Electroweak model-independent tests for SU(3) symmetry in hadronic B decays

Xiao-Gang He, Jenq-Yuan Leou, and Chung-Yi Wu

Physics Department, National Taiwan University, Taipei, Taiwan 10764, Republic of China

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We study the effects of new physics beyond the standard model on SU(3) symmetry in charmless hadronic two body B decays. It is found that several equalities for some of the decay amplitudes if small annihilation contributions are neglected, such as $A(B_d(B_u) \rightarrow \pi^+ \pi^-, \pi^+ K^- (\pi^- \bar{K}^0)) = A(B_s \rightarrow K^+ \pi^-, K^- K^+ (K^0 \bar{K}^0))$, $A(B_d \rightarrow \pi^+ \rho^-, \pi^- \rho^+, K^- \rho^+, \pi^+ K^{*-}) = A(B_s \rightarrow K^+ \rho^-, \pi^- K^{*+}, K^- K^{*+}, K^+ K^{*-})$, $A(B_d(B_u) \rightarrow \rho^+ \rho^-, \rho^+ K^{*-} (\rho^- \bar{K}^{*0})) = A(B_s \rightarrow K^{*+} \rho^-, K^{*-} K^{*+} (K^{*0} \bar{K}^{*0}))$, predicted by SU(3) symmetry in the SM are not affected by new physics. These relations provide important electroweak model independent tests for SU(3) symmetry in B decays. We also discuss how the assumption of the smallness of annihilation contributions can be tested experimentally.

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The calculations of hadronic B decays are difficult due to our poor understanding of QCD hadronic physics. Model calculations can account for some of the measured B decays, but not all of them [1,2]. The results from model calculations are, in any case, far from the desired accuracy needed to test the standard model (SM) and models beyond. To overcome some of the difficulties, several groups have proposed to use SU(3) symmetry to study charmless hadronic two body B decays and have obtained some interesting results [3–7]. At present SU(3) symmetry is not well tested, although it has been shown to be consistent with data from $B_u \rightarrow D^0 \pi^- (K^-)$ decays [5]. In order to have a good understanding of the underlying theory of B decays using information extracted from SU(3) symmetry considerations, it is important to know to what precision the SU(3) symmetry is valid. To achieve this, one should not only obtain testable relations predicted by SU(3) symmetry in the SM, but also make sure that the relations are not modified by new physics beyond the SM for electroweak interactions. In this paper we study the effects of new physics beyond the SM on SU(3) symmetry predictions in some of the charmless hadronic two body B decays into SU(3) octet pseudoscalars P and/or vectors V . We find that, indeed, it is possible to test SU(3) symmetry in a way which is independent of models for electroweak interactions.

We find some equalities for hadronic B decays using SU(3) symmetry when small annihilation contributions are neglected. These are the following relations for $B \rightarrow PP$ modes:

$$\begin{aligned} A(B_d(B_u) \rightarrow \pi^+ \pi^-, \pi^+ K^- (\pi^- \bar{K}^0)) \\ = A(B_s \rightarrow K^+ \pi^-, K^- K^+ (K^0 \bar{K}^0)), \end{aligned} \quad (1)$$

and the following relations for $B \rightarrow PV$ and $B \rightarrow VV$ modes:

$$\begin{aligned} A(B_d \rightarrow \pi^+ \rho^-, \pi^- \rho^+, K^- \rho^+, \pi^+ K^{*-}) \\ = A(B_s \rightarrow K^+ \rho^-, \pi^- K^{*+}, K^- K^{*+}, K^+ K^{*-}), \\ A(B_d(B_u) \rightarrow \rho^+ \rho^-, \rho^+ K^{*-} (\rho^- \bar{K}^{*0})) \\ = A(B_s \rightarrow K^{*+} \rho^-, K^{*-} K^{*+} (K^{*0} \bar{K}^{*0})). \end{aligned} \quad (2)$$

Further we find that these equalities are not affected by new physics beyond the SM. We note that the relations in Eqs. (1) and (2) always involve charmless hadronic B_s decays which have not been measured. However, these decay modes have relatively large branching ratios (10^{-5} – 10^{-6}) from model calculations and are expected to be measured at the Collider Detector at Fermilab (CDF), D0, DESY HERA-B, CERN LHC-B, and BTeV. When all related modes are measured SU(3) symmetry will be tested in an electroweak model independent way. In the following we provide more details.

