SU(3) Predictions of $B \rightarrow PP$ Decays in the Standard Model

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With SU(3) symmetry one only needs 13 hadronic parameters to describe $B \rightarrow PP$ decays in the Standard Model. When annihilation contributions are neglected, only 7 hadronic parameters are needed. These parameters can be determined from existing experimental data and some unmeasured branching ratios and CP asymmetries of the type $B \rightarrow PP$ can be predicted. In this talk we present SU(3) predictions of branching ratios and CP asymmetries for $B \rightarrow PP$ decays in the Standard Model.

1. SU(3) Parameters for $B \rightarrow PP$

In Standard Model (SM) the decay amplitudes for B meson to two pseudoscalar SU(3) octet mesons, $B \rightarrow PP$, have both tree and penguin contributions and can be written as[1]

$$A(B \rightarrow PP) = \langle PP | H_{eff}^q | B \rangle = \frac{G_F}{\sqrt{2}} [V_{ub} V_{uq}^* T(q) + V_{tb} V_{tq}^* P(q)]. \quad (1)$$

As far as the SU(3) structure is concerned, the effective Hamiltonian contains $\bar{3}$, 6, and $\bar{15}$ representations of SU(3) which define several invariant amplitudes[2],

$$T = A_3^T B_i(\bar{3})^i (M_i^k M_k^l) + C_3^T B_i M_k^i M_j^k (\bar{3})^j + A_6^T B_i(6)_k^{ij} M_j^l M_i^k + C_6^T B_i M_j^i (6)_l^{jk} M_k^l + A_{15}^T B_i(\bar{15})_k^{ij} M_j^k M_i^l + C_{15}^T B_i M_j^i (\bar{15})_l^{jk} M_k^l,$$

where $B_i = (B_u, B_d, B_s)$ and M is the SU(3) pseudoscalar octet. $C_6 - A_6$ always appear together. We use C_6 only.

In general there are both tree and penguin amplitudes $C(A)_{3,6,\bar{15}}^{T,P}$. In the SM the amplitudes $C(A)_{6,\bar{15}}^T$ and $C(A)_{6,\bar{15}}^P$ are related by the Wilson coefficients c_i of the relevant operators[3,4],

$$\frac{C_6^P}{C_6^T} = -\frac{3}{2} \frac{c_9^{tc} - c_{10}^{tc}}{c_1 - c_2 - 3(c_9^{uc} - c_{10}^{uc})/2},$$

$$\frac{C_{15}^P(A_{15}^P)}{C_{15}^T(A_{15}^T)} = -\frac{3}{2} \frac{c_9^{tc} + c_{10}^{tc}}{c_1 + c_2 - 3(c_9^{uc} + c_{10}^{uc})/2}. \quad (2)$$

Without loss of generality, one can set C_3^P to be real. One only needs to use 13 real independent parameters to describe $B \rightarrow PP$ in the SM which we choose to be[3,4]

$$C_3^P, C_3^T e^{i\delta_3}, C_6^T e^{i\delta_6}, C_{15}^T e^{i\delta_{15}},$$

$$A_3^T e^{i\delta_{A_3^T}}, A_3^P e^{i\delta_{A_3^P}}, A_{15}^T e^{i\delta_{A_{15}^T}}. \quad (3)$$

The phases are defined such that all $C_i^{T,P}$ are real positive numbers. The amplitudes A_i correspond to annihilation contributions and are expected to be small. If they are neglected, there are only 7 independent hadronic parameters.

Using available data on, $B \rightarrow K\pi, \pi\pi, KK$, one can obtain information about the hadronic parameters and predict other decay branching ratios and CP violating asymmetries. We carried out a χ^2 analysis[5] assuming SU(3) symmetry with the $B \rightarrow PP$ data[6] shown in Table 1. In our numerical analysis the KM matrix elements $V_{us} = \lambda$, $V_{cb} = A\lambda^2$, $V_{ub} = |V_{ub}| \exp(-i\gamma)$ fixed by $\lambda = 0.2196$, $A = 0.835$ and $|V_{ub}| = 0.09|V_{cb}|$ [7] and $\gamma = 59^\circ$ as determined from other data[5]. We study both the cases with and without annihilation contributions and compare the results.

