eV seesaw with four generations

Wei-Shu Hou^{a,*}, Andrea Soddu^b

^a Department of Physics, National Taiwan University, Taipei, Taiwan 10617, ROC

^b Department of Particle Physics, Weizmann Institute of Science, Rehovot 76100, Israel

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Abstract

We extend the eV seesaw scenario to four lepton generations. The LSND anomaly is taken as the right-handed seesaw scale, i.e., $m_R \sim \text{eV}$. The fourth generation then gives a heavy pseudo-Dirac neutrino which largely decouples from other generations, and is relatively stable. One effectively has a $3 + 3$ solution to the LSND anomaly, where we illustrate with numerical solutions. Our study seems to indicate that the third mixing angle $\sin^2 \theta_{13}$ may be less than 0.01.

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1. Introduction

Neutrino oscillation measurements have become more and more accurate. The latest combined results [1] read $\Delta m_{\odot}^2 = (7.9 \pm 0.7) \times 10^{-5} \text{ eV}^2$ and $\Delta m_{\text{atm}}^2 = (2.4_{-0.6}^{+0.5}) \times 10^{-3} \text{ eV}^2$, for the mass squared differences of the large mixing angle solution (LMA) to the solar neutrino problem [2,3], and for atmospheric neutrinos [4–6], respectively. Both mass differences are sub-eV, but the neutrino mass scale is not yet certain.

Neutrino data also hint at the possibility of more than three massive, mostly active neutrinos. The Liquid Scintillator Neutrino Detector (LSND) Collaboration [7] has reported evidence for a $\bar{\nu}_e$ flux 30 meters away from a source of $\bar{\nu}_\mu$, produced in $\pi^+ \rightarrow \mu^+ \nu_\mu$ with subsequent $\mu^+ \rightarrow \bar{\nu}_\mu + e^+ + \nu_e$ decay. The unexpected flux can be explained if there is a small probability $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = (0.26 \pm 0.08)\%$ for a neutrino produced as a $\bar{\nu}_\mu$ to be detected as a $\bar{\nu}_e$ [8]. This LSND anomaly has yet to be confirmed, and the MiniBooNE experiment at Fermilab [9] should very soon give a definitive confirmation or refutation. Numerous attempts to solve the LSND puzzle, however, have already been proposed [10]. It has been shown that oscillations

with extra sterile neutrinos can fit the LSND anomaly [11]. But it has also been pointed out that extra sterile neutrinos could be in conflict with Big Bang Nucleosynthesis (BBN) [12], as well as SN1987A supernova neutrino events [13].

In a recent work [14], it has been argued that, if the right-handed neutrino Majorana scale m_R is of $\mathcal{O}(\text{eV})$, adequate fits to the LSND data can be obtained. This “eV seesaw” scenario runs against theoretical arguments in favor of a very large m_R . To name a few such arguments: the canonical seesaw mechanism [15–17] with $m_R \sim 10^{14} \text{ GeV}$ can elegantly explain why neutrino masses are so small, even with lepton Yukawa couplings that are of order one; thermal leptogenesis [18] points to $m_R \gtrsim 10^{10} \text{ GeV}$ [19]. However, as stressed in Ref. [14], nothing *experimental* is really known about the magnitude of m_R , except perhaps the LSND result, which is at eV scale.

The purpose of this Letter is to show that the eV seesaw proposed in Ref. [14] can be straightforwardly incorporated in a four generation scenario.

The Standard Model (SM) with a sequential fourth generation (SM4) is not ruled out by electroweak precision measurements, if one allows the extra active neutrino to have mass close to 50 GeV [20,21]. To avoid bounds from direct search at LEP II [22], mixing of the fourth heavy neutrino with the three light neutrinos should be small ($\lesssim 10^{-6}$). It is clear that in standard seesaw with $m_R \sim 10^{15} \text{ GeV}$, an extra generation is hard to

* Corresponding author.

E-mail address: wshou@phys.ntu.edu.tw (W.-S. Hou).

accommodate. (A different approach to predict a light sterile neutrino in the presence of a fourth generation is the so-called “flipped seesaw” [23].) All four (mostly) active neutrinos will then be light, contradicting the invisible Z width which measures only three light neutrinos. But taking m_R at scale $\mathcal{O}(\text{eV})$, one can now have a sufficiently heavy fourth neutrino.