In the SM the quark level effective Hamiltonian for charmless hadronic B decays, including QCD corrections, can be written as

$$\begin{aligned} H_{\text{eff}}^q = \frac{G_F}{\sqrt{2}} \left[V_{ub} V_{uq}^* (c_1 O_1 + c_2 O_2) \right. \\ \left. - \sum_{i=3}^{12} (V_{ub} V_{uq}^* c_i^{uc} + V_{tb} V_{tq}^* c_i^{tc}) O_i^q \right], \end{aligned}$$

where $c_{1,2}$ and $c_i^{jk} = c_i^j - c_i^k$ are the Wilson coefficients which have been evaluated in various renormalization schemes [8]. Here j, k indicate the internal quarks for loop induced operators. q can be d or s depending on whether the decays are $\Delta S=0$ or $\Delta S=1$. The specific form of the operators can be found in Ref. [8]. Since we will only need to know their SU(3) structures, we will suppress Lorentz structure and treat the Fierz transformed and untransformed forms the same in our later discussions.

The operators $O_{3,4,5,6,11}$ all have a simple SU(3) structure and transform as $\bar{3}$. The complication comes from the fact that $O_{1,2,7,8,9,10,12}$ are not a single SU(3) irreducible representation. They contain two $\bar{3}$'s, one 6, and one $\bar{15}$. Although $O_{7,8,9,10}$ are electroweak type with small Wilson coefficients, they are important for B decays and must be kept [9]. For illustration we give the detailed decomposition of $O_2 = \bar{u}b\bar{q}u$ for $q=d$ and have

$$\begin{aligned}
\bar{u}b\bar{d}u &= \frac{1}{8} \{ 3[\bar{u}b\bar{d}u + \bar{d}b\bar{d}d + \bar{s}b\bar{d}s]_{\bar{3}} - [\bar{d}b(\bar{u}u + \bar{d}d + \bar{s}s)]_{\bar{3}} \\
&\quad + 2[\bar{u}b\bar{d}u - \bar{d}b\bar{u}u + \bar{d}b\bar{s}s - \bar{s}b\bar{d}s]_6 \\
&\quad + [3\bar{u}b\bar{d}u + 3\bar{d}b\bar{u}u - 2\bar{d}b\bar{d}d - \bar{d}b\bar{s}s - \bar{s}b\bar{d}s]_{\bar{15}} \} \\
&= \frac{1}{8} [3H(\bar{3}) - H(\bar{3}') + 2H(6) + H(\bar{15})], \tag{3}
\end{aligned}$$

where $H(i)$ are matrices in SU(3) flavor space. With the identification of $u=1$, $d=2$, and $s=3$, the nonzero entries of $H(i)$ are given by [5]

$$\begin{aligned}
H(\bar{3}^{(')})^2 &= 1, \quad H(6)_1^{12} = H(6)_3^{23} = 1, \\
H(\bar{15})_1^{12} &= 3, \quad H(\bar{15})_2^{22} = -2, \quad H(\bar{15})_3^{32} = -1. \tag{4}
\end{aligned}$$

$H(6)$ and $H(\bar{15})$ are antisymmetric and symmetric in exchanging the upper two indices, respectively.

For $q=s$ case, we have [5]

$$\begin{aligned}
H(\bar{3}^{(')})^2 &= 1, \quad H(6)_1^{13} = H(6)_2^{32} = 1, \\
H(\bar{15})_1^{13} &= 3, \quad H(\bar{15})_3^{33} = -2, \quad H(\bar{15})_2^{32} = -1. \tag{5}
\end{aligned}$$

The $H(i)$ matrices for other operators can be obtained in a similar way.

