Effects of extra dimensions on unitarity and the Higgs boson mass

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We study the unitarity constraint on two body Higgs boson elastic scattering in the presence of extra dimensions. The contributions from the exchange of spin-2 and spin-0 Kaluza-Klein states can have a large effect on the partial wave amplitude. The unitarity condition restricts the maximal allowed value for the ratio $r = \sqrt{s}/M_S$ of the center of mass energy $\sqrt{s}$ to the gravity scale M_S to be less than one. Although the constraint on the standard Higgs boson mass for r of order one is considerably relaxed, for small r the constraint is similar to that in the standard model. The resulting bound on the Higgs boson mass is not dramatically altered if perturbative calculations are required to be valid up to the maximal allowed value for r .

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It has recently been proposed that gravitational effects can become large at a scale M_S near the weak scale due to effects from extra dimensions [1,2], which is quite different from the traditional thought that gravitational effects only become large at the Planck scale $M_{Pl} = 1/\sqrt{G_N} \approx 1.22 \times 10^{19}$ GeV. In this proposal the total space-time is $D = 4 + n$. The relation between the scale M_S and the scale M_{Pl} , assuming all extra dimensions are compactified with the same size R , is given by $M_{Pl}^2 \sim R^n M_S^{2+n}$. With M_S near a TeV and $n = 1$, R would be too large. However, with n larger than or equal to 2, the theory is not ruled out for $M_S \sim 1$ TeV. The lower bound for M_S is constrained, typically, to be of order one TeV from present experimental data [3–7]. Future experiments will provide more stringent constraints [3–7]. There are many interesting phenomena due to the presence of extra dimensions [3–9]. In this paper we study effects from extra dimensions on the unitarity condition of partial wave amplitude in elastic two body Higgs boson scattering, and implications for the validity of perturbative calculations and for allowed Higgs boson mass.

In the minimal standard model (SM) there is a neutral Higgs boson H resulting from spontaneous symmetry breaking of $SU(2)_L \times U(1)_Y$ to $U(1)_{em}$ due to the Higgs mechanism. The mechanism for spontaneous symmetry breaking is not well understood. There is no experimental evidence favoring any particular mechanism, such as the Higgs mechanism. The discovery of the Higgs boson and understanding of its properties are fundamentally important [10]. Many methods have been proposed to produce and to study the properties of Higgs bosons [10]. One of the most important issues is its mass. At present the lower bound on SM Higgs boson mass m_H is set by the CERN e^+e^- collider LEP II to be 95.5 GeV at 95% C.L. [11]. There are many theoretical constraints on the Higgs boson masses. The constraint from unitarity of partial wave amplitudes of longitudinal gauge boson and/or Higgs boson scatterings provide some of the interesting upper bounds on the mass [12–15].

In the presence of extra dimensions there are additional contributions to gauge and/or Higgs boson scatterings due to exchanges of spin-2 and spin-0 graviton excitations (the Kaluza-Klein states). The effects of these Kaluza-Klein (KK)

states can affect the partial wave amplitudes significantly if the ratio $r = \sqrt{s}/M_S$ of the center of mass frame energy $\sqrt{s}$ and the gravity scale M_S is close to or larger than one. The unitarity condition for partial wave amplitude constrains r to be less than one if perturbative calculations are valid. The allowed range for Higgs boson mass can also be different from that in the SM depending on the value of r . We find that effects from extra dimensions affect the Higgs boson scattering $HH \rightarrow HH$ the most. In the following we will concentrate on this process, and will comment on other processes at the end.

In the SM, the scattering amplitude for the process $HH \rightarrow HH$ at the tree level is given by [13]

$$M_{SM}(s, t) = 3\sqrt{2}G_F m_H^2 \left(1 + \frac{3m_H^2}{s - m_H^2} + \frac{3m_H^2}{t - m_H^2} + \frac{3m_H^2}{u - m_H^2} \right), \quad (1)$$

where s , t , and $u = 4m_H^2 - s - t$ are the Mandelstam variables.

From the above expression, one obtains the $J=0$ partial wave amplitude [13]

$$\begin{aligned} a_0^{SM} &= \frac{1}{16\pi} \left(\frac{4p_i p_f}{s} \right)^{1/2} \frac{1}{s - 4m_H^2} \int_{-(s-4m_H^2)}^0 M_{SM}(s, t) dt \\ &= \frac{G_F m_H^2}{8\sqrt{2}\pi} \sqrt{1 - \frac{4m_H^2}{s}} \left[3 + \frac{9m_H^2}{s - m_H^2} - \frac{18m_H^2}{s - 4m_H^2} \ln \right. \\ &\quad \left. \times \left(\frac{s}{m_H^2} - 3 \right) \right]. \end{aligned} \quad (2)$$

In the above, $p_{i,f}$ are the momentum of the initial and final Higgs boson in the center of mass frame, respectively.

