pi_k^m H_m^l \epsilon^{ijk} + s_{82} \bar{R}_{b,i} S_{c,jl} \times \Pi_m^i H_k^l \epsilon^{jkm} + s_{83} \bar{R}_{b,i} S_{c,jl} \Pi_m^l H_k^i \epsilon^{jkm}]. \quad (16)$$

The amplitudes for $S_b \rightarrow R_c \Pi$ can be obtained by interchanging the indices b and c , and treating the processes as the charge conjugated ones in the second equation of Eq. (16).

The amplitudes for $S_b \rightarrow S_c \Pi$ can be written as

$$H(S_b \rightarrow S_c \Pi) = V_{cb}^* V_{uq} [s_1 \bar{S}_b^{ij} S_{c,ij} \Pi_k^l H_l^k + s_2 \bar{S}_b^{ij} S_{c,kl} \Pi_k^i H_l^j + s_3 \bar{S}_b^{ij} S_{c,kj} \Pi_k^l H_l^i + s_4 \bar{S}_b^{ij} S_{c,kj} \Pi_k^l H_l^i]. \quad (17)$$

We list the results in Tables III, IV, and V. One can easily generalize the above formulation to the case with the vector meson nonet ($\rho^{0,\pm}, K^{*,0,\pm}, \omega, \phi$).

B. $R_b(S_b) \rightarrow R_c(S_c) D(D^*)$

The effective Hamiltonian for these processes is given by

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{cq} (c_1 O_1^c + c_2 O_2^c). \quad (18)$$

Here $O_1^c = \bar{b} \gamma^\mu (1 - \gamma_5) c \bar{c} \gamma_\mu (1 - \gamma_5) q$ and $O_2^c = \bar{c} \gamma^\mu (1 - \gamma_5) q \bar{b} \gamma_\mu (1 - \gamma_5) c$. In the above we have neglected small penguin contributions. This effective Hamiltonian transforms as a triplet 3 with nonzero entries