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Table 1

Experimental data on the branching ratios and CP asymmetries obtained by averaging different data with the assumption that they obey uncorrelated Gaussian distribution. The branching ratios for $B \rightarrow PP$ are shown in units of 10^{-6} .

Branching ratio and CP asymmetries	Averaged Value
$Br(B_d \rightarrow \pi^+ \pi^-)$	5.2 ± 0.6
$Br(B_u \rightarrow \pi^- \pi^0)$	4.9 ± 1.1
$Br(B_d \rightarrow K^+ \pi^-)$	18.6 ± 1.1
$Br(B_u \rightarrow K^- \pi^0)$	11.5 ± 1.3
$Br(B_u \rightarrow K^0 \pi^-)$	17.9 ± 1.7
$Br(B_d \rightarrow K^0 \pi^0)$	8.8 ± 2.3
$Br(B_u \rightarrow K^- K^0)$	0 ± 0.8
$Br(B_d \rightarrow K^0 K^0)$	1.8 ± 2.5
$Br(B_d \rightarrow \pi^0 \pi^0)$	1.7 ± 0.9
$Br(B_d \rightarrow K^+ K^-)$	0 ± 0.3
$A_{K^-\pi^0}^{B_d}$	-0.05 ± 0.09
$A_{K^-\pi^+}^{B_d}$	-0.05 ± 0.05
$A_{K^0\pi^-}^{B_u}$	0.04 ± 0.08
$A_{\pi^+\pi^-}^{B_d}$	0.42 ± 0.22
$A_{\pi^-\pi^0}^{B_u}$	0.13 ± 0.21

2. Without Annihilation Contributions

For the case without annihilation contributions, there are only 7 hadronic parameters. The results for the best fit values and their 1σ ranges are shown in Table 2.

Using the above determined hadronic parameters, one can easily obtain the branching ratios and CP asymmetries for other $B \rightarrow PP$. We used the following definition for the CP violating rate asymmetry,

$$A_{PP}^{B_i} = \frac{\Gamma(B_i \rightarrow PP) - \Gamma(\bar{B}_i \rightarrow \bar{P}\bar{P})}{\Gamma(B_i \rightarrow PP) + \Gamma(\bar{B}_i \rightarrow \bar{P}\bar{P})}. \quad (4)$$

In general P can be any one of the SU(3) pseudoscalar octet mesons, π , K and η_8 . Here we will limit our study to $P = \pi, K$ to avoid complications associated with η_1 and η_8 mixings. In this case there are total 16 decay modes. Among them the decay amplitudes for $B_d \rightarrow K^- K^+$, $B_s \rightarrow \pi^- \pi^+, \pi^0 \pi^0$ only receive annihilation con-

Table 2

The best fit values and their 1σ errors of the hadronic parameters using all data in Table 1 with annihilation terms set to be zero.

	best value	error
C_3^P	0.138	0.003
C_3^T	0.248	0.111
C_6^T	0.155	0.112
C_{15}^T	0.142	0.014
δ_3	38.10^0	29.69^0
δ_6	83.17^0	35.97^0
δ_{15}	4.78^0	17.84^0

tributions. Since we have neglected annihilation contributions they would have vanishing branching ratios. At present none of them have been measured experimentally. The present bound on $B_d \rightarrow K^- K^+$ is consistent with this prediction.

The predictions for the branching ratios of $B_s \rightarrow K^+ \pi^-, K^0 \pi^0, K^- K^+, K^0 \bar{K}^0$ decays are shown in Table 3. These decay modes are predicted to be large and can be measured at hadron colliders. The standard model and SU(3) flavor symmetry can be tested.