The Higgs boson mass is constrained if one requires that the absolute value of a_0 does not violate the unitarity condition. There are many discussions of how to implement unitarity conditions [14]. For our purpose of demonstrating possible large effects of extra dimensions, we use a weak condition $|a_0| < 1$ and work with the tree level amplitude to obtain a conservative bound. Applying this condition for $s \gg m_H^2$, one obtains

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$$m_H^2 < \frac{8\sqrt{2}\pi}{3G_F} = 1010 \text{ GeV}. \quad (3)$$

If m_H is substantially less than the above bound, the magnitude of the amplitude is well within the bound everywhere.

With extra dimensions, there are new contributions to $HH \rightarrow HH$ due to the exchange of KK states. Using the Feynman rules derived in Ref [6], we obtain

$$\begin{aligned} M_{NEW}(s, t) = & \frac{\kappa^2}{2} \left\{ \frac{1}{s - m_l^2} [(2m_H^2 - t)^2 + (2m_H^2 - u)^2 - \frac{2}{3}(s - 2m_H^2)^2 - \frac{4}{3}m_H^2 s] + \frac{1}{t - m_l^2} [(2m_H^2 - s)^2 + (2m_H^2 - u)^2 \right. \\ & \left. - \frac{2}{3}(t - 2m_H^2)^2 - \frac{4}{3}m_H^2 t] + \frac{1}{u - m_l^2} [(2m_H^2 - s)^2 + (2m_H^2 - t)^2 - \frac{2}{3}(u - 2m_H^2)^2 - \frac{4}{3}m_H^2 u] \right\} \\ & + \kappa^2 \left\{ \frac{2(n-1)}{3(n+2)} \left(\frac{(s + 2m_H^2)^2}{s - m_H^2} + \frac{(t + 2m_H^2)^2}{t - m_H^2} + \frac{(u + 2m_H^2)^2}{u - m_H^2} \right) \right\}. \end{aligned} \quad (4)$$

The first and the second terms are due to exchanges of spin-2 and spin-0 KK states, respectively.

Summing over all intermediate KK states and projecting out the $J=0$ partial wave amplitude, we obtain

$$\begin{aligned} a_0^{NEW} = & \sqrt{1 - \frac{4m_H^2}{s}} \left\{ \frac{2}{3(n+2)} + \frac{11s - 12m_H^2}{3nM_S^2} - \frac{2}{3(s - 4m_H^2)} \left(M_S^2 Fn(4) + (6s - 8m_H^2)Fn(2) + \frac{6s(s - 4m_H^2) + 16m_H^2}{M_S^2} Fn(0) \right) \right\} \\ & + \frac{4(n-1)}{3(n+2)} \left\{ \frac{1}{2} (s + 2m_H^2)^2 \frac{s^{n/2-1}}{M_S^{n+2}} [-i\pi + 2In(M_S/\sqrt{s})] + \frac{2}{n+2} + \frac{12m_H^2 - s}{nM_S^2} - \frac{2}{s - 4m_H^2} \left(M_S^2 Fn(4) + 4m_H^2 Fn(2) \right. \right. \\ & \left. \left. + \frac{4m_H^4}{M_S^2} Fn(0) \right) \right\} \sqrt{1 - \frac{4m_H^2}{s}}, \end{aligned} \quad (5)$$

where

$$\begin{aligned} In(x) &= \int_0^x \frac{y^{n-1}}{1-y^2} dy, \\ Fn(\delta) &= \int_0^1 x^{n+\delta-1} \ln \left[\left(\frac{s - 4m_H^2}{M_S^2} + x^2 \right) / x^2 \right] dx. \end{aligned} \quad (6)$$

In the above we have used $\kappa^2 = 16\pi G_N = 16\pi(4\pi)^{n/2} \Gamma(n/2) R^{-n} M_S^{-(n+2)}$ to relate G_N to M_S .

In the expression for a_0^{NEW} there are several constant terms which look dangerously large and are all canceled by terms proportional to $Fn(4)$. In the large M_S limit, a_0^{NEW} is proportional to $1/M_S^4$ and approaches zero as M_S goes to infinity with s and m_H kept finite. In this limit the theory reduces to the SM. However, when the ratio $r = \sqrt{s}/M_S$ approaches one, the real and imaginary parts of a_0^{NEW} both become of order one and can violate the unitarity condition $|a_0| < 1$ even if m_H is small. This indicates that the applicability of the effective theory, perturbatively, should be in the range $\sqrt{s} < M_S$. We remark that a_0^{NEW} only becomes sensitive to m_H for small M_S . With sufficiently large M_S , a_0^{NEW} by itself does not provide interesting information for m_H . However since the SM is sensitive to Higgs boson mass, the total contribution will provide information about m_H . To have a better understanding of the details we consider $|a_0|$ as