$$q=d, H_2=1; \quad q=s, H_3=1. \quad (19)$$

The $SU(3)_f$ invariant amplitudes can be written as

$$H[R_b(S_b) \rightarrow R_c(S_c) D] = V_{cb}^* V_{cq} [r_{31} \bar{R}_b^i R_{c,i} H_j D^j + r_{32} \bar{R}_b^i H_i R_{c,j} D^j + r_{33} \bar{R}_b^i R_{c,j} D^j + r_{34} \bar{R}_b^i R_{c,j} D^j + r_{35} \bar{R}_b^i R_{c,j} D^j + r_{36} \bar{R}_b^i R_{c,j} D^j + r_{37} \bar{R}_b^i R_{c,j} D^j + r_{38} \bar{R}_b^i R_{c,j} D^j + r_{39} \bar{R}_b^i R_{c,j} D^j + r_{40} \bar{R}_b^i R_{c,j} D^j + r_{41} \bar{R}_b^i R_{c,j} D^j + r_{42} \bar{R}_b^i R_{c,j} D^j + r_{43} \bar{R}_b^i R_{c,j} D^j + r_{44} \bar{R}_b^i R_{c,j} D^j + r_{45} \bar{R}_b^i R_{c,j} D^j + r_{46} \bar{R}_b^i R_{c,j} D^j + r_{47} \bar{R}_b^i R_{c,j} D^j + r_{48} \bar{R}_b^i R_{c,j} D^j + r_{49} \bar{R}_b^i R_{c,j} D^j + r_{50} \bar{R}_b^i R_{c,j} D^j + r_{51} \bar{R}_b^i R_{c,j} D^j + r_{52} \bar{R}_b^i R_{c,j} D^j + r_{53} \bar{R}_b^i R_{c,j} D^j + r_{54} \bar{R}_b^i R_{c,j} D^j + r_{55} \bar{R}_b^i R_{c,j} D^j + r_{56} \bar{R}_b^i R_{c,j} D^j + r_{57} \bar{R}_b^i R_{c,j} D^j + r_{58} \bar{R}_b^i R_{c,j} D^j + r_{59} \bar{R}_b^i R_{c,j} D^j + r_{60} \bar{R}_b^i R_{c,j} D^j + r_{61} \bar{R}_b^i R_{c,j} D^j + r_{62} \bar{R}_b^i R_{c,j} D^j + r_{63} \bar{R}_b^i R_{c,j} D^j + r_{64} \bar{R}_b^i R_{c,j} D^j + r_{65} \bar{R}_b^i R_{c,j} D^j + r_{66} \bar{R}_b^i R_{c,j} D^j + r_{67} \bar{R}_b^i R_{c,j} D^j + r_{68} \bar{R}_b^i R_{c,j} D^j + r_{69} \bar{R}_b^i R_{c,j} D^j + r_{70} \bar{R}_b^i R_{c,j} D^j + r_{71} \bar{R}_b^i R_{c,j} D^j + r_{72} \bar{R}_b^i R_{c,j} D^j + r_{73} \bar{R}_b^i R_{c,j} D^j + r_{74} \bar{R}_b^i R_{c,j} D^j + r_{75} \bar{R}_b^i R_{c,j} D^j + r_{76} \bar{R}_b^i R_{c,j} D^j + r_{77} \bar{R}_b^i R_{c,j} D^j + r_{78} \bar{R}_b^i R_{c,j} D^j + r_{79} \bar{R}_b^i R_{c,j} D^j + r_{80} \bar{R}_b^i R_{c,j} D^j + r_{81} \bar{R}_b^i R_{c,j} D^j + r_{82} \bar{R}_b^i R_{c,j} D^j + r_{83} \bar{R}_b^i R_{c,j} D^j + r_{84} \bar{R}_b^i R_{c,j} D^j + r_{85} \bar{R}_b^i R_{c,j} D^j + r_{86} \bar{R}_b^i R_{c,j} D^j + r_{87} \bar{R}_b^i R_{c,j} D^j + r_{88} \bar{R}_b^i R_{c,j} D^j + r_{89} \bar{R}_b^i R_{c,j} D^j + r_{90} \bar{R}_b^i R_{c,j} D^j + r_{91} \bar{R}_b^i R_{c,j} D^j + r_{92} \bar{R}_b^i R_{c,j} D^j + r_{93} \bar{R}_b^i R_{c,j} D^j + r_{94} \bar{R}_b^i R_{c,j} D^j + r_{95} \bar{R}_b^i R_{c,j} D^j + r_{96} \bar{R}_b^i R_{c,j} D^j + r_{97} \bar{R}_b^i R_{c,j} D^j + r_{98} \bar{R}_b^i R_{c,j} D^j + r_{99} \bar{R}_b^i R_{c,j} D^j + r_{100} \bar{R}_b^i R_{c,j} D^j + r_{101} \bar{R}_b^i R_{c,j} D^j + r_{102} \bar{R}_b^i R_{c,j} D^j + r_{103} \bar{R}_b^i R_{c,j} D^j + r_{104} \bar{R}_b^i R_{c,j} D^j + r_{105} \bar{R}_b^i R_{c,j} D^j + r_{106} \bar{R}_b^i R_{c,j} D^j + r_{107} \bar{R}_b^i R_{c,j} D^j + r_{108} \bar{R}_b^i R_{c,j} D^j + r_{109} \bar{R}_b^i R_{c,j} D^j + r_{110} \bar{R}_b^i R_{c,j} D^j + r_{111} \bar{R}_b^i R_{c,j} D^j + r_{112} \bar{R}_b^i R_{c,j} D^j + r_{113} \bar{R}_b^i R_{c,j} D^j + r_{114} \bar{R}_b^i R_{c,j} D^j + r_{115} \bar{R}_b^i R_{c,j} D^j + r_{116} \bar{R}_b^i R_{c,j} D^j + r_{117} \bar{R}_b^i R_{c,j} D^j + r_{118} \bar{R}_b^i R_{c,j} D^j + r_{119} \bar{R}_b^i R_{c,j} D^j + r_{120} \bar{R}_b^i R_{c,j} D^j + r_{121} \bar{R}_b^i R_{c,j} D^j + r_{122} \bar{R}_b^i R_{c,j} D^j + r_{123} \bar{R}_b^i R_{c,j} D^j + r_{124} \bar{R}_b^i R_{c,j} D^j + r_{125} \bar{R}_b^i R_{c,j} D^j + r_{126} \bar{R}_b^i R_{c,j} D^j + r_{127} \bar{R}_b^i R_{c,j} D^j + r_{128} \bar{R}_b^i R_{c,j} D^j + r_{129} \bar{R}_b^i R_{c,j} D^j + r_{130} \bar{R}_b^i R_{c,j} D^j + r_{131} \bar{R}_b^i R_{c,j} D^j + r_{132} \bar{R}_b^i R_{c,j} D^j + r_{133} \bar{R}_b^i R_{c,j} D^j + r_{134} \bar{R}_b^i R_{c,j} D^j + r_{135} \bar{R}_b^i R_{c,j} D^j + r_{136} \bar{R}_b^i R_{c,j} D^j + r_{137} \bar{R}_b^i R_{c,j} D^j + r_{138} \bar{R}_b^i R_{c,j} D^j + r_{139} \bar{R}_b^i R_{c,j} D^j + r_{140} \bar{R}_b^i R_{c,j} D^j + r_{141} \bar{R}_b^i R_{c,j} D^j + r_{142} \bar{R}_b^i R_{c,j} D^j + r_{143} \bar{R}_b^i R_{c,j} D^j + r_{144} \bar{R}_b^i R_{c,j} D^j + r_{145} \bar{R}_b^i R_{c,j} D^j + r_{146} \bar{R}_b^i R_{c,j} D^j + r_{147} \bar{R}_b^i R_{c,j} D^j + r_{148} \bar{R}_b^i R_{c,j} D^j + r_{149} \bar{R}_b^i R_{c,j} D^j + r_{150} \bar{R}_b^i R_{c,j} D^j + r_{151} \bar{R}_b^i R_{c,j} D^j + r_{152} \bar{R}_b^i R_{c,j} D^j + r_{153} \bar{R}_b^i R_{c,j} D^j + r_{154} \bar{R}_b^i R_{c,j} D^j + r_{155} \bar{R}_b^i R_{c,j} D^j + r_{156} \bar{R}_b^i R_{c,j} D^j + r_{157} \bar{R}_b^i R_{c,j} D^j + r_{158} \bar{R}_b^i R_{c,j} D^j + r_{159} \bar{R}_b^i R_{c,j} D^j + r_{160} \bar{R}_b^i R_{c,j} D^j + r_{161} \bar{R}_b^i R_{c,j} D^j + r_{162} \bar{R}_b^i R_{c,j} D^j + r_{163} \bar{R}_b^i R_{c,j} D^j + r_{164} \bar{R}_b^i R_{c,j} D^j + r_{165} \bar{R}_b^i R_{c,j} D^j + r_{166} \bar{R}_b^i R_{c,j} D^j + r_{167} \bar{R}_b^i R_{c,j} D^j + r_{168} \bar{R}_b^i R_{c,j} D^j + r_{169} \bar{R}_b^i R_{c,j} D^j + r_{170} \bar{R}_b^i R_{c,j} D^j + r_{171} \bar{R}_b^i R_{c,j} D^j + r_{172} \bar{R}_b^i R_{c,j} D^j + r_{173} \bar{R}_b^i R_{c,j} D^j + r_{174} \bar{R}_b^i R_{c,j} D^j + r_{175} \bar{R}_b^i R_{c,j} D^j + r_{176} \bar{R}_b^i R_{c,j} D^j + r_{177} \bar{R}_b^i R_{c,j} D^j + r_{178} \bar{R}_b^i R_{c,j} D^j + r_{179} \bar{R}_b^i R_{c,j} D^j + r_{180} \bar{R}_b^i R_{c,j} D^j + r_{181} \bar{R}_b^i R_{c,j} D^j + r_{182} \bar{R}_b^i R_{c,j} D^j + r_{183} \bar{R}_b^i R_{c,j} D^j + r_{184} \bar{R}_b^i R_{c,j} D^j + r_{185} \bar{R}_b^i R_{c,j} D^j + r_{186} \bar{R}_b^i R_{c,j} D^j + r_{187} \bar{R}_b^i R_{c,j} D^j + r_{188} \bar{R}_b^i R_{c,j} D^j + r_{189} \bar{R}_b^i R_{c,j} D^j + r_{190} \bar{R}_b^i R_{c,j} D^j + r_{191} \bar{R}_b^i R_{c,j} D^j + r_{192} \bar{R}_b^i R_{c,j} D^j + r_{193} \bar{R}_b^i R_{c,j} D^j + r_{194} \bar{R}_b^i R_{c,j} D^j + r_{195} \bar{R}_b^i R_{c,j} D^j + r_{196} \bar{R}_b^i R_{c,j} D^j + r_{197} \bar{R}_b^i R_{c,j} D^j + r_{198} \bar{R}_b^i R_{c,j} D^j + r_{199} \bar{R}_b^i R_{c,j} D^j + r_{200} \bar{R}_b^i R_{c,j} D^j + r_{201} \bar{R}_b^i R_{c,j} D^j + r_{202} \bar{R}_b^i R_{c,j} D^j + r_{203} \bar{R}_b^i R_{c,j} D^j + r_{204} \bar{R}_b^i R_{c,j} D^j + r_{205} \bar{R}_b^i R_{c,j} D^j + r_{206} \bar{R}_b^i R_{c,j} D^j + r_{207} \bar{R}_b^i R_{c,j} D^j + r_{208} \bar{R}_b^i R_{c,j} D^j + r_{209} \bar{R}_b^i R_{c,j} D^j + r_{210} \bar{R}_b^i R_{c,j} D^j + r_{211} \bar{R}_b^i R_{c,j} D^j + r_{212} \bar{R}_b^i R_{c,j} D^j + r_{213} \bar{R}_b^i R_{c,j} D^j + r_{214} \bar{R}_b^i R_{c,j} D^j + r_{215} \bar{R}_b^i R_{c,j} D^j + r_{216} \bar{R}_b^i R_{c,j} D^j + r_{217} \bar{R}_b^i R_{c,j} D^j + r_{218} \bar{R}_b^i R_{c,j} D^j + r_{219} \bar{R}_b^i R_{c,j} D^j + r_{220} \bar{R}_b^i R_{c,j} D^j + r_{221} \bar{R}_b^i R_{c,j} D^j + r_{222} \bar{R}_b^i R_{c,j} D^j + r_{223} \bar{R}_b^i R_{c,j} D^j + r_{224} \bar{R}_b^i R_{c,j} D^j + r_{225} \bar{R}_b^i R_{c,j} D^j + r_{226} \bar{R}_b^i R_{c,j} D^j + r_{227} \bar{R}_b^i R_{c,j} D^j + r_{228} \bar{R}_b^i R_{c,j} D^j + r_{229} \bar{R}_b^i R_{c,j} D^j + r_{230} \bar{R}_b^i R_{c,j} D^j + r_{231} \bar{R}_b^i R_{c,j} D^j + r_{232} \bar{R}_b^i R_{c,j} D^j + r_{233} \bar{R}_b^i R_{c,j} D^j + r_{234} \bar{R}_b^i R_{c,j} D^j + r_{235} \bar{R}_b^i R_{c,j} D^j + r_{236} \bar{R}_b^i R_{c,j} D^j + r_{237} \bar{R}_b^i