The CP asymmetries for some of the decays are shown in Table 4. In the SU(3) limit there are some relations between rate differences defined as, $\Delta_{PP}^{B_i} = \Gamma(B_i \rightarrow PP) - \Gamma(\bar{B}_i \rightarrow \bar{P}\bar{P})$, between $\Delta S = 0$ and $\Delta S = -1$ modes due to a unique feature of the SM in the KM matrix element that $Im(V_{ub}V_{ud}^*V_{tb}^*V_{td}) = -Im(V_{ub}V_{us}^*V_{tb}^*V_{ts})$. One has the following relations[2,8],

$$\begin{aligned} \Delta_{\pi^+\pi^-}^{B_d} &= \Delta_{K^+\pi^-}^{B_s} = -\Delta_{\pi^+K^-}^{B_d}, \\ \Delta_{\pi^0\pi^0}^{B_d} &= \Delta_{K^-\pi^0}^{B_s} = -\Delta_{\pi^0\bar{K}^0}^{B_d}. \end{aligned} \quad (5)$$

As can be seen from Table 4 that the best fit values for $A_{PP}^{B_i}$ can be large with several of them to be more than 30%, such as the asymmetries for $B_s \rightarrow K^+ \pi^-, K^0 \pi^0$, and $B_d \rightarrow \pi^0 \pi^0, \pi^+ \pi^-$. The size of $A_{PP}^{B_i}$ for these modes are large, but can be easily understood from the fact that they all have relatively small branching ratios. Using the above relations, one would obtain

$$\begin{aligned}
A_{\pi^+\pi^-}^{B_d} &= A_{K^+\pi^-}^{B_s} \\
&= -A_{\pi^+K^-}^{B_d} \frac{Br(B_d \rightarrow \pi^+ K^-)}{Br(B_d \rightarrow \pi^+ \pi^-)}, \\
A_{\pi^0\pi^0}^{B_d} &= A_{K^0\pi^0}^{B_s} \\
&= -A_{\pi^0\bar{K}^0}^{B_d} \frac{Br(B_d \rightarrow \pi^0 \bar{K}^0)}{Br(B_d \rightarrow \pi^0 \pi^0)}. \quad (6)
\end{aligned}$$

In all the above cases the ratios of the branching ratios are larger than one, a small $A_{PP}^{B_i}$ of the decay mode on the right hand side can induce a large $A_{PP}^{B_i}$ for the decay modes on the left hand side. These predictions can provide interesting tests for the SM.

Table 3

Predictions of branching ratios and 1σ ranges without annihilation terms in units of 10^{-6} .

	best value	range
$B_s \rightarrow K^+\pi^-$	4.8	(5.3 , 4.2)
$B_s \rightarrow K^0\pi^0$	1.2	(2.0 , 0.7)
$B_s \rightarrow K^-K^+$	17.4	(18.3 , 16.5)
$B_s \rightarrow K^0\bar{K}^0$	16.8	(17.9 , 15.8)

3. With Annihilation Contributions

In the analyses of the previous sections we have neglected annihilation contributions to $B \rightarrow PP$ decays. In this section we study the effects of the annihilation terms on $B \rightarrow PP$ decays. In this case we would have total 13 parameters. From Table 1 we see that there are 15 experimental data points. In principle, the 13 hadronic parameters under consideration can be determined. In Tables 5, 6 and 7 we show the results on the hadronic parameters, and some of the $B \rightarrow PP$ branching ratios and CP asymmetries.

From Table 5 we see that the size of the best fit annihilation parameters A_i are small compared the non-annihilation terms $C_{3,15}$. This confirms the expectation that annihilation contributions

Table 4

Predictions of CP asymmetry without annihilation terms.

	best value	range
$A_{K^-K^0}^{B_u}$	-0.09	(0.85, -0.91)
$A_{\pi^+\pi^-}^{B_d}$	0.32	(0.46, 0.18)
$A_{\pi^0\pi^0}^{B_d}$	0.37	(0.64, -0.58)
$A_{\bar{K}^0K^0}^{B_d}$	-0.09	(0.85, -0.91)
$A_{\pi^+K^-}^{B_u}$	0.00	(0.05, -0.04)
$A_{\pi^0K^-}^{B_u}$	-0.01	(0.06, -0.10)
$A_{\pi^+K^-}^{B_d}$	-0.09	(-0.05, -0.13)
$A_{\pi^0\bar{K}^0}^{B_d}$	-0.06	(0.06, -0.13)
$A_{K^+\pi^-}^{B_s}$	0.32	(0.46, 0.18)
$A_{K^0\pi^0}^{B_s}$	0.37	(0.64, -0.58)
$A_{K^-K^+}^{B_s}$	-0.09	(-0.05, -0.13)
$A_{K^0\bar{K}^0}^{B_s}$	0.00	(0.05, -0.04)

are small. The allowed ranges are large and therefore can not rule out the possibility of having significant annihilation contributions. We have to wait improved experiments to obtain more precise information.