At the hadronic level, the decay amplitudes can be written as

$$A(B \rightarrow ij) = V_{ub}V_{uq}^* T^{\text{SM}} + V_{tb}V_{tq}^* P^{\text{SM}}. \tag{6}$$

Here T^{SM} and P^{SM} both have $\bar{3}$, 6, and $\bar{15}$ components through operators associated with $c_{1,2}$, c_i^{uc} , and c_i^{tc} . Since we are only concerned with the SU(3) structure, the detailed coefficients are not important, that is, difference operators having the same SU(3) irreducible representations can be combined together and denoted by certain SU(3) invariant amplitudes. The amplitudes T^{SM} can be written in the following form for $B \rightarrow PP$ decays [5]:

$$\begin{aligned}
T^{\text{SM}} &= A_{\bar{3}}^T B_i H(\bar{3})^i (M_k^l M_j^l) + C_{\bar{3}}^T B_i M_k^i M_j^k H(\bar{3})^j \\
&\quad + A_6^T B_i H(6)_k^{ij} (M_j^l M_i^l) + C_6^T B_i M_j^i H(6)_l^{jk} M_k^l \\
&\quad + A_{\bar{15}}^T B_i H(\bar{15})_k^{ij} (M_j^l M_i^l) + C_{\bar{15}}^T B_i M_j^i H(\bar{15})_l^{jk} M_k^l, \tag{7}
\end{aligned}$$

where M_j^i is the pseudoscalar octet P . B_i is the B -meson SU(3) triplet (B_u , B_d , B_s). In the case for $B \rightarrow PP$ decays A_6 and C_6 always appear together in the form $C_6 - A_6$ [5]. We will eliminate A_6 in the expressions. P^{SM} can be obtained similarly.

The amplitudes A_i correspond to annihilation contributions which can be seen from Eq. (7) where B_i is contracted with one of the indices in H matrices. These contributions are much smaller than the amplitudes C_i from model calculations [2,5]. The smallness of these annihilation amplitudes

can be tested by measuring the branching ratios for $B_d \rightarrow K^+ K^-$, $B_s \rightarrow \pi^+ \pi^-$, $\pi^0 \pi^0$ which are proportional to $A_{\bar{3}} + A_{\bar{15}}$ [4,5]. We will work with the assumption that annihilation contributions are small and can be neglected. Future experiments will decide if this assumption is valid [4–6]. Expanding Eq. (7) one obtains the decay amplitudes in terms of A_i and C_i . The relevant decay amplitudes are given by

$$\begin{aligned}
T_{B_d \rightarrow \pi^+ \pi^-, B_s \rightarrow K^- K^+}^{\text{SM}} &= 2A_{\bar{3}}^T + A_{\bar{15}}^T + C_{\bar{3}}^T + C_6^T + 3C_{\bar{15}}^T, \\
T_{B_s \rightarrow K^+ \pi^-, B_d \rightarrow \pi^+ K^-}^{\text{SM}} &= -A_{\bar{15}}^T + C_{\bar{3}}^T + C_6^T + 3C_{\bar{15}}^T, \\
T_{B_u \rightarrow \pi^- \bar{K}^0}^{\text{SM}} &= 3A_{\bar{15}}^T + C_{\bar{3}}^T - C_6^T - C_{\bar{15}}^T, \\
T_{B_s \rightarrow K^0 \bar{K}^0}^{\text{SM}} &= 2A_{\bar{3}}^T - 3A_{\bar{15}}^T + C_{\bar{3}}^T - C_6^T - C_{\bar{15}}^T. \tag{8}
\end{aligned}$$

In naive quark diagram analysis, when annihilation contributions are neglected, $B_u(B_s) \rightarrow \pi^- \bar{K}^0 (K^- \bar{K}^0)$ do not have contributions from $O_{1,2}$ [3,4]. This is not true when going beyond naive quark diagram analysis and it needs to be tested [5]. Our results, however, do not rely on whether they are zero or not.

For $B \rightarrow VV$, the decay amplitudes can be obtained from $B \rightarrow PP$ by a simple replacement of the corresponding final states. The decay amplitudes for $B \rightarrow PV$ are more complicated because the fact that there are two terms, except for A_3 , for each of the terms in Eq. (7). For example, the term corresponding to $C_{\bar{3}}^T B_i M_k^i M_j^k H(\bar{3})^j$ becomes $C_{\bar{3}}^V B_i V_k^i V_j^k H(\bar{3})^j$ and $C_{\bar{3}}^M B_i M_k^i V_j^k H(\bar{3})^j$. Although there are relations between $A_6^{V,M}$ and $C_6^{V,M}$, they do not always appear together as $C_6 - A_6$ as for $B \rightarrow PP$. We will need to keep all of them. The details for the whole amplitudes can be found in Ref. [6]. The smallness of annihilation contributions for $B \rightarrow VV$ and $B \rightarrow PV$ can, again, be tested by measuring some pure annihilation decays such as $B_d \rightarrow K^{*+} K^{*-}$, $K^+ K^{*-}$, $K^- K^{*+}$, $B_s \rightarrow \rho^+ \rho^-$, $\rho^0 \rho^0$, $\pi^+ \rho^-$, $\pi^- \rho^+$, $\pi^0 \rho^0$.