R_{c,j} D^j + r_{238} \bar{R}_b^i R_{c,j} D^j + r_{239} \bar{R}_b^i R_{c,j} D^j + r_{240} \bar{R}_b^i R_{c,j} D^j + r_{241} \bar{R}_b^i R_{c,j} D^j + r_{242} \bar{R}_b^i R_{c,j} D^j + r_{243} \bar{R}_b^i R_{c,j} D^j + r_{244} \bar{R}_b^i R_{c,j} D^j + r_{245} \bar{R}_b^i R_{c,j} D^j + r_{246} \bar{R}_b^i R_{c,j} D^j + r_{247} \bar{R}_b^i R_{c,j} D^j + r_{248} \bar{R}_b^i R_{c,j} D^j + r_{249} \bar{R}_b^i R_{c,j} D^j + r_{250} \bar{R}_b^i R_{c,j} D^j + r_{251} \bar{R}_b^i R_{c,j} D^j + r_{252} \bar{R}_b^i R_{c,j} D^j + r_{253} \bar{R}_b^i R_{c,j} D^j + r_{254} \bar{R}_b^i R_{c,j} D^j + r_{255} \bar{R}_b^i R_{c,j} D^j + r_{256} \bar{R}_b^i R_{c,j} D^j + r_{257} \bar{R}_b^i R_{c,j} D^j + r_{258} \bar{R}_b^i R_{c,j} D^j + r_{259} \bar{R}_b^i R_{c,j} D^j + r_{260} \bar{R}_b^i R_{c,j} D^j + r_{261} \bar{R}_b^i R_{c,j} D^j + r_{262} \bar{R}_b^i R_{c,j} D^j + r_{263} \bar{R}_b^i R_{c,j} D^j + r_{264} \bar{R}_b^i R_{c,j} D^j + r_{265} \bar{R}_b^i R_{c,j} D^j + r_{266} \bar{R}_b^i R_{c,j} D^j + r_{267} \bar{R}_b^i R_{c,j} D^j + r_{268} \bar{R}_b^i R_{c,j} D^j + r_{269} \bar{R}_b^i R_{c,j} D^j + r_{270} \bar{R}_b^i R_{c,j} D^j + r_{271} \bar{R}_b^i R_{c,j} D^j + r_{272} \bar{R}_b^i R_{c,j} D^j + r_{273} \bar{R}_b^i R_{c,j} D^j + r_{274} \bar{R}_b^i R_{c,j} D^j + r_{275} \bar{R}_b^i R_{c,j} D^j + r_{276} \bar{R}_b^i R_{c,j} D^j + r_{277} \bar{R}_b^i R_{c,j} D^j + r_{278} \bar{R}_b^i R_{c,j} D^j + r_{279} \bar{R}_b^i R_{c,j} D^j + r_{280} \bar{R}_b^i R_{c,j} D^j + r_{281} \bar{R}_b^i R_{c,j} D^j + r_{282} \bar{R}_b^i R_{c,j} D^j + r_{283} \bar{R}_b^i R_{c,j} D^j + r_{284} \bar{R}_b^i R_{c,j} D^j + r_{285} \bar{R}_b^i R_{c,j} D^j + r_{286} \bar{R}_b^i R_{c,j} D^j + r_{287} \bar{R}_b^i R_{c,j} D^j + r_{288} \bar{R}_b^i R_{c,j} D^j + r_{289} \bar{R}_b^i R_{c,j} D^j + r_{290} \bar{R}_b^i R_{c,j} D^j + r_{291} \bar{R}_b^i R_{c,j} D^j + r_{292} \bar{R}_b^i R_{c,j} D^j + r_{293} \bar{R}_b^i R_{c,j} D^j + r_{294} \bar{R}_b^i R_{c,j} D^j + r_{295} \bar{R}_b^i R_{c,j} D^j + r_{296} \bar{R}_b^i R_{c,j} D^j + r_{297} \bar{R}_b^i R_{c,j} D^j + r_{298} \bar{R}_b^i R_{c,j} D^j + r_{299} \bar{R}_b^i R_{c,j} D^j + r_{300} \bar{R}_b^i R_{c,j} D^j + r_{301} \bar{R}_b^i R_{c,j} D^j + r_{302} \bar{R}_b^i R_{c,j} D^j + r_{303} \bar{R}_b^i R_{c,j} D^j + r_{304} \bar{R}_b^i R_{c,j} D^j + r_{305} \bar{R}_b^i R_{c,j} D^j + r_{306} \bar{R}_b^i R_{c,j} D^j + r_{307} \bar{R}_b^i R_{c,j} D^j + r_{308} \bar{R}_b^i R_{c,j} D^j + r_{309} \bar{R}_b^i R_{c,j} D^j + r_{310} \bar{R}_b^i R_{c,j} D^j + r_{311} \bar{R}_b^i R_{c,j} D^j + r_{312} \bar{R}_b^i R_{c,j} D^j + r_{313} \bar{R}_b^i R_{c,j} D^j + r_{314} \bar{R}_b^i R_{c,j} D^j + r_{315} \bar{R}_b^i R_{c,j} D^j + r_{316} \bar{R}_b^i R_{c,j} D^j + r_{317} \bar{R}_b^i R_{c,j} D^j + r_{318} \bar{R}_b^i R_{c,j} D^j + r_{319} \bar{R}_b^i R_{c,j} D^j + r_{320} \bar{R}_b^i R_{c,j} D^j + r_{321} \bar{R}_b^i R_{c,j} D^j + r_{322} \bar{R}_b^i R_{c,j} D^j + r_{323} \bar{R}_b^i R_{c,j} D^j + r_{324} \bar{R}_b^i R_{c,j} D^j + r_{325} \bar{R}_b^i R_{c,j} D^j + r_{326} \bar{R}_b^i R_{c,j} D^j + r_{327} \bar{R}_b^i R_{c,j} D^j + r_{328} \bar{R}_b^i R_{c,j} D^j + r_{329} \bar{R}_b^i R_{c,j} D^j + r_{330} \bar{R}_b^i R_{c,j} D^j + r_{331} \bar{R}_b^i R_{c,j} D^j + r_{332} \bar{R}_b^i R_{c,j} D^j + r_{333} \bar{R}_b^i R_{c,j} D^j + r_{334} \bar{R}_b^i R_{c,j} D^j + r_{335} \bar{R}_b^i R_{c,j} D^j + r_{336} \bar{R}_b^i R_{c,j} D^j + r_{337} \bar{R}_b^i R_{c,j} D^j + r_{338} \bar{R}_b^i R_{c,j} D^j + r_{339} \bar{R}_b^i R_{c,j} D^j + r_{340} \bar{R}_b^i R_{c,j} D^j + r_{341} \bar{R}_b^i R_{c,j} D^j + r_{342} \bar{R}_b^i R_{c,j} D^j + r_{343} \bar{R}_b^i R_{c,j} D^j + r_{344} \bar{R}_b^i R_{c,j} D^j + r_{345} \bar{R}_b^i R_{c,j} D^j + r_{346} \bar{R}_b^i R_{c,j} D^j + r_{347} \bar{R}_b^i R_{c,j} D^j + r_{348} \bar{R}_b^i R_{c,j} D^j + r_{349} \bar{R}_b^i R_{c,j} D^j + r_{350} \bar{R}_b^i R_{c,j} D^j + r_{351} \bar{R}_b^i R_{c,j} D^j + r_{352} \bar{R}_b^i R_{c,j} D^j + r_{353} \bar{R}_b^i R_{c,j} D^j + r_{354} \bar{R}_b^i R_{c,j} D^j + r_{355} \bar{R}_b^i R_{c,j} D^j + r_{356} \bar{R}_b^i R_{c,j} D^j + r_{357} \bar{R}_b^i R_{c,j} D^j + r_{358} \bar{R}_b^i R_{c,j} D^j + r_{359} \bar{R}_b^i R_{c,j} D^j + r_{360} \bar{R}_b^i R_{c,j} D^j + r_{361} \bar{R}_b^i R_{c,j} D^j + r_{362} \bar{R}_b^i R_{c,j} D^j + r_{363} \bar{R}_b^i R_{c,j} D^j + r_{364} \bar{R}_b^i R_{c,j} D^j + r_{365} \bar{R}_b^i R_{c,j} D^j + r_{366} \bar{R}_b^i R_{c,j} D^j + r_{367} \bar{R}_b^i R_{c,j} D^j + r_{368} \bar{R}_b^i R_{c,j} D^j + r_{369} \bar{R}_b^i R_{c,j} D^j + r_{370} \bar{R}_b^i R_{c,j} D^j + r_{371} \bar{R}_b^i R_{c,j} D^j + r_{372} \bar{R}_b^i R_{c,j} D^j + r_{373} \bar{R}_b^i R_{c,j} D^j + r_{374} \bar{R}_b^i R_{c,j} D^j + r_{375} \bar{R}_b^i R_{c,j} D^j + r_{376} \bar{R}_b^i R_{c,j} D^j + r_{377} \bar{R}_b^i R_{c,j} D^j + r_{378} \bar{R}_b^i R_{c,j} D^j + r_{379} \bar{R}_b^i R_{c,j} D^j + r_{380} \bar{R}_b^i R_{c,j} D^j + r_{381} \bar{R}_b^i R_{c,j} D^j + r_{382} \bar{R}_b^i R_{c,j} D^j + r_{383} \bar{R}_b^i R_{c,j} D^j + r_{384} \bar{R}_b^i R_{c,j} D^j + r_{385} \bar{R}_b^i R_{c,j} D^j + r_{386} \bar{R}_b^i R_{c,j} D^j + r_{387} \bar{R}_b^i R_{c,j} D^j + r_{388} \bar{R}_b^i R_{c,j} D^j + r_{389} \bar{R}_b^i R_{c,j} D^j + r_{390} \bar{R}_b^i R_{c,j} D^j + r_{391} \bar{R}_b^i R_{c,j} D^j + r_{392} \bar{R}_b^i R_{c,j} D^j + r_{393} \bar{R}_b^i R_{c,j} D^j + r_{394} \bar{R}_b^i R_{c,j} D^j + r_{395} \bar{R}_b^i R_{c,j} D^j + r_{396} \bar{R}_b^i R_{c,j} D^j + r_{397} \bar{R}_b^i R_{c,j} D^j + r_{398} \bar{R}_b^i R_{c,j} D^j + r_{399} \bar{R}_b^i R_{c,j} D^j + r_{400} \bar{R}_b^i R_{c,j} D^j + r_{401} \bar{R}_b^i R_{c,j} D^j + r_{402} \bar{R}_b^i R_{c,j} D^j + r_{403} \bar{R}_b^i R_{c,j} D^j + r_{404} \bar{R}_b^i R_{c,j} D^j + r_{405} \bar{R}_b^i R_{c,j} D^j + r_{406} \bar{R}_b^i R_{c,j} D^j + r_{407} \bar{R}_b^i R_{c,j} D^j + r_{408} \bar{R}_b^i R_{c,j} D^j + r_{409} \bar{R}_b^i R_{c,j} D^j + r_{410} \bar{R}_b^i R_{c,j} D^j + r_{411} \bar{R}_b^i R_{c,j} D^j + r_{412} \bar{R}_b^i R_{c,j} D^j + r_{413} \bar{R}_b^i R_{c,j} D^j + r_{414} \bar{R}_b^i R_{c,j} D^j + r_{415} \bar{R}_b^i R_{c,j} D^j + r_{416} \bar{R}_b^i R_{c,j} D^j + r_{417} \bar{R}_b^i R_{c,j} D^j + r_{418} \bar{R}_b^i R_{c,j} D^j + r_{419} \bar{R}_b^i R_{c,j} D^j + r_{420} \bar{R}_b^i R_{c,j} D^j + r_{421} \bar{R}_b^i R_{c,j} D^j + r_{422} \bar{R}_b^i R_{c,j} D^j + r_{423} \bar{R}_b^i R_{c,j} D^j + r_{424} \bar{R}_b^i R_{c,j} D^j + r_{425} \bar{R}_b^i R_{c,j} D^j + r_{426} \bar{R}_b^i R_{c,j} D^j + r_{427} \bar{R}_b^i R_{c,j} D^j + r_{428} \bar{R}_b^i R_{c,j} D^j + r_{429} \bar{R}_b^i R_{c,j} D^j + r_{430} \bar{R}_b^i R_{c,j} D^j + r_{431} \bar{R}_b^i R_{c,j} D^j + r_{432} \bar{R}_b^i R_{c,j} D^j + r_{433} \bar{R}_b^i R_{c,j} D^j + r_{434} \bar{R}_b^i R_{c,j} D^j + r_{435} \bar{R}_b^i R_{c,j} D^j + r_{436} \bar{R}_b^i R_{c,j} D^j + r_{437} \bar{R}_b^i R_{c,j} D^j + r_{438} \bar{R}_b^i R_{c,j} D^j + r_{439} \bar{R}_b^i R_{c,j} D^j + r_{440} \bar{R}_b^i R_{c,j} D^j + r_{441} \bar{R}_b^i R_{c,j} D^j + r_{442} \bar{R}_b^i R_{c,j} D^j + r_{443} \bar{R}_b^i R_{c,j} D^j + r_{444} \bar{R}_b^i R_{c,j} D^j + r_{445} \bar{R}_b^i R_{c,j} D^j + r_{446} \bar{R}_b^i R_{c,j} D$$