In the following we will show that, by taking $m_R \sim \mathcal{O}(\text{eV})$, the fourth neutrino is pseudo-Dirac and heavy. It will not affect the invisible Z width, and largely decouples from lower generations. Aside from three mostly active light neutrinos, three sterile neutrinos with mass $\gtrsim \text{eV}$ is predicted. A numerical analysis gives results consistent with the LSND as well as solar and atmospheric data. It seems that $\sin^2 \theta_{13}$ cannot be large.

2. Pseudo-Dirac fourth neutrino

Following Ref. [14] but allowing for a possible 4th generation, the 8×8 neutrino mass matrix M is given by

$$M = M_D + \Delta M_R + \delta M_D, \quad (1)$$

in a form suggestive of mass hierarchies. In the basis where the 4×4 Dirac mass matrix is diagonal, the dominant Dirac mass for the 4th generation arises from

$$M_D = m_D \begin{pmatrix} 0 & I_4 \\ I_4 & 0 \end{pmatrix}, \quad (2)$$

where $m_D \sim 50 \text{ GeV}$, 0 and I_4 are 4×4 matrices with zero elements, except 1 in 44 element of I_4 . The right-handed Majorana mass matrix is given by

$$\Delta M_R = m_R \begin{pmatrix} 0 & 0 \\ 0 & r \end{pmatrix}, \quad (3)$$

where $m_R \sim \text{eV}$ [14], and r is a 4×4 symmetric matrix with elements $r_{ij} \sim 1$. The third matrix is

$$\delta M_D = m_R \begin{pmatrix} 0 & \varepsilon \\ \varepsilon & 0 \end{pmatrix}, \quad (4)$$

which is pinned more to the m_R scale, with

$$\varepsilon = \begin{pmatrix} \epsilon_1 & 0 & 0 & 0 \\ 0 & \epsilon_2 & 0 & 0 \\ 0 & 0 & \epsilon_3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (5)$$

where ϵ_4 has been absorbed into m_D . Clearly, with $m_R/m_D \equiv x \sim 10^{-10}$ and ϵ_i considerably less than 1, ΔM_R and δM_D can be treated as perturbations to M_D .

We note that the neutrino mass matrix of Eq. (1) could arise from very small deviations from a democratic structure for the Dirac contribution [24], and lepton number is assumed to be only slightly violated. The latter requires $m_R \sim 0$ [14], i.e., symmetry is enhanced in the limit of $\Delta M_R \rightarrow 0$. Having smaller elements in δM_D compared to m_R is purely phenomenological [14].

The dominant M_D has six null eigenvalues, plus two eigenvalues $\pm m_D$. For the eigenvectors associated with zero eigenvalues, one can choose $e_i^{(0)} = (0, \dots, 0, 1, 0, \dots, 0)$ with 1 in

the i th position for $i = 1, 2, 3$ and $i = 5, 6, 7$. For the eigenvalues $-m_D$ and m_D , the corresponding eigenvectors are $e_{4,8}^{(0)} = (0, 0, 0, \mp 1/\sqrt{2}, 0, 0, 0, 1/\sqrt{2})$. At zeroth order in x , the states $e_4^{(0)}$ and $e_8^{(0)}$ combine into a pure Dirac state of mass m_D . When linear corrections in x are considered, the perturbed states e_4 and e_8 with $e_i = e_i^{(0)} + x e_i^{(1)}$ have masses which differ by $\mathcal{O}(x)$, and they now correspond to a pseudo-Dirac neutrino with mass $\sim m_D$, which we denote as N (the charged partner is denoted E , with $m_E \gtrsim 100 \text{ GeV}$ [25]).