From Eq. (8) and the tables in Ref. [6], we find that when annihilation contributions are neglected (setting all A_i to zero), the equalities in Eqs. (1) and (2) hold in the SM. These relations can be used to test whether SU(3) is a good symmetry for B decays in the SM. We emphasize that these relations are true regardless of whether there are large rescattering effects as long as SU(3) is a good symmetry.

The relations in Eqs. (1) and (2) hold in SU(3) symmetry limit. There are SU(3) breaking effects. These include differences in phase space for $B_{u,d}$ and B_s decays, and also in the decay amplitudes. It is not possible to reliably calculate the breaking effects in the decay amplitudes at present. Factorization approximation gives, for example, $A[B_d(B_u) \rightarrow \pi^+ \pi^-, \pi^+ K^- (\pi^- \bar{K}^0)] = (F_0^{B_d \pi}/F_0^{B_s K}) A[B_s \rightarrow K^+ \pi^-, K^- K^+ (K^0 \bar{K}^0)]$, where $F_0^{B_{ij}}$ are transition form factors. Model calculations indicate that the ratio $F_0^{B_{u,d} \pi}/F_0^{B_s K}$ is close to 1. Deviation from one for this ratio would be an indication of SU(3) breaking in B decays. Estimates of SU(3) breaking effects for other decays can be obtained in a similar way. As already mentioned that accurate

theoretical calculations are very difficult to carry out, we, therefore, will not attempt to obtain precise theoretical predictions of SU(3) symmetry breaking effects, but to study if the relations in Eqs. (1) and (2) are modified by new physics beyond the SM and further to study if SU(3) symmetry and its breaking can be determined in an electroweak model independent way.

In the presence of new physics beyond the SM, there are new operators in addition to the ones already present in the SM. As far as SU(3) structure is concerned, there are two types of new operators which can appear at the four quark level for charmless hadronic $\Delta S=0$ and $\Delta S=1$ B decays. These operators are

$$O_{\bar{q}dd} = \bar{q}b\bar{d}d, \quad O_{\bar{q}ss} = \bar{q}b\bar{s}s. \quad (9)$$

These operators can naturally appear in extensions of the SM. For example the term $U_R D_R D_R$ in R -parity violating supersymmetric models [10] can induce $O_{\bar{q}dd, \bar{q}ss}$ by exchanging s -up quarks with sizeable contributions to B decays. In the same model exchanging s -down quarks can also generate $\bar{u}b\bar{q}u$ type of operators which has the same SU(3) structure as the operators $O_{1,2}$ in the SM, but with difference Lorentz structure.

One may wonder if considering operators just up to dimension 6 are sufficient. Higher order operators may have more complicated structure due to higher SU(3) irreducible representations. However, because exchange of gluons, soft or hard, will not change the SU(3) structure, the contributions of higher order operators with different SU(3) structures will have additional suppression factors from loop integrals or additional propagators of electroweak types in a model and can be neglected. Thus considering operators up to dimension 6 is sufficient for our purpose.

The operators $O_{\bar{d}dd}$ and $O_{\bar{s}ss}$ contain only $\bar{3}^{(')}$ and $\bar{15}$. The nonzero entries of $H(\bar{3}^{(')})$ are the same as the corresponding ones in the SM with an appropriate normalizations. The nonzero entries of $H(\bar{15})$ are given by

$$\begin{aligned} H^{\bar{d}dd}(\bar{15})_1^{12} &= H^{\bar{d}dd}(\bar{15})_3^{23} = -2, \quad H^{\bar{d}dd}(\bar{15})_2^{22} = 4, \\ H^{\bar{s}ss}(\bar{15})_1^{13} &= H^{\bar{s}ss}(\bar{15})_2^{23} = -2, \quad H^{\bar{s}ss}(\bar{15})_3^{33} = 4. \end{aligned} \quad (10)$$