TABLE III. $SU(3)$ decay amplitudes for $R_b \rightarrow R_c(S_c)\Pi$. The Lorentz structure of the bispinor product is of the form $\bar{R}_c(\bar{S}_c)(1 + b\gamma_5)R_b$. Here b is a parameter.

U_b^+ decay	$\Delta S=0$		$\Delta S=1$
$U_c^0 \pi^+$	$r_{81} + r_{83}$	$U_c^0 K^+$	$r_{81} + r_{83}$
$T_c^0 K^+$	$s_{81} - s_{82}$	$N_c^0 K^+$	$\frac{1}{\sqrt{2}}(s_{81} - s_{82})$
$N_c^0 \pi^+$	$-\frac{1}{\sqrt{2}}(s_{81} - s_{82})$	$\Theta_c^0 \pi^+$	$-(s_{81} - s_{82})$
D_b^0 decay			
$U_c^0 K^0$	$\frac{1}{\sqrt{2}}(r_{82} - r_{83})$	$U_c^0 \pi^0$	r_{83}
$U_c^0 \eta$	$\frac{1}{\sqrt{6}}(r_{82} + r_{83})$	$D_c^- K^+$	r_{81}
$D_c^- \pi^+$	$r_{81} + r_{82}$	$N_c^- K^+$	$-\frac{1}{\sqrt{2}}s_{81}$
$S_c^- K^+$	r_{82}	$N_c^0 K^0$	$-\frac{1}{\sqrt{2}}s_{82}$
$T_c^- K^+$	$-\frac{1}{\sqrt{2}}(s_{81} + s_{83})$	$\Theta_c^0 \pi^0$	$\frac{1}{\sqrt{2}}(s_{81} - s_{82})$
$T_c^0 K^0$	$-(s_{82} + s_{83})$	$\Theta_c^0 \eta$	$\frac{1}{\sqrt{6}}(s_{81} + s_{82})$
$N_c^- \pi^+$	$\frac{1}{\sqrt{2}}s_{83}$		
$N_c^0 \pi^0$	$\frac{1}{2}(s_{81} - s_{82} - s_{83})$		
$N_c^0 \eta$	$\frac{1}{2\sqrt{3}}(s_{81} + s_{82} + 3s_{83})$		
$\Theta_c^0 \bar{K}^0$	s_{83}		
S_b^0 decay			
$U_c^0 \bar{K}^0$	r_{83}	$U_c^0 \pi^0$	$\frac{1}{\sqrt{2}}r_{82}$
$S_c^- \pi^+$	r_{81}	$U_c^0 \eta$	$\frac{1}{\sqrt{6}}(2r_{82} - r_{83})$
$T_c^- \pi^+$	$\frac{1}{\sqrt{2}}s_{81}$	$D_c^- \pi^+$	r_{82}
$T_c^0 \pi^0$	$-\frac{1}{\sqrt{2}}s_{81}$	$S_c^- K^+$	$r_{81} + r_{82}$
$T_c^0 \eta$	$-\frac{1}{\sqrt{6}}(s_{81} - 2s_{82})$	$T_c^- K^+$	$-\frac{1}{\sqrt{2}}s_{83}$
$N_c^0 \bar{K}^0$	$\frac{1}{\sqrt{2}}s_{82}$	$T_c^0 K^0$	$-s_{83}$
		$N_c^- \pi^+$	$\frac{1}{\sqrt{2}}(s_{81} + s_{83})$
		$N_c^0 \pi^0$	$-\frac{1}{2}(s_{81} + s_{83})$
		$N_c^0 \eta$	$-\frac{1}{2\sqrt{3}}(s_{81} - 2s_{82} - 3s_{83})$
		$\Theta_c^0 \bar{K}^0$	$s_{82} + s_{83}$