a function of $r = \sqrt{s}/M_S$ and m_H for four typical cases: (a) $M_S = 5$ TeV and $n = 2$; (b) $M_S = 5$ TeV and $n = 7$; (c) $M_S = 100$ TeV and $n = 2$; and (d) $M_S = 100$ TeV and $n = 7$, for illustrations. For cases (c) and (d), the results reduce to the SM if $\sqrt{s}$ is not too close to M_S , while for cases (a) and (b) the effects from extra dimensions can be large. The results are shown in Figs. 1 and 2.

From Figs. 1 and 2 we see that in all cases where r approaches one, a_0 becomes large and can violate the unitarity condition setting the perturbative range (the maximal allowed range r_{max} for r where $|a_0| = 1$) of the theory to be 0.81, 0.96, 0.81, and 0.96 for cases (a), (b), (c), and (d) with $m_H = 100$ GeV, respectively. With larger m_H , the allowed range for r_{max} can be larger, as can be seen from Figs. 1 and 2, due to the cancellation between contributions from the SM and extra dimensions, and the total a_0 is sensitive to m_H as mentioned before. As long as $\sqrt{s}$ is much larger than $2m_H$, r_{max} is not sensitive to M_S but is sensitive to the number of extra dimensions n . We have studied different values for n up to 7 and find that in all cases r_{max} is constrained to be less than one, but varies from case to case. Violation of the unitarity condition for $\sqrt{s} > M_S$ is not a big surprise because M_S serves as a cutoff where gravity becomes strong. The calculation here provides a specific example. We stress, however, that results obtained using perturbative calculations with $\sqrt{s}$ close to M_S are not reliable.

It has been shown [14] that in the SM at one loop level, for a given Higgs boson mass, there is a critical scale $\sqrt{s_c}$

FIG. 1. $|a_0|$ (vertical axis) as a function of $r = \sqrt{s}/M_S$ and m_H (GeV) for $M_S = 5$ TeV with $n=2$ and $n=7$, respectively. Cases (a) and (b) are shown at the top and bottom, respectively. The allowed parameter space with $|a_0| < 1$ are located at the left-lower corners indicated by the dented regions.

and that when s is larger than s_c , the unitarity condition is violated (s_c decreases as m_H increases). With extra dimensions, we obtain the energy scale s_{max} , below which the unitarity condition was reached earlier for small M_S and m_H . If a stronger requirement on $|a_0|$ is made as discussed in Ref. [14], the allowed r_{max} will be smaller. The bound we obtained is a conservative one.

The amplitude a_0 is also sensitive to the Higgs boson mass m_H as mentioned earlier. In all cases the allowed range for the Higgs boson masses is altered. In some cases the changes can be dramatic, as in case (c), in certain ranges of r . For small r the bound on the Higgs boson mass is similar to that in the SM. A Higgs boson mass bound with smaller r is tighter than other ranges except for r close to its maximal, as can be seen from Figs. 1 and 2. The bound on Higgs boson mass must be the smallest in the whole valid range of r for perturbative calculation. Therefore the bound on the Higgs boson mass is not dramatically affected compared with the SM case, which is effectively determined with the small value for r in our cases.

We also checked some other gauge and/or Higgs boson scattering processes. For example, the contribution to HH

FIG. 2. The same as Fig. 1, but with $M_S = 100$ TeV. Cases (c) and (d) are shown at top and bottom, respectively. For $\sqrt{s}$ much smaller than M_S ($r \ll 1$) the theory reduces to the SM.

$\rightarrow VV$ from extra dimensions in the limit neglecting the mass of the gauge boson V is given by

$$a_0^{NEW}(HH \rightarrow VV) = \frac{1}{12} \frac{s^{n/2-1}}{M_S^{n+2}} [-i\pi + 2\ln(M_S/\sqrt{s})] s^2 \times \left(1 - \frac{4m_H^2}{s} \right)^{5/4}. \quad (7)$$

This is smaller than that given for $HH \rightarrow HH$. In the unitarity consideration for $HH \rightarrow HH$ we obtain a stronger constraint.

In conclusion we have shown that effects from extra dimensions can have large contributions to $HH \rightarrow HH$. The range valid for perturbative calculations with extra dimensions is limited by the scale M_S . Results obtained with $\sqrt{s} > M_S$ are not reliable. Although Higgs boson mass bound can be drastically affected for certain ranges of $\sqrt{s}$, the overall bound is not modified significantly if one requires the perturbative calculation for $HH \rightarrow HH$ to be valid up to the maximal allowed value of r .

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