The operators $O_{\bar{s}dd}$ and $O_{\bar{d}ss}$ contain $\bar{3}^{(')}$, 6, and $\bar{15}$. The nonzero entries of $H(\bar{3}^{(')})$ can again be normalized to be the same as the corresponding ones in the SM. The nonzero entries of 6 and $\bar{15}$ are

$$\begin{aligned} H^{\bar{d}ss}(6)_3^{23} &= H^{\bar{d}ss}(6)_1^{12} = 1, \\ H^{\bar{d}ss}(\bar{15})_3^{23} &= 3, \quad H^{\bar{d}ss}(\bar{15})_2^{22} = -2, \quad H^{\bar{d}ss}(\bar{15})_1^{12} = -1, \\ H^{\bar{s}dd}(6)_1^{13} &= H^{\bar{s}dd}(6)_2^{32} = 1, \\ H^{\bar{s}dd}(\bar{15})_2^{23} &= 3, \quad H^{\bar{s}dd}(\bar{15})_3^{33} = -2, \quad H^{\bar{s}dd}(\bar{15})_1^{13} = -1. \end{aligned} \quad (11)$$

With these new operators the charmless hadronic B decay amplitudes will be modified. Normalizing to the SM amplitudes, we can write the total amplitudes as

$$\begin{aligned} A(B \rightarrow ij) &= V_{ub}V_{uq}^* T^{\text{SM}} + V_{tb}V_{tq}^* P^{\text{SM}} + a^{\bar{q}uu} P^{\bar{q}uu} \\ &\quad + a^{\bar{q}dd} P^{\bar{q}dd} + a^{\bar{q}ss} P^{\bar{q}ss}, \end{aligned} \quad (12)$$

where a^i indicate the coefficients due to new physics beyond the SM, and $P^{\bar{q}ll} = \langle ij | O_{\bar{q}ll} | B \rangle$. Here we have also included the contributions from operators of the form $O_{\bar{q}uu}$ which have the same SU(3) structure as $O_{1,2}$ in the SM, but are due to new physics and also may have difference Lorentz structures.

Following the same procedure as for the SM discussed before, one can obtain the decay amplitudes in terms of the SU(3) invariant amplitudes. The decay amplitudes due to the new operators for the relevant $B \rightarrow PP$ modes are given as below.

For $\Delta S=0$ decay modes, we have

$$\begin{aligned} P_{B_d \rightarrow \pi^+ \pi^-}^{\bar{d}dd} &= 2A_{\bar{3}}^{\bar{d}dd} + 2A_{\bar{15}}^{\bar{d}dd} + C_{\bar{3}}^{\bar{d}dd} - 2C_{\bar{15}}^{\bar{d}dd}, \\ P_{B_s \rightarrow K^+ \pi^-}^{\bar{d}dd} &= -2A_{\bar{15}}^{\bar{d}dd} + C_{\bar{3}}^{\bar{d}dd} - 2C_{\bar{15}}^{\bar{d}dd}, \\ P_{B_d \rightarrow \pi^+ \pi^-}^{\bar{d}ss} &= 2A_{\bar{3}}^{\bar{d}ss} - 3A_{\bar{15}}^{\bar{d}ss} + C_{\bar{3}}^{\bar{d}ss} + C_6^{\bar{d}ss} - C_{\bar{15}}^{\bar{d}ss}, \\ P_{B_s \rightarrow K^+ \pi^-}^{\bar{d}ss} &= 3A_{\bar{15}}^{\bar{d}ss} + C_{\bar{3}}^{\bar{d}ss} + C_6^{\bar{d}ss} - C_{\bar{15}}^{\bar{d}ss}. \end{aligned} \quad (13)$$