The conjugate operators $\tilde{O}_{1,2}$ in Eq. (14) are of the form $\bar{q}b\bar{u}c$. These operators can induce $B \rightarrow \bar{N} + R_c(S_c)$. It contains an $SU(3)$ triplet and an antisixtet. The nonzero entries are

$$q=d, \quad H(3c)_3=1, \quad H(6c)^{12}=H(6c)^{21}=1;$$

$$q=s, \quad H(3c)_2=-1, \quad H(6c)^{13}=H(6c)^{31}=1. \quad (22)$$

One can write down $SU(3)_f$ decay amplitudes for $B \rightarrow \bar{N}R_c(S_c)$ as

$$H(R_c) = V_{ub}V_{cq}^*[\tilde{r}_{31}\bar{B}_iR_{cj}\bar{N}_l^iH_k\epsilon^{ilk} + \tilde{r}_{32}\bar{B}_iR_{cj}\bar{N}_l^jH_k\epsilon^{ilk} \\ + \tilde{r}_{61}\bar{B}_iR_{cj}\bar{N}_k^iH^{jk} + \tilde{r}_{62}\bar{B}_iR_{cj}\bar{N}_k^jH^{ik}],$$

$$H(S_c) = V_{ub}V_{cq}^*[\tilde{s}_{31}\bar{B}_iS_c^{ij}\bar{N}_j^iH_k + \tilde{s}_{32}\bar{B}_iS_c^{jk}\bar{N}_j^iH_k \\ + \tilde{s}_{61}\bar{B}_iS_c^{ij}\bar{N}_l^kH^{lm}\epsilon_{jkm} + \tilde{s}_{62}\bar{B}_iS_c^{jk}\bar{N}_l^iH^{lm}\epsilon_{jlm}]. \quad (23)$$

The hadronic parameters $\tilde{r}_{ij}$ are expected to be similar, $\Gamma[B \rightarrow \bar{N}R_c(S_c)]$ would be smaller by a factor of $|V_{cb}V_{uq}^*|^2/|V_{ub}V_{cq}^*|^2$ compared with $\Gamma(B \rightarrow N\bar{R}_c(\bar{S}_c))$. The details are listed in Tables VII, VIII, and IX.

IV. DISCUSSIONS AND CONCLUSIONS

If the recently discovered state Θ^+ is interpreted as a pentaquark bound state with an $\bar{s}$ and four light quarks, heavy pentaquarks with the $\bar{s}$ replaced by a $\bar{b}$ or a $\bar{c}$ should exist. H1 experiment at HERA-B has obtained some evidence for Θ_c . These states form $SU(3)_f$ triplets $R_{c,b}$ and antisixtets $S_{c,b}$. R_c and S_c can also be produced from B decays at B factories. If pentaquarks $R_{c,b}$ and $S_{c,b}$ are kinematically allowed to decay through strong interactions, one can use Tables I and II to relate different decay widths, and to test the model.

At present there is only some evidence for $\Theta_c \rightarrow D^{*-}p$. Using the decay width $\Gamma=12$ MeV obtained from H1, we determine c_{SND}^2 to be 1.712 and 0.167 for Θ_c with positive and negative parities, respectively. One expects c_{SNB}^2 to be similar to c_{SND}^2 . Using Table II, we can obtain other decay widths which can be tested in the future. We have nothing much to say about the size of the couplings except that we expect the couplings c_{R_bNB} and c_{S_bND} to be about the same as c_{R_cND} and c_{S_cND} , respectively. If one extends $SU(3)_f$ to $SU(4)_f$, the strong couplings can be related in principle to the ones involving just light pentaquarks [12]. We will not consider this possibility here.

We have parametrized some of the two body hadronic weak decays of heavy pentaquarks in terms of $SU(3)_f$ invariant amplitudes. From the tables obtained we see that there are several relations among different decay modes. For example, for $\Delta S=0$ processes of the type $S_b \rightarrow S_c\Pi$, from Table V we obtain

TABLE IV. $SU(3)$ decay amplitudes for $S_b \rightarrow S_c \Pi$ with $\Delta S=0$. The Lorentz structure is similar to Table III.

T_b^-	$T_c^- \pi^+$	$T_c^- \pi^0$	$T_c^- \eta$	$T_c^0 \pi^-$	$N_c^- \bar{K}^0$	$N_c^0 K^-$
	$s_1 + s_3$	$\frac{1}{2}(s_2 - s_3 + s_4)$	$\frac{1}{2\sqrt{3}}(s_2 + s_3 + s_4)$	s_2	$\frac{1}{\sqrt{2}}s_3$	$\frac{1}{\sqrt{2}}s_2$
T_b^0	$T_c^- \pi^+$	$T_c^0 \pi^0$	$T_c^0 \eta$	$N_c^0 \bar{K}^0$		
	$\frac{1}{2}(s_1 + s_2 + s_4)$	$-\frac{1}{2}s_2$	$\frac{1}{2\sqrt{3}}s_2$	$\frac{1}{2}s_2$		
T_b^+	$T_c^0 \pi^+$					
	$s_1 + s_4$					
N_b^0	$T_c^- K^+$	$T_c^0 K^0$	$N_c^- \pi^+$	$N_c^0 \pi^0$	$N_c^0 \eta$	$\Theta_c^0 \bar{K}^0$
	$\frac{1}{2}(s_2 + s_4)$	$\frac{1}{\sqrt{2}}s_2$	$\frac{1}{2}(2s_1 + s_3)$	$-\frac{1}{2\sqrt{2}}(s_3 - s_4)$	$-\frac{1}{\sqrt{6}}(s_2 - s_3 - s_4)$	$\frac{1}{\sqrt{2}}s_3$
N_b^+	$T_c^0 K^+$	$N_c^0 \pi^+$				
	$\frac{1}{\sqrt{2}}s_4$	$\frac{1}{2}(2s_1 + s_4)$				
Θ_b^+	$N_c^0 K^+$	$\Theta_c^0 \pi^+$				
	$\frac{1}{\sqrt{2}}s_4$	s_1				

$$\begin{aligned}
\Gamma(T_b^- \rightarrow T_c^0 \pi^-) &= 2\Gamma(T_b^- \rightarrow N_c^0 K^-) \\
&= 4\Gamma(T_b^0 \rightarrow T_c^0 \pi^0) = 12\Gamma(T_b^0 \rightarrow T_c^0 \eta) \\
&= 4\Gamma(T_b^0 \rightarrow N_c^0 \bar{K}^0) = 2\Gamma(N_b^0 \rightarrow T_c^0 K^0), \\
\Gamma(N_b^+ \rightarrow T_c^0 K^+) &= \Gamma(\Theta_b^+ \rightarrow N_c^0 K^+). \quad (24)
\end{aligned}$$

More relations can be read off from the tables. These relations can be used to study the properties of heavy pen-

taquarks and test the model provided that the decays have substantial branching ratios which require knowledge about the size of the $SU(3)_f$ invariant amplitudes.

Theoretical calculations of the decay amplitudes are very difficult since multiquarks are involved. However, for certain decays, the structure is very simple and can be related to experimentally measured modes, for example, $\Theta_b^+ \rightarrow \Theta_c^0 \pi^+$ can be related to $\Lambda_b \rightarrow \Lambda_c^+ \pi^-$.

In $\Theta_b^+ \rightarrow \Theta_c^0 \pi^+$ decay, the main contributions is due to

TABLE V. $SU(3)$ decay amplitudes for $S_b \rightarrow S_c \Pi$ with $\Delta S=1$. The Lorentz structure is similar to that in Table III.