For the six null eigenvalues of M_D , one can apply perturbation theory with degeneracies. A linear combination of the degenerate unperturbed states $e_i^{(0)}$ diagonalizes the $\Delta M_R + \delta M_D$ perturbation. That is, by diagonalizing the 6×6 perturbation matrix

$$M^{(3)} = m_R \begin{pmatrix} 0 & 0 & 0 & \epsilon_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \epsilon_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \epsilon_3 \\ \epsilon_1 & 0 & 0 & r_{11} & r_{12} & r_{13} \\ 0 & \epsilon_2 & 0 & r_{21} & r_{22} & r_{23} \\ 0 & 0 & \epsilon_3 & r_{31} & r_{32} & r_{33} \end{pmatrix}, \quad (6)$$

one obtains the corrections of $\mathcal{O}(x)$ to the mass eigenvalues, and the correct eigenstates at order zero. For the effect of the 4th generation neutrino through the right-handed sector, r_{ij} , one has linear corrections in x proportional to r_{44} to the eigenvalues $\pm m_D$ for e_4 and e_8 as already stated, and $\mathcal{O}(x^2)$ corrections to all the other eigenvalues. The big hierarchy between the matrix elements M_{48} , $M_{84} \cong m_D$ and all the others allow the fourth generation to largely decouple from the other three. As stated in the introduction, this is also required by direct search limits that demand very small mixings between N and the light neutrino flavors.

Having reduced the problem to a 6×6 case, the analysis performed in [14] suggests that one could find a solution to the LSND puzzle. Our main goal here is to confirm the possibility of an existing solution, and to gain some insight on what could be a plausible scenario.

3. 3 + 3 neutrino model

We set to zero all phases for simplicity, since there are already too many parameters. We define U' to be the rotation matrix which diagonalizes $M^{(3)}$. Having started in the basis where the Dirac neutrino mass matrix $M_D + \delta M_D$ is diagonal, one still has the freedom to perform a rotation U'' in the left sector

$$U'' = \begin{pmatrix} c_1 c_3 & s_1 c_3 & s_3 & 0 & 0 & 0 \\ -s_1 c_2 - c_1 s_2 s_3 & c_1 c_2 - s_1 s_2 s_3 & s_2 c_3 & 0 & 0 & 0 \\ s_1 s_2 - c_1 c_2 s_3 & -c_1 s_2 - s_1 c_2 s_3 & c_2 c_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \quad (7)$$

We assume no mixing between the fourth and first three generation charged leptons, as already discussed. For the right sector, a rotation will just change r_{ij} to r'_{ij} , resulting in no change to our numerical analysis.

Table 1

Best fit values, 2σ and 3σ intervals for three-flavor neutrino oscillation parameters from global data, including solar, atmospheric, reactor (KamLAND and CHOOZ) and accelerator (K2K) experiments, taken from Ref. [28]

	Best fit	2σ	3σ
$\Delta m_{21}^2 (10^{-5} \text{ eV}^2)$	8.1	7.5–8.7	7.2–9.1
$\Delta m_{31}^2 (10^{-3} \text{ eV}^2)$	2.2	1.7–2.9	1.4–3.3
$\sin^2 \theta_{12}$	0.30	0.25–0.34	0.23–0.38
$\sin^2 \theta_{23}$	0.50	0.38–0.64	0.34–0.68
$\sin^2 \theta_{13}$	0.0	≤ 0.028	≤ 0.047

The probability for a neutrino, produced with flavor α and energy E , to be detected as a neutrino of flavor β after travelling a distance L is [25]

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{j>i}^n U_{\alpha j} U_{\beta j} U_{\alpha i} U_{\beta i} \sin^2 x_{ji}, \quad (8)$$

where $\alpha = e, \mu, \tau, s_i$ with s_i the sterile neutrino flavors, $U = U''U'$, and $x_{ji} = 1.27 \Delta m_{ji}^2 L/E$ with $\Delta m_{ji}^2 \equiv m_j^2 - m_i^2$. Applying Eq. (8) [26] in the “3 active plus 3 sterile neutrino” (3+3) case, using the approximations $x_{21} = x_{31} = x_{32} = 0$ and $x_{i1} = x_{i2} = x_{i3}$ for $i = 4, 5, 6$, one obtains