For the two pairs of $\Delta S=1$ decay modes, we have

$$\begin{aligned} P_{B_d \rightarrow \pi^+ K^-}^{\bar{s}ss} &= -2A_{\bar{15}}^{\bar{s}ss} + C_{\bar{3}}^{\bar{s}ss} - 2C_{\bar{15}}^{\bar{s}ss}, \\ P_{B_s \rightarrow K^+ K^-}^{\bar{s}ss} &= 2A_{\bar{3}}^{\bar{s}ss} + 2A_{\bar{15}}^{\bar{s}ss} + C_{\bar{3}}^{\bar{s}ss} - 2C_{\bar{15}}^{\bar{s}ss}, \\ P_{B_d \rightarrow \pi^+ K^-}^{\bar{s}dd} &= 3A_{\bar{15}}^{\bar{s}dd} + C_{\bar{3}}^{\bar{s}dd} + C_6^{\bar{s}dd} - C_{\bar{15}}^{\bar{s}dd}, \\ P_{B_s \rightarrow K^+ K^-}^{\bar{s}dd} &= 2A_{\bar{3}}^{\bar{s}dd} - 3A_{\bar{15}}^{\bar{s}dd} + C_{\bar{3}}^{\bar{s}dd} + C_6^{\bar{s}dd} - C_{\bar{15}}^{\bar{s}dd} \end{aligned} \quad (14)$$

and

$$\begin{aligned} P_{B_u \rightarrow \pi^- \bar{K}^0}^{\bar{s}ss} &= -2A_{\bar{15}}^{\bar{s}ss} + C_{\bar{3}}^{\bar{s}ss} - 2C_{\bar{15}}^{\bar{s}ss}, \\ P_{B_s \rightarrow K^0 \bar{K}^0}^{\bar{s}ss} &= 2A_{\bar{3}}^{\bar{s}ss} + 2A_{\bar{15}}^{\bar{s}ss} + C_{\bar{3}}^{\bar{s}ss} - 2C_{\bar{15}}^{\bar{s}ss}, \\ P_{B_u \rightarrow \pi^- \bar{K}^0}^{\bar{s}dd} &= -A_{\bar{15}}^{\bar{s}dd} + C_{\bar{3}}^{\bar{s}dd} - C_6^{\bar{s}dd} + 3C_{\bar{15}}^{\bar{s}dd}, \\ P_{B_s \rightarrow K^0 \bar{K}^0}^{\bar{s}dd} &= 2A_{\bar{3}}^{\bar{s}dd} + A_{\bar{15}}^{\bar{s}dd} + C_{\bar{3}}^{\bar{s}dd} - C_6^{\bar{s}dd} + 3C_{\bar{15}}^{\bar{s}dd}. \end{aligned} \quad (15)$$

From the above expressions for $P^{\bar{q}dd, \bar{q}ss}$, we clearly see that when annihilation contributions are neglected the equalities in Eq. (1) hold. In fact the new operators have zero contri-

TABLE I. SU(3) invariant amplitudes for $B \rightarrow PV$ decays in models beyond SM. The factors $a_i^{V,M}$ and $b_i^{V,M}$ are defined through $Pq\bar{d}d, q\bar{s}s = \sum_i [a_i^V A_i^V + a_i^M A_i^M + b_i^V C_i^V + b_i^M C_i^M]$ as in Ref. [6]. It is understood that A_i and C_i associated with each type of operators are different.