T_b^-	$T_c^- K^+$	$T_c^- K^0$	$N_c^- \pi^0$	$N_c^- \eta$	$N_c^- \pi^-$	$\Theta_c^0 K^-$
	$s_1 + s_3$	$\frac{1}{\sqrt{2}}s_3$	$\frac{1}{2}(s_2 + s_4)$	$\frac{1}{2\sqrt{3}}(s_2 - s_3 + s_4)$	$\frac{1}{\sqrt{2}}s_2$	s_2
T_b^0	$T_c^- K^+$	$N_c^- \pi^+$	$N_c^0 \pi^0$	$N_c^0 \eta$	$\Theta_c^0 \bar{K}^0$	
	$\frac{1}{2}s_1$	$\frac{1}{2}(s_2 + s_4)$	$-\frac{1}{2\sqrt{2}}s_2$	$\frac{1}{2\sqrt{6}}s_2$	$\frac{1}{\sqrt{2}}s_2$	
T_b^+	$T_c^0 K^+$	$N_c^0 \pi^+$				
	s_1	$\frac{1}{\sqrt{2}}s_4$				
N_b^0	$N_c^- K^+$	$N_c^0 K^0$	$\Theta_c^0 \pi^0$	$\Theta_c^0 \eta$		
	$\frac{1}{2}(2s_1 + s_2 + s_3 + s_4)$	$\frac{1}{2}(s_2 + s_3)$	$\frac{1}{2}s_4$	$-\frac{1}{\sqrt{3}}(s_2 + s_3 - s_4)$		
N_b^+	$N_c^0 K^+$	$\Theta_c^0 \pi^+$				
	$\frac{1}{2}(2s_1 + s_4)$	$\frac{1}{\sqrt{2}}s_4$				
Θ_b^+	$\Theta_c^0 K^+$					
	$s_1 + s_4$					

TABLE VI. $SU(3)$ decay amplitudes for $S_b \rightarrow S_c D$. The Lorentz structure is similar to that in Table III. For $S_b \rightarrow S_c D^*$, the Lorentz structure of the bi-spinor product is of the form $\bar{S}_c \gamma_\mu (1 + b \gamma_5) S_b$.

$\Delta S=0$	$\Delta S=1$	
U_b^+	$r_{31}U_c^0D_d$	$r_{31}U_c^0D_s$
D_b^0	$(r_{31}+r_{32})D_c^-D_d+r_{32}U_c^0D_u+r_{32}S_c^-D_s$	$r_{31}D_c^-D_s$
S_b^0	$r_{31}S_c^-D_d$	$(r_{31}+r_{32})S_c^-D_s+r_{32}U_c^0D_u+r_{31}D_c^-D_d$
U_b^+	$rT_c^0D_s-\frac{1}{\sqrt{2}}rN_c^0D_d$	$\frac{1}{\sqrt{2}}rN_c^0D_s-r\Theta_c^0D_d$
D_b^0	$\frac{1}{\sqrt{2}}rN_c^0D_u-\frac{1}{\sqrt{2}}rT_c^-D_s$	$-\frac{1}{\sqrt{2}}rN_c^-D_s+r\Theta_c^0D_u$
S_b^0	$\frac{1}{\sqrt{2}}rT_c^-D_d-rT_c^0D_u$	$\frac{1}{\sqrt{2}}rN_c^-D_d-\frac{1}{\sqrt{2}}rN_c^0D_u$
T_b^-	$s_{31}T_c^-D_d+\frac{1}{\sqrt{2}}s_{32}T_c^-D_u$	$s_{31}T_c^-D_s+\frac{1}{\sqrt{2}}s_{32}N_c^-D_u$
T_b^0	$(s_{31}+\frac{1}{2}s_{32})T_c^-D_d+\frac{1}{\sqrt{2}}s_{32}T_c^0D_u$	$s_{31}T_c^-D_s+\frac{1}{2}s_{32}N_c^0D_u+\frac{1}{2}s_{32}N_c^-D_d$
T_b^+	$(s_{31}+s_{32})T_c^0D_d$	$s_{31}T_c^0D_s+\frac{1}{\sqrt{2}}s_{32}N_c^0D_d$
N_b^0	$s_{31}N_c^-D_d+\frac{1}{2}s_{32}N_c^0D_u+\frac{1}{2}s_{32}T_c^-D_s$	$(s_{31}+\frac{1}{2}s_{32})N_c^-D_s+\frac{1}{\sqrt{2}}s_{32}N_c^0D_u$
N_b^+	$(s_{31}+\frac{1}{2}s_{32})N_c^0D_d+\frac{1}{\sqrt{2}}s_{32}T_c^0D_s$	$(s_{31}+\frac{1}{2}s_{32})N_c^0D_s+\frac{1}{\sqrt{2}}s_{32}\Theta_c^0D_d$
Θ_b^+	$s_{31}\Theta_c^0D_d+\frac{1}{\sqrt{2}}s_{32}N_c^0D_s$	$(s_{31}+s_{32})\Theta_c^0D_s$

the factorized matrix elements where π is emitted from two light quarks in the effective Hamiltonian, the transition of Θ_b^+ to Θ_c^0 is due to the transition of a $\bar{b}$ quark to a $\bar{c}$ quark, and the structure of the rest of the four light quarks in the pentaquarks is basically preserved. Based on this intuitive picture, Ref. [18] relates $\Theta_b^+ \rightarrow \Theta_c^0 \pi^+$ to $\Lambda_b \rightarrow \Lambda_c^+ \pi^-$ using heavy quark effective theory, and concludes that the branching ratios for these two processes are similar. The decay rate $\Gamma(\Theta_b^+ \rightarrow \Theta_c^0 \pi^+)$ is estimated to be about $2.5\Gamma(B^0 \rightarrow D^- \pi^-)$. This prediction can be tested at future collider experiments.

From Table IV, we see that $\Theta_b^+ \rightarrow \Theta_c^0 \pi^+$ is proportional to the $SU(3)_f$ invariant amplitude s_1 , one therefore can use the estimate in Ref. [18] to obtain an estimate for s_1 . The invariant amplitudes $s_{2,3,4}$ involve more complicated topology and are much harder to estimate. Although it is difficult to know all decay amplitudes, knowing s_1 one can make some useful predictions. The branching ratios for processes in Tables IV and V which just depend on s_1 are therefore known. Up to mass splitting corrections in phase space, we obtain

$$\Gamma(T_b^0 \rightarrow T_c^- K^+) \sim \frac{1}{4} \frac{|V_{us}|^2}{|V_{ud}|^2} \Gamma(\Theta_b^+ \rightarrow \Theta_c^0 \pi^+),$$

$$\Gamma(T_b^+ \rightarrow T_c^0 K^+) \sim \frac{|V_{us}|^2}{|V_{ud}|^2} \Gamma(\Theta_b^+ \rightarrow \Theta_c^0 \pi^+). \quad (25)$$

Using the heavy quark effective theory, one can also relate several other $SU(3)_f$ invariant amplitudes to $\Lambda_b \rightarrow \Lambda_c \pi$ by realizing the fact that all amplitudes for the heavy pentaquark transitions of the form $R_{b,i} R_c^i$, $S_{b,ij} S_c^{ij}$ have similar factorization structure, and their strength should also be similar. We therefore expect that $s_1 \sim r_{81} \sim r_{31} \sim s_{31}$.

One then has, up to mass splitting corrections in phase space, the following relations:

TABLE VII. $SU(3)$ decay amplitudes for $B \rightarrow N \bar{R}_c(\bar{S}_c)$. The Lorentz structure should be understood to be $\bar{N}(1+b\gamma_5)R_c(S_c)$ since weak interaction can have S - and P -wave amplitudes.

