Evidence for factorization in three-body $\bar{B} \rightarrow D^{(*)} K^- K^0$ decays

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Motivated by recent experimental results, we use a factorization approach to study the three-body $\bar{B} \rightarrow D^{(*)} K^- K^0$ decay modes. Two mechanisms are proposed for kaon pair production: current produced (from a vacuum) and transition (from the B meson). The $\bar{B}^0 \rightarrow D^{(*)+} K^- K^0$ decay is governed solely by the current-produced mechanism. As the kaon pair can be produced only by the vector current, the matrix element can be extracted from $e^+e^- \rightarrow K\bar{K}$ processes via isospin relations. The decay rates obtained this way are in good agreement with experiment. Both current-produced and transition processes contribute to $B^- \rightarrow D^{(*)0} K^- K^0$ decays. By using QCD counting rules and the measured $B^- \rightarrow D^{(*)0} K^- K^0$ decay rates, the measured decay spectra can be understood.

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I. INTRODUCTION

The Belle Collaboration reported recently the first observation of $\bar{B} \rightarrow D^{(*)} K^- K^{(*)0}$ decays [1], with branching fractions at the level of 10^{-4} – 10^{-3} . An angular analysis of the $K^- K^{(*)0}$ subsystem reveals that $K^- K^0$ and $K^- K^{*0}$ meson pairs are dominantly $J^P=1^-$ and $J^P=1^+$, respectively. While there is no sign of decay via resonance for the $K^- K^0$ pair, data suggest a dominant $a_1(1260)$ resonance contribution in the production of the $K^- K^{*0}$ pair. Both the $K^- K^{*0}$ and the $K^- K^0$ pair mass spectra of the $D^{(*)} K^- K^{*0}$ and $D^0 K^- K^0$ modes show a maximum near threshold.

These processes can be described by conventional $b \rightarrow c\bar{u}d$ diagrams with additional $s\bar{s}$ pair creation. For example, in the $\bar{B}^0 \rightarrow D^{(*)+} K^- K^{(*)0}$ case, the spectator quark ($\bar{d}$) ends up in the $D^{(*)+}$ meson, while the $\bar{u}d$ pair ends up in the kaon pair. This is similar to the observed three-body baryonic mode $B^0 \rightarrow D^{*-} p\bar{n}$ [2], where the nucleon-antinucleon pair replaces the $K^- K^{(*)0}$ pair above. A generalized factorization approach [3] has been applied to study this three-body baryonic mode, where the amplitude is factorized into a current-produced $p\bar{n}$ part and a $B^0 \rightarrow D^{*-}$ transition part.

A crucial ingredient in the factorization approach to the $B^0 \rightarrow D^{*-} p\bar{n}$ mode is the knowledge of the timelike nucleon form factors. Although timelike axial nucleon form factor data are still not available, hence making the axial vector current contribution incalculable, the vector current induced nucleon form factors can be obtained via isospin rotation from their e.m. counterparts, where data are quite abundant. By utilizing nucleon e.m. form factor data, it was shown that the vector current contribution can account for up to 60% of the observed $B^0 \rightarrow D^{*-} p\bar{n}$ rate. The predicted $p\bar{n}$ mass spectrum shows a threshold enhancement effect as expected [4]. This can be seen as rooted in the near-threshold behavior of the nucleon form factors, whose appearance can be traced back to the application of factorization and QCD counting rules, which has been confirmed in the nucleon e.m. data. The total rate might be fully understood once the axial

nucleon form factor becomes available. Alternatively, it has been proposed that one will be able to extract the axial nucleon form factor from future $B^0 \rightarrow D^{*-} p\bar{n}$ data. A factorization and pole model approach has recently been employed to study the $B \rightarrow \bar{D}^{(*)} p\bar{n}, \bar{D}^{(*)} p\bar{p}$ modes [5], and some information on the axial form factor is extracted.

With success in the three-body baryonic modes, and the encouragingly similar threshold enhancement behavior in the $\bar{B} \rightarrow D^{(*)} K^- K^{(*)0}$ modes [1], we apply the factorization approach to study these three-body decays. For the $\bar{B}^0 \rightarrow D^{(*)+} K^- K^0$ modes, the fact that the $K^- K^0$ pair is observed only in the $J^P=1^-$ state already supports the factorization picture, since *only* the vector current can produce a kaon pair under factorization. With no axial current contribution, the amplitude can be predicted by using kaon e.m. form factors through isospin relations. This is in contrast with the $B \rightarrow D^{*-} p\bar{n}$ case where the axial current also contributes. Thus, with no tuning of parameters, the $\bar{B}^0 \rightarrow D^{(*)+} K^- K^0$ modes provide a useful testing ground of the factorization approach.

On the other hand, for the $B^- \rightarrow D^{(*)0} K^- K^0$ modes, the kaon pair can also be produced from a B meson by a current induced transition. The situation is similar to $B \rightarrow p\bar{p}K$ transitions in the $B \rightarrow p\bar{p}K$ case [6]. Since the B to kaon pair transition form factors are not known, we use a parametrization motivated by QCD counting rules, and determine these parameters from total decay rates. In other words, our approach is less predictive for these modes compared to $\bar{B}^0 \rightarrow D^{(*)+} K^- K^0$. However, the predicted decay spectra are closely related to the QCD counting rules, which can be tested experimentally.

In this work we shall concentrate on the $\bar{B} \rightarrow D^{(*)} K^- K^0$ modes, while making only some comments on the $\bar{B} \rightarrow D^{(*)} K^- K^{*0}$ modes. The experimental data indicate that the $K^- K^{*0}$ subsystem is in a 1^+ state and dominated by the $a_1(1260)$ resonance; thus it originates from the axial current. But, unlike the vector current case for $K^- K^0$ production, we do not have independent information on the axial form factors, so we have no control over these modes.