The branching ratios for $B_d \rightarrow K^-K^+$, $B_s \rightarrow \pi^+\pi^-$, $\pi^0\pi^0$ which only receive contribution from annihilation are not vanishing any more. The branching ratios are expected to be small. From Table 6, we indeed find that these branching ratios are smaller than others.

It is interesting to note that although the annihilation amplitudes are small, in certain decay modes, such as $B_s \rightarrow K^+K^-$ and $B_s \rightarrow K^0\bar{K}^0$, the effects can be significant, the branching ratios almost doubled. This is because that although A_3^P is small compared with $C_{3,15}$, it is comparable with C_6^T , but enhanced by a KM factor $|V_{ts}^*V_{us}/V_{ub}V_{us}^*|$. These modes provide good places to study the annihilation contributions.

4. Discussions and Conclusions

In this talk we have presented SU(3) predictions of branching ratios and CP asymmetries for some $B \rightarrow PP$ decays in the Standard Model. There can be large CP violation

Table 5

The best fit values and their errors for the hadronic parameters with annihilation terms.

	best value	error
C_3^P	0.138	0.004
C_5^P	0.208	0.181
C_6^P	0.043	0.206
C_{15}^P	0.141	0.014
δ_3	31.65^0	57.7^0
δ_6	97.74^0	147.97^0
δ_{15}	8.54^0	21.67^0
A_3^P	0.025	0.042
A_5^P	0.061	0.143
A_{15}^P	0.036	0.075
$\delta_{A_3^T}$	73.46^0	107.18^0
$\delta_{A_5^P}$	-13.78^0	89.52^0
$\delta_{A_{15}^T}$	-131.12^0	180.51^0

Table 6

Predictions of branching ratios and 1σ range errors with annihilation terms in units of 10^{-6} .

	best value	range
$B_s \rightarrow K^+\pi^-$	4.1	(6.7, 2.6)
$B_s \rightarrow K^0\pi^0$	1.1	(2.2, 0.4)
$B_s \rightarrow K^-K^+$	31.8	(51.9, 7.1)
$B_s \rightarrow K^0\bar{K}^0$	30.9	(50.7, 6.5)
$B_s \rightarrow \pi^-\pi^+$	2.3	(9.5, 0.0)
$B_s \rightarrow \pi^0\pi^0$	1.1	(4.7, 0.0)

in $B_d \rightarrow \pi^0\pi^0, \pi^+\pi^-$ and $B_s \rightarrow K^0\pi^0, K^-\pi^-$. Also several B_s decays can have large branching ratios. We presented results obtained with and without annihilation contributions. The results indicate that the annihilation contributions are in general small, but can still have large effect in several $B_s \rightarrow K^+K^-, K^0\bar{K}^0$ decay modes.

In SM predictions for branching ratios and CP asymmetries are possible because that SU(3) symmetry relates different decay modes. This symmetry is expected to be broken. In that case more parameters are needed to describe the decays. The effects of SU(3) breaking have to be

Table 7

Predictions of CP asymmetry with annihilation terms.

	best value	error range
$A_{K^-\bar{K}^0}^{B_u}$	-0.24	(0.82, -0.96)
$A_{\pi^0\pi^0}^{B_d}$	0.19	(0.72, -0.99)
$A_{K^0\bar{K}^0}^{B_d}$	0.78	(1.00, -1.00)
$A_{K^+\pi^-}^{B_s}$	0.26	(0.54, 0.03)
$A_{K^0\pi^0}^{B_s}$	0.12	(0.68, -0.86)
$A_{K^0\bar{K}^0}^{B_s}$	-0.02	(0.13, -0.12)
$A_{K^-\bar{K}^+}^{B_s}$	-0.06	(-0.02, -0.24)
$A_{K^0\bar{K}^0}^{B_s}$	-0.05	(0.18, -0.22)

further studied. We however expect that the general feature will not be altered[5] dramatically, and results obtained here can still provide some guidance in searching for large CP asymmetries and branching ratios in $B \rightarrow PP$ decays. Future experiments on $B \rightarrow PP$ will provide valuable information about the Standard Model.

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