B_u decay	$\Delta S=0$	$\Delta S=-1$
$\bar{U}_c^0 \Sigma^-$	$\tilde{r}_{81} + \tilde{r}_{83}$	$\bar{U}_c^0 \Xi^-$ $\tilde{r}_{81} + \tilde{r}_{83}$
$\bar{T}_c^0 \Xi^-$	$\tilde{s}_{81} - \tilde{s}_{82}$	$\bar{N}_c^0 \Xi^-$ $\frac{1}{\sqrt{2}}(\tilde{s}_{81} - \tilde{s}_{82})$
$\bar{N}_c^0 \Sigma^-$	$-\frac{1}{\sqrt{2}}(\tilde{s}_{81} - \tilde{s}_{82})$	$\bar{\Theta}_c^0 \Sigma^-$ $-(\tilde{s}_{81} - \tilde{s}_{82})$
B_d decay		
$\bar{U}_c^0 \Sigma^0$	$-\frac{1}{\sqrt{2}}(\tilde{r}_{81} - \tilde{r}_{82})$	$\bar{U}_c^0 \Xi^0$ $\tilde{r}_{81}$
$\bar{U}_c^0 \Lambda$	$\frac{1}{\sqrt{6}}(\tilde{r}_{81} + \tilde{r}_{82})$	$\bar{D}_c^- \Xi^-$ $\tilde{r}_{83}$
$\bar{D}_c^- \Sigma^-$	$\tilde{r}_{82} + \tilde{r}_{83}$	$\bar{N}_c^- \Xi^-$ $-\frac{1}{\sqrt{2}}\tilde{s}_{81}$
$\bar{S}_c^- \Xi^-$	$\tilde{r}_{82}$	$\bar{N}_c^0 \Xi^0$ $-\frac{1}{\sqrt{2}}\tilde{s}_{82}$
$\bar{T}_c^- \Xi^-$	$-\frac{1}{\sqrt{2}}(\tilde{s}_{81} + \tilde{s}_{83})$	$\bar{\Theta}_c^0 \Sigma^0$ $\frac{1}{\sqrt{2}}(\tilde{s}_{81} - \tilde{s}_{82})$
$\bar{T}_c^0 \Xi^0$	$-(\tilde{s}_{82} + \tilde{s}_{83})$	$\bar{\Theta}_c^0 \Lambda$ $\frac{1}{\sqrt{6}}(\tilde{s}_{81} + \tilde{s}_{82})$
$\bar{N}_c^- \Sigma^-$	$\frac{1}{\sqrt{2}}\tilde{s}_{83}$	
$\bar{N}_c^0 \Sigma^0$	$\frac{1}{2}(\tilde{s}_{81} - \tilde{s}_{82} - \tilde{s}_{83})$	
$\bar{N}_c^0 \Lambda$	$\frac{1}{2\sqrt{3}}(\tilde{s}_{81} + \tilde{s}_{82} + 3\tilde{s}_{83})$	
$\bar{\Theta}_c^0 n$	$\tilde{s}_{83}$	
B_s decay		
$\bar{U}_c^0 n$	$\tilde{r}_{81}$	$\bar{U}_c^0 \Sigma^0$ $\frac{1}{\sqrt{2}}\tilde{r}_{82}$
$\bar{S}_c^- \Sigma^-$	r_{83}	$\bar{U}_c^0 \Lambda$ $-\frac{1}{\sqrt{6}}(2\tilde{r}_{81} - \tilde{r}_{82})$
$\bar{T}_c^- \Sigma^-$	$\frac{1}{\sqrt{2}}\tilde{s}_{81}$	$\bar{D}_c^- \Sigma^-$ $\tilde{r}_{82}$
$\bar{T}_c^0 \Sigma^0$	$-\frac{1}{\sqrt{2}}\tilde{s}_{81}$	$\bar{S}_c^- \Xi^-$ $\tilde{r}_{82} + \tilde{r}_{83}$
$\bar{T}_c^0 \Lambda$	$-\frac{1}{\sqrt{6}}(\tilde{s}_{81} - 2\tilde{s}_{82})$	$\bar{T}_c^- \Xi^-$ $-\frac{1}{\sqrt{2}}\tilde{s}_{83}$
$\bar{N}_c^0 n$	$\frac{1}{\sqrt{2}}\tilde{s}_{82}$	$\bar{T}_c^0 \Xi^0$ $-\tilde{s}_{83}$
		$\bar{N}_c^- \Sigma^-$ $\frac{1}{\sqrt{2}}(\tilde{s}_{81} + \tilde{s}_{83})$
		$\bar{N}_c^0 \Sigma^0$ $-\frac{1}{2}(\tilde{s}_{81} + \tilde{s}_{83})$
		$\bar{N}_c^0 \Lambda$ $-\frac{1}{2\sqrt{3}}(\tilde{s}_{81} - 2\tilde{s}_{82} - 3\tilde{s}_{83})$
		$\bar{\Theta}_c^0 n$ $\tilde{s}_{82} + \tilde{s}_{83}$

TABLE VIII. $SU(3)$ decay amplitudes for $B \rightarrow \bar{N}(R_c, S_c)$ with $\Delta S=0$. The Lorentz structure is of the form $\bar{R}_c(\bar{S}_c)(1+b\gamma_5)N$.

B_u	$\Sigma^+ U_c^0$	$(\tilde{r}_{31} + \tilde{r}_{32} + \tilde{r}_{61} + \tilde{r}_{62})$
	$\Sigma^0 D_c^-$	$-\frac{1}{\sqrt{2}}(\tilde{r}_{31} + \tilde{r}_{32} - \tilde{r}_{61} + \tilde{r}_{62})$
	$\bar{\Lambda} D_c^-$	$-\frac{1}{\sqrt{6}}(\tilde{r}_{31} - \tilde{r}_{32} + 2\tilde{r}_{33} - \tilde{r}_{61} - \tilde{r}_{62})$
	$\Xi^0 S_c^-$	$\tilde{r}_{32} - \tilde{r}_{33} + \tilde{r}_{62}$
	$\Xi^- T_c^-$	$\tilde{s}_{31} - \tilde{s}_{61} - \tilde{s}_{62}$
	$\Xi^0 T_c^-$	$\frac{1}{\sqrt{2}}(\tilde{s}_{31} + \tilde{s}_{61} - \tilde{s}_{62})$
	$\Sigma^0 N_c^-$	$\frac{1}{2}(\tilde{s}_{32} + 2\tilde{s}_{61} + \tilde{s}_{62})$
	$\bar{\Lambda} N_c^-$	$-\frac{1}{2\sqrt{3}}(2\tilde{s}_{31} - \tilde{s}_{32} - 3\tilde{s}_{62})$
	$\Sigma^+ N_c^0$	$\frac{1}{\sqrt{2}}(\tilde{s}_{32} + \tilde{s}_{62})$
	$\bar{p} \Theta_c^0$	$\tilde{s}_{32} + \tilde{s}_{62}$
B_d	$\Sigma^0 U_c^0$	$-\frac{1}{\sqrt{2}}(\tilde{r}_{31} + \tilde{r}_{32} + \tilde{r}_{61} - \tilde{r}_{62})$
	$\bar{\Lambda} U_c^0$	$\frac{1}{\sqrt{6}}(\tilde{r}_{31} - \tilde{r}_{32} + 2\tilde{r}_{33} + \tilde{r}_{61} + \tilde{r}_{62})$
	$\Sigma^- D_c^-$	$-(\tilde{r}_{31} + \tilde{r}_{32} - \tilde{r}_{61} - \tilde{r}_{62})$
	$\Xi^- S_c^-$	$-(\tilde{r}_{32} - \tilde{r}_{33} + \tilde{r}_{62})$
	$\Xi^- T_c^-$	$\frac{1}{\sqrt{2}}(\tilde{s}_{31} - \tilde{s}_{61} + \tilde{s}_{62})$
	$\Xi^0 T_c^0$	$\tilde{s}_{31} + \tilde{s}_{61} + \tilde{s}_{62}$
	$\Sigma^- N_c^-$	$\frac{1}{\sqrt{2}}(\tilde{s}_{32} - \tilde{s}_{62})$
	$\bar{\Lambda} N_c^0$	$-\frac{1}{2\sqrt{3}}(2\tilde{s}_{31} - \tilde{s}_{32} + 3\tilde{s}_{62})$
	$\Sigma^0 N_c^0$	$-\frac{1}{2}(\tilde{s}_{32} - 2\tilde{s}_{61} - \tilde{s}_{62})$
	$\bar{n} \Theta_c^0$	$\tilde{s}_{32} - \tilde{s}_{62}$
B_s	$\Xi^0 U_c^0$	$\tilde{r}_{31} + \tilde{r}_{33} + \tilde{r}_{61}$
	$\Xi^- D_c^-$	$-(\tilde{r}_{31} + \tilde{r}_{33} - \tilde{r}_{61})$
	$\Xi^- N_c^-$	$\frac{1}{\sqrt{2}}(\tilde{s}_{31} + \tilde{s}_{32} - \tilde{s}_{61})$
	$\Xi^0 N_c^0$	$\frac{1}{\sqrt{2}}(\tilde{s}_{31} + \tilde{s}_{32} + \tilde{s}_{61})$
	$\Sigma^0 \Theta_c^0$	$\sqrt{2}\tilde{s}_{61}$
	$\bar{\Lambda} \Theta_c^0$	$-\sqrt{\frac{2}{3}}(\tilde{s}_{31} + \tilde{s}_{32})$

TABLE IX. $SU(3)$ decay amplitudes for $B \rightarrow \bar{N}(R_c, S_c)$ with $\Delta S = -1$. The Lorentz structure is the same as Table VIII.