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta) &= \delta_{\alpha\beta} + 4[U_{\alpha 4}^2(U_{\beta 4}^2 - \delta_{\alpha\beta}) \sin^2 x_{41} \\ &+ U_{\alpha 5}^2(U_{\beta 5}^2 - \delta_{\alpha\beta}) \sin^2 x_{51} + U_{\alpha 6}^2(U_{\beta 6}^2 - \delta_{\alpha\beta}) \sin^2 x_{61} \\ &+ U_{\alpha 4} U_{\beta 4} U_{\alpha 5} U_{\beta 5} (\sin^2 x_{41} + \sin^2 x_{51} - \sin^2 x_{54}) \\ &+ U_{\alpha 4} U_{\beta 4} U_{\alpha 6} U_{\beta 6} (\sin^2 x_{41} + \sin^2 x_{61} - \sin^2 x_{64}) \\ &+ U_{\alpha 5} U_{\beta 5} U_{\alpha 6} U_{\beta 6} (\sin^2 x_{51} + \sin^2 x_{61} - \sin^2 x_{65})], \end{aligned} \quad (9)$$

where orthogonality of U has been used. Expressions for the mixing angles are given by [27]

$$\begin{aligned} \tan^2 \theta_{12} &\equiv \frac{|U_{e2}|^2}{|U_{e1}|^2}, & \tan^2 \theta_{23} &\equiv \frac{|U_{\mu 3}|^2}{|U_{\tau 3}|^2}, \\ \sin^2 \theta_{13} &\equiv |U_{e3}|^2. \end{aligned} \quad (10)$$

To perform our numerical analysis, we build the χ^2 by using the three-flavor neutrino oscillation parameters Δm_{ji}^2 and $\sin^2 \theta_{ij}$ taken from Ref. [28], which is compiled from global data including solar, atmospheric, reactor (KamLAND and CHOOZ) and accelerator (K2K) experiments. These are given in Table 1. We also include the LSND result of $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = (0.26 \pm 0.08)\%$, and require $\Delta m_{41}^2 \sim 1 \text{ eV}^2$, for a total of seven inputs. As one clearly has too many parameters to make a proper fit, we just minimize the χ^2 built with these quantities by letting the twelve parameters of the model vary.

As an illustration, we find $s_1 = -0.57$, $s_2 = 0.98$, $s_3 = 0.80$, which give

$$U'' = \begin{pmatrix} 0.49 & -0.34 & 0.80 & 0 & 0 & 0 \\ -0.54 & 0.60 & 0.58 & 0 & 0 & 0 \\ -0.69 & -0.72 & 0.11 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad (11)$$

$$M^{(3)} = (5 \text{ eV}) \begin{pmatrix} 0 & 0 & 0 & 0.051 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.004 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.031 \\ 0.051 & 0 & 0 & 1.195 & 0.861 & 1.038 \\ 0 & 0.004 & 0 & 0.861 & 0.968 & 0.878 \\ 0 & 0 & 0.031 & 1.038 & 0.878 & 1.264 \end{pmatrix}, \quad (12)$$

from which, together with Eq. (6), one can read the values for the rest of the parameters coming from the minimization process. The eigenvalues of $M^{(3)}$ are

$$(m_1, m_2, m_3) = -(7 \times 10^{-5}, 9 \times 10^{-3}, 0.048) \text{ eV},$$

$$(m_4, m_5, m_6) = (1, 1.2, 15) \text{ eV}, \quad (13)$$

and the associated rotation matrix is

$$U' = \begin{pmatrix} -0.06 & -0.99 & -0.11 & 0 & 0 & 0 \\ -0.38 & 0.12 & -0.91 & 0.01 & -0.06 & 0.05 \\ 0.90 & -0.02 & -0.37 & -0.17 & 0.05 & 0.12 \\ -0.19 & 0 & 0.09 & -0.75 & 0.16 & 0.60 \\ 0.05 & -0.01 & 0.07 & 0.22 & -0.83 & 0.49 \\ 0.01 & 0 & 0 & 0.60 & 0.52 & 0.61 \end{pmatrix}. \quad (14)$$

From Eq. (13) one gets $\Delta m_{21}^2 = 8.1 \times 10^{-5} \text{ eV}^2$ and $\Delta m_{31}^2 = 2.3 \times 10^{-3} \text{ eV}^2$, which, of course, is in good agreement with data. The requirements for the neutrino mass splitting applied in Ref. [26] for the 3+2 case, $0.1 \text{ eV}^2 \leq \Delta m_{41}^2 \leq \Delta m_{51}^2 \leq 100 \text{ eV}^2$, are also satisfied. This guarantees that the approximations used to derive Eq. (9) are valid.