Decay modes	$a_{\bar{3}}^V$	a_6^V	a_6^M	a_{15}^V	a_{15}^M	$b_{\bar{3}}^V$	$b_{\bar{3}}^M$	b_6^V	b_6^M	b_{15}^V	b_{15}^M
$P_{B_d \rightarrow \pi^- \rho^+}^{\bar{d}dd}$	1	0	0	-2	4	1	0	0	0	-2	0
$P_{B_s \rightarrow \pi^- K^{*+}}^{\bar{d}dd}$	0	0	0	0	-2	1	0	0	0	-2	0
$P_{B_d \rightarrow \pi^- \rho^+}^{\bar{d}ss}$	1	-1	0	-1	-2	1	0	1	0	-1	0
$P_{B_s \rightarrow \pi^- K^{*+}}^{\bar{d}ss}$	0	0	-1	0	3	1	0	1	0	-1	0
$P_{B_d \rightarrow \pi^+ \rho^-}^{\bar{d}dd}$	1	0	0	4	-2	0	1	0	0	0	-2
$P_{B_s \rightarrow K^+ \rho^-}^{\bar{d}dd}$	0	0	0	-2	0	0	1	0	0	0	-2
$P_{B_d \rightarrow \pi^+ \rho^-}^{\bar{d}ss}$	1	0	-1	-2	-1	0	1	0	1	0	-1
$P_{B_s \rightarrow K^+ \rho^-}^{\bar{d}ss}$	0	-1	0	3	0	0	1	0	1	0	-1
$P_{B_d \rightarrow \rho^+ K^-}^{\bar{s}ss}$	0	0	0	0	-2	1	0	0	0	-2	0
$P_{B_s \rightarrow K^- K^{*+}}^{\bar{s}ss}$	1	0	0	-2	4	1	0	0	0	-2	0
$P_{B_d \rightarrow \rho^+ K^-}^{\bar{s}dd}$	0	0	-1	0	3	1	0	1	0	-1	0
$P_{B_s \rightarrow K^- K^{*+}}^{\bar{s}dd}$	1	-1	0	-1	-2	1	0	1	0	-1	0
$P_{B_d \rightarrow \pi^+ K^{*-}}^{\bar{s}ss}$	0	0	0	-2	0	0	1	0	0	0	-2
$P_{B_s \rightarrow K^+ K^{*-}}^{\bar{s}ss}$	1	0	0	4	-2	0	1	0	0	0	-2
$P_{B_d \rightarrow \pi^+ K^{*-}}^{\bar{s}dd}$	0	-1	0	3	0	0	1	0	1	0	-1
$P_{B_s \rightarrow K^+ K^{*-}}^{\bar{s}dd}$	1	0	-1	-2	-1	0	1	0	1	0	-1

butions from C_i to the above amplitudes, except $B_u(B_s) \rightarrow \pi^- \bar{K}^0 (K^0 \bar{K}^0)$, in the naive quark diagram analysis. Rescattering effects may generate nonzero contributions. Our results are, however, independent of whether these contributions are large or not. This also applies to related $B \rightarrow VV, PV$ modes.

The decay amplitudes for $B \rightarrow VV$ can be obtained, again, by a simple replacement of the corresponding final states. The amplitudes for $B \rightarrow PV$ are given in Table I. We can see that the SU(3) predictions of the relations in Eq. (2) are not modified if annihilation amplitudes are neglected. We con-

clude that the relations in Eqs. (1) and (2) are independent of new physics beyond the SM when small annihilation contributions are neglected. Measurements of these relations provide true tests of SU(3) symmetry in charmless hadronic two body B decays. There are also other interesting relations, with some of them depending on electroweak model which can be tested experimentally. A more detailed study of related relations and applications to new physics beyond the SM will be presented elsewhere [10].

The branching ratios for the decays in the relations in Eqs. (1) and (2) are in the range of 10^{-5} – 10^{-6} and some of the decay modes of $B_{u,d}$ have been measured. Although none of the B_s decay modes involved have been measured, they are expected to be measured at future hadron collider experiments such as CDF, D0, HERA-B, LHC-B, and BTeV. With 10^8 mesons for each of the $B_{u,d,s}$, most of the relations discussed can be tested at a level better than 10%. Combined studies, such as taking the sum of some of the branching ratios of the $B_{u,d}$ decays on the left-hand side of the equalities and the corresponding decay modes for B_s decays on the right-hand side, can also be carried out to increase the statistics and have earlier tests.

We emphasize that the relations discussed here provide electroweak model independent tests for SU(3) symmetry in hadronic B decays and therefore important information about the QCD hadronic dynamics. Only when these relations are established, one can have confidence in using SU(3) relations to extract important parameters, such as the phase angle γ , in the SM and to test models beyond the SM using other relations predicted by SU(3) symmetry which are electroweak model dependent [7]. For a full consistent test of SU(3) symmetry, one also needs to make sure that annihilations are indeed small. It is therefore also important to measure some pure annihilation decays such as

$$B_d \rightarrow K^+ K^-, K^{*+} K^{*-}, K^+ K^{*-}, K^- K^{*+},$$

$$B_s \rightarrow \pi^+ \pi^-, \pi^0 \pi^0, \rho^+ \rho^-, \rho^0 \rho^0, \pi^+ \rho^-, \pi^- \rho^+, \pi^0 \rho^0$$

to good precision. We urge our experimental colleagues to measure these decays and the decays in Eqs. (1) and (2) to carry out electroweak model independent tests for SU(3) symmetry in B decays.

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