B_u	$\bar{p}U_c^0$	$\tilde{r}_{31} + \tilde{r}_{32} + \tilde{r}_{61} + \tilde{r}_{62}$
	$\bar{n}D_c^-$	$\tilde{r}_{32} - \tilde{r}_{33} + \tilde{r}_{62}$
	$\bar{\Sigma}^0 S_c^-$	$-\frac{1}{\sqrt{2}}(\tilde{r}_{31} + \tilde{r}_{33} - \tilde{r}_{61})$
	$\bar{\Lambda} S_c^-$	$-\frac{1}{\sqrt{6}}(\tilde{r}_{31} + 2\tilde{r}_{32} - \tilde{r}_{33} - \tilde{r}_{61} + 2\tilde{r}_{62})$
	$\bar{\Sigma}^- T_c^-$	$-(\tilde{s}_{31} - \tilde{s}_{61} - \tilde{s}_{62})$
	$\bar{\Sigma}^0 T_c^-$	$-\frac{1}{2}(\tilde{s}_{31} - \tilde{s}_{32} - \tilde{s}_{61} - 2\tilde{s}_{62})$
	$\bar{\Lambda} T_c^-$	$-\frac{1}{2\sqrt{3}}(\tilde{s}_{31} + \tilde{s}_{32} + 3\tilde{s}_{61})$
	$\bar{\Sigma}^+ T_c^0$	$-(\tilde{s}_{32} + \tilde{s}_{62})$
	$\bar{n}N_c^-$	$-\frac{1}{\sqrt{2}}(\tilde{s}_{31} + \tilde{s}_{61} - \tilde{s}_{62})$
	$\bar{p}N_c^0$	$-\frac{1}{\sqrt{2}}(\tilde{s}_{32} + \tilde{s}_{62})$
B_d	$\bar{n}U_c^0$	$(\tilde{r}_{31} + \tilde{r}_{33} + \tilde{r}_{61})$
	$\bar{\Sigma}^- S_c^-$	$-(\tilde{r}_{31} + \tilde{r}_{33} - \tilde{r}_{61})$
	$\bar{\Sigma}^- T_c^-$	$-\frac{1}{\sqrt{2}}(\tilde{s}_{31} + \tilde{s}_{32} - \tilde{s}_{61})$
	$\bar{\Sigma}^0 T_c^0$	$\frac{1}{\sqrt{2}}(\tilde{s}_{31} + \tilde{s}_{32} - \tilde{s}_{61})$
	$\bar{\Lambda} T_c^0$	$-\frac{1}{\sqrt{6}}(\tilde{s}_{31} + \tilde{s}_{32} + 3\tilde{s}_{61})$
	$\bar{n}N_c^0$	$-\frac{1}{\sqrt{2}}(\tilde{s}_{31} + \tilde{s}_{32} + \tilde{s}_{61})$
B_s	$\bar{\Sigma}^0 U_c^0$	$-\frac{1}{\sqrt{2}}(\tilde{r}_{32} - \tilde{r}_{33} - \tilde{r}_{62})$
	$\bar{\Lambda} U_c^0$	$-\frac{1}{\sqrt{6}}(2\tilde{r}_{31} + \tilde{r}_{32} + \tilde{r}_{33} + 2\tilde{r}_{61} - \tilde{r}_{62})$
	$\bar{\Sigma}^- D_c^-$	$-(\tilde{r}_{32} - \tilde{r}_{33} - \tilde{r}_{62})$
	$\bar{\Xi}^- S_c^-$	$-(\tilde{r}_{31} + \tilde{r}_{32} - \tilde{r}_{61} - \tilde{r}_{62})$
	$\bar{\Xi}^- T_c^-$	$-\frac{1}{\sqrt{2}}(\tilde{s}_{32} - \tilde{s}_{62})$
	$\bar{\Xi}^0 T_c^0$	$-(\tilde{s}_{32} - \tilde{s}_{62})$
	$\bar{\Sigma}^0 N_c^0$	$\frac{1}{2}(\tilde{s}_{31} - \tilde{s}_{61} + \tilde{s}_{62})$
	$\bar{\Lambda} N_c^0$	$-\frac{1}{2\sqrt{3}}(\tilde{s}_{31} - 2\tilde{s}_{32} + 3\tilde{s}_{61} + 3\tilde{s}_{62})$
	$\bar{\Sigma}^- N_c^-$	$-\frac{1}{\sqrt{2}}(\tilde{s}_{31} - \tilde{s}_{61} + \tilde{s}_{62})$
	$\bar{n}\bar{\Theta}_c^0$	$-(\tilde{s}_{31} + \tilde{s}_{61} + \tilde{s}_{62})$

$$\begin{aligned}
\Gamma(\Theta_b^+ \rightarrow \Theta_c^0 \pi^+) &\sim \Gamma(S_b^0 \rightarrow S_c^- \pi^+) \sim \frac{|V_{ud}|^2}{|V_{us}|^2} \Gamma(D_b^0 \rightarrow D_0^- K^+) \\
&\sim \frac{|V_{ud}|^2}{|V_{cd}|^2} [\Gamma(U_b^+ \rightarrow U_c^0 D_d), \\
&\Gamma(S_b^0 \rightarrow S_c^- D_d)] \sim \frac{|V_{ud}|^2}{|V_{cs}|^2} [\Gamma(U_b^+ \rightarrow U_c^0 D_s), \\
&\Gamma(U_b^+ \rightarrow D_c^- D_s), \Gamma(S_b^0 \rightarrow D_c^- D_d)] \\
&\sim \frac{|V_{ud}|^2}{|V_{cd}|^2} [\Gamma(T_b^- \rightarrow T_c^- D_d), \\
&\Gamma(N_b^0 \rightarrow N_c^- D_d), \Gamma(\Theta_b^+ \rightarrow \Theta_c^0 D_d)] \\
&\sim \frac{|V_{ud}|^2}{|V_{cs}|^2} [\Gamma(T_b^- \rightarrow T_c^- D_s), \\
&\Gamma(T_b^0 \rightarrow T_c^- D_s), \Gamma(T_b^+ \rightarrow T_c^0 D_s)]. \quad (26)
\end{aligned}$$

The above relations also hold for processes with replacing D by D^* .

Pentaquark properties can also be studied at B factories. We have studied $B \rightarrow N\bar{R}_c(\bar{S}_c)$ and $B \rightarrow \bar{N}R_c(S_c)$ decays. From Tables VII, VIII, and IX, we see that there are several relations. For example,

$$\begin{aligned}
\Gamma(B_u \rightarrow \bar{U}_c^0 \Sigma^-) &= \frac{|V_{ud}|^2}{|V_{us}|^2} \Gamma(B_u \rightarrow \bar{U}_c \Xi^-), \\
\Gamma(B_u \rightarrow \bar{p}U_c^0) &= \frac{|V_{ud}|^2}{|V_{us}|^2} \Gamma(B_u \rightarrow \bar{\Sigma}^+ U_c^0). \quad (27)
\end{aligned}$$

Should the heavy pentaquarks be discovered, these relations can also provide important information. The decay amplitudes for $B \rightarrow N\bar{R}_c(\bar{S}_c)$ and $B \rightarrow \bar{N}R_c(S_c)$ are difficult to estimate. We are not able to provide any reliable estimate, except that we expect them to be smaller than $B \rightarrow N\bar{\Lambda}_c$ amplitudes.

Using the same formulation, one can also study B decays into an ordinary baryon N and a light pentaquark, such as a Θ^+ in the antidecuplet. We, however, expect that the branching ratios are smaller than $B \rightarrow N\bar{N}$. Since $B \rightarrow N\bar{N}$ have small branching ratios, it may be difficult to study $B \rightarrow N\bar{\Theta}^+$ experimentally. This situation may change if one studies three body decays of B with a light pentaquark in the final states, such as $B \rightarrow Dp(n)\Theta$. The $B \rightarrow DK$ decay has a branching ratio of order a few times 10^{-4} . The K has a strong coupling to $p(n)\Theta^+$ which can be determined from the Θ^+ decay. With Γ_Θ of Θ^+ being about one MeV, one can obtain a branching ratio for $B \rightarrow Dp(n)\Theta$ as large as 10^{-6} which is within the reach of near future B factories.

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