From Eqs. (11) and (14), we obtain the full rotation matrix $U = U''U'$, i.e.,

$$U = \begin{pmatrix} 0.82 & -0.54 & -0.04 & -0.14 & 0.06 & 0.08 \\ 0.33 & 0.60 & -0.71 & -0.09 & -0.01 & 0.10 \\ 0.42 & 0.59 & 0.69 & -0.03 & 0.05 & -0.03 \\ -0.19 & 0 & 0.09 & -0.75 & 0.16 & 0.60 \\ 0.05 & -0.01 & 0.07 & 0.22 & -0.84 & 0.49 \\ 0.01 & 0 & 0.01 & 0.60 & 0.52 & 0.61 \end{pmatrix}. \quad (15)$$

Using $x_{ji} = 1.27 \Delta m_{ji}^2 (\text{eV}^2) L(\text{m}) / E(\text{MeV})$ with $L/E \sim 1$, together with Eq. (9) for the $\mu \rightarrow e$ case, Eqs. (13) and (15), one obtains $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = 0.15\%$, which is within 2σ from the LSND central value.

$M^{(3)}$ in Eq. (12) can be viewed as deviating from

$$M^{(3)} = m_R \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}, \quad (16)$$

which has five zero eigenvalues and one nonzero eigenvalue equal to $3m_R \sim 15 \text{ eV}$, and diagonalized by

$$U' = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}. \quad (17)$$

Together with U'' of Eq. (7), one has

$$U = \begin{pmatrix} c_1 c_3 & s_1 c_3 & s_3 & 0 & 0 & 0 \\ -s_1 c_2 - c_1 s_2 s_3 & c_1 c_2 - s_1 s_2 s_3 & s_2 c_3 & 0 & 0 & 0 \\ s_1 s_2 - c_1 c_2 s_3 & -c_1 s_2 - s_1 c_2 s_3 & c_2 c_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}. \quad (18)$$

Applying this to Eq. (9) for the $\mu \rightarrow e$ case, one sees that the transition probability $P(\nu_\mu \rightarrow \nu_e)$ would vanish. Deviations from Eq. (16) as realized in Eq. (12) not only produces the needed neutrino mass spectrum, finite transition probability $P(\nu_\mu \rightarrow \nu_e)$ is also achieved.

In our analysis we did not take into account the null short-baseline experiments (NSBL). Let us compare our results with the ones obtained by doing a full analysis in Ref. [26] for the $(3+2)$ case. Using the rotation matrix elements $U_{e4} = -0.14$, $U_{\mu 4} = -0.09$, $U_{e5} = 0.06$, $U_{\mu 5} = -0.01$, $U_{e6} = 0.08$ and $U_{\mu 6} = 0.10$ as one can read from Eq. (15), together with Eq. (9), respectively, for $\mu \rightarrow e$, $e \rightarrow e$ and $\mu \rightarrow \mu$, we obtain $P(\nu_\mu \rightarrow \nu_e) = 0.0015$, $P(\nu_e \rightarrow \nu_e) = 0.89$ and $P(\nu_\mu \rightarrow \nu_\mu) = 0.93$ in the $L/E \sim 1$ approximation. Our calculated values for oscillation appearance and disappearance probabilities can now be compared with the ones obtained by using the best fit values for the rotation matrix elements in the $(3+2)$ case as in Ref. [26], $U_{e4} = 0.121$, $U_{\mu 4} = 0.204$, $U_{e5} = 0.036$ and $U_{\mu 5} = 0.224$. In the $L/E \sim 1$ approximation these give $P(\nu_\mu \rightarrow \nu_e) = 0.0021$, $P(\nu_e \rightarrow \nu_e) = 0.95$ and $P(\nu_\mu \rightarrow \nu_\mu) = 0.84$. We remark that, although a full analysis is needed to tell if our model is able to accommodate both NSBL and LSND data, our predictions seem to be not too far away from the results of Ref. [26].

From Eqs. (10) and (15), we find $\sin^2 \theta_{12} = 0.30$, $\sin^2 \theta_{23} = 0.52$, and

$$\sin^2 \theta_{13} = 0.0018. \quad (19)$$

As expected, the values for $\sin^2 \theta_{12}$ and $\sin^2 \theta_{23}$ are in good agreement with data, but the as yet unmeasured $\sin^2 \theta_{13}$ turns out to be rather small.

4. In search of sizable $\sin^2 \theta_{13}$

To investigate the possibility for a bigger value of $\sin^2 \theta_{13}$, we restrict the χ^2 to the four inputs of $\sin^2 \theta_{ij}$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$. The mass spectrum is not affected by the rotation of Eq. (7). With ϵ_i and r_{ij} given as in Eq. (12), we first fix $s_1 = -0.57$ and perform a χ^2 fit vs s_2 and s_3 . We iterate with fixing $s_2 = 0.98$ ($s_3 = 0.8$) and minimize χ^2 vs s_1 and s_3 (s_1 and s_2). We find for both cases of fixing s_1 and s_3 to the values found in previous section, $\sin^2 \theta_{23}$ is quite strongly dependent on s_2 , and the value around 0.98 is preferred. We thus illustrate with fixing $s_2 = 0.98$.

In Fig. 1 we show the contour plot of χ^2 vs s_1, s_3 . The three different shaded regions should not be interpreted as the 1σ , 2σ and 3σ regions, since we have fixed the rest of the parameters to the best fit values. But they still give an indication of

Fig. 1. Contour-plot of χ^2 vs the mixing angles s_1 and s_3 , with ϵ_i and r_{ij} as in Eq. (12), and $s_2 = 0.98$ held fixed. The regions in different shades are only indicative, and should not be interpreted as the 1σ , 2σ and 3σ regions, as the rest of the parameters are fixed at the best fit values.

Fig. 2. $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$, $\sin^2 \theta_{12}$, $\sin^2 \theta_{23}$ and $\sin^2 \theta_{13}$ vs. s_1 and s_3 , corresponding to the lower right solution in Fig. 1, with ϵ_i and r_{ij} fixed as in Eq. (12), and $s_2 = 0.98$.

variations around the best fit region under the above assumptions.

In Fig. 2 we plot the four quantities $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$, $\sin^2 \theta_{12}$, $\sin^2 \theta_{23}$ and $\sin^2 \theta_{13}$ vs s_1 and s_3 , for the solution on the lower right of Fig. 1. The same is plotted in Fig. 3 for the upper left solution. Again, ϵ_i and r_{ij} are fixed as in Eq. (12), and s_2 is held fixed at 0.98. We see that $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ can reach the one σ region and $\sin^2 \theta_{12}$ is well within range. However, to push $\sin^2 \theta_{13}$ beyond 0.01, $\sin^2 \theta_{23}$ seems to wander away from maximal mixing of 0.5, and values at ~ 0.4 or 0.6 has to be tolerated. We note further that the sensitivity of $\sin^2 \theta_{23}$ is to s_1 , rather than s_3 .

We conclude that $\sin^2 \theta_{13}$ greater than 0.01 is possible, but seemingly not preferred. It is not clear whether this is an artefact of not being able to do a real fit. Note that we have not

Fig. 3. Same as Fig. 2, but corresponding to upper left solution in Fig. 1.

checked explicitly whether constraints from short baseline disappearance experiments are fully satisfied.

5. Discussion and conclusion

It is tempting to consider whether mixing between the fourth and the first three light charged lepton generations could modify the situation with $\sin^2 \theta_{13}$. But as already mentioned in the introduction, one needs to satisfy both bounds from direct search at LEP II and electroweak precision measurements. We have pursued a numerical study, but find that, if we wish to keep the mixings sufficiently small so that the fourth active heavy neutrino will be semi-stable, no important change with respect to the no-mixing case is observed. We note that the heavy neutrino could be heavier than 50 GeV and still with suppressed mixing to lower generations, but then one would have to face electroweak precision constraints. We note in passing that semi-stable heavy neutrinos are still of interest [29] to dark matter search experiments, as a fourth heavy lepton was once a leading dark matter candidate.

In summary, we have extended the eV seesaw scenario to four lepton generations. Taking the LSND scale as the right-handed seesaw scale $m_R \sim \text{eV}$, one has a heavy pseudo-Dirac neutrino with mass $m_N \sim 50 \text{ GeV}$, which largely decouples from other generations, and is relatively stable. One effectively has a $3 + 3$ solution to the LSND anomaly, where we illustrate with numerical solutions. As a possible outcome, our numerical study indicates that the third mixing angle, $\sin^2 \theta_{13}$, seems to be less than 0.01.

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