FIG. 1. (a) The current-produced ($\mathcal{J}$) and (b) the transition ($\mathcal{T}$) diagrams for $\bar{B}^0 \rightarrow D^+ K^- K^0$ and $B^- \rightarrow D^0 K^- K^0$ decays, respectively.

This paper is organized as follows. In the next section we lay down the formalism of the factorization approach. We show how to extract kaon form factors from e.m. data in Sec. III, and parametrize the transition form factor. In Sec. IV we show the results of our calculation, and in the last section we present some discussion before conclusions are drawn.

II. FACTORIZATION FORMALISM

The relevant effective Hamiltonian for the $b \rightarrow c$ transition is

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* [c_1(\mu) \mathcal{O}_1^c(\mu) + c_2(\mu) \mathcal{O}_2^c(\mu)], \quad (1)$$

where $c_i(\mu)$ are the Wilson coefficients and V_{cb} and V_{ud} are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. The four-quark operators $\mathcal{O}_i$ are products of two $V-A$ currents, i.e., $\mathcal{O}_1^c = (\bar{c}b)_{V-A} (\bar{d}u)_{V-A}$ and $\mathcal{O}_2^c = (\bar{d}b)_{V-A} (\bar{c}u)_{V-A}$.

With the factorization ansatz, the decay amplitudes for $B \rightarrow D^{(*)} K^- K^0$ are given by

$$\begin{aligned} \mathcal{A}(D^{(*)+} K^- K^0) &= \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* a_1 \langle D^{(*)+} | (\bar{c}b)_{V-A} | \bar{B}^0 \rangle \\ &\quad \times \langle K^- K^0 | (\bar{d}u)_{V-A} | 0 \rangle, \\ \mathcal{A}(D^{(*)0} K^- K^0) &= \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* [a_1 \langle D^{(*)+} | (\bar{c}b)_{V-A} | \bar{B}^0 \rangle \\ &\quad \times \langle K^- K^0 | (\bar{d}u)_{V-A} | 0 \rangle \\ &\quad + a_2 \langle K^- K^0 | (\bar{d}b)_{V-A} | B^- \rangle \\ &\quad \times \langle D^{(*)0} | (\bar{c}u)_{V-A} | 0 \rangle], \end{aligned} \quad (2)$$

where the effective coefficients are expressed as $a_1 = c_1 + c_2/3$ and $a_2 = c_2 + c_1/3$ if naive factorization is used. The factorized amplitudes consist of products of two matrix elements: the case of the B to D transition times the current-produced kaon pair is called “current produced” (denoted as $\mathcal{J}$), while the case of the B to kaon pair transition times the current-produced D is called the “transition” (denoted as $\mathcal{T}$). The two cases are depicted in Fig. 1. Note that the $\bar{B}^0$

$\rightarrow D^{(*)+} K^- K^0$ decay can only be current produced, while $B^- \rightarrow D^{(*)0} K^- K^0$ receives both contributions.

The $\langle D^{(*)} | V-A | B \rangle$ and $\langle D^{(*)} | V-A | 0 \rangle$ matrix elements are familiar and can be parametrized in the standard way. We shall adopt the Bauer-Stech-Wirbel (BSW) [7] and the Melikhov-Stech (MS) [8] models for comparison. The matrix elements $\langle KK | V-A | 0 \rangle$ and $\langle KK | V-A | B \rangle$ are less familiar. They are parametrized as [7,9]

$$\langle K_1(p_1) K_2(p_2) | V^\mu | 0 \rangle = (p_1 - p_2)^\mu F_1^{KK}(q^2), \quad (3)$$

$$\begin{aligned} \langle K_1(p_1) K_2(p_2) | (V-A)_\mu | B(p_B) \rangle &= i w_- (q^2) (p_2 - p_1)_\mu \\ &\quad + h(q^2) \epsilon_{\mu\nu\alpha\beta} p_B^\nu q^\alpha (p_2 - p_1)^\beta, \end{aligned} \quad (4)$$

where $q \equiv p_1 + p_2$, and we have dropped $m_1^2 - m_2^2$ dependent terms in Eq. (3) by assuming isospin symmetry. Since $\langle K^- K^0 | (\bar{d}u)_A | 0 \rangle = 0$ from Lorentz covariance and parity, only the vector current contributes to Eq. (3), which explains the experimental observation of $J^P = 1^-$ for the $K^- K^0$ pair in $\bar{B}^0 \rightarrow D^{(*)+} K^- K^0$. The data also show that the kaon pair for the $B^- \rightarrow D^{(*)0} K^- K^0$ modes is also in a 1^- configuration [1]. Since the transition amplitude now also contributes, we have used this experimental fact to drop the $(p_B - q)_\mu$ and q_μ dependent terms in Eq. (4), as they would lead to other quantum numbers for the kaon pair. This greatly simplifies the work.

We now show how to use data on timelike kaon e.m. form factors and an isospin relation to obtain $F_1^{KK}(q^2)$. We then give a simple parametrization for the $B \rightarrow KK$ transition form factors motivated by QCD counting rules.

III. CURRENT-PRODUCED AND TRANSITION $K\bar{K}$ FORM FACTORS

A. Isospin relation and kaon electromagnetic form factor

The $D^{(*)+} K^- K^0$ modes contain only current-produced amplitudes $\mathcal{J}$, as can be seen from Eq. (2) and is depicted in Fig. 1. Since only the vector current contributes, we can use kaon e.m. form factor data to obtain the weak vector form factor via the isospin relation

$$F_1^{KK}(q^2) = F_{K^+}(q^2) - F_{K^0}(q^2), \quad (5)$$

where F_{K^+}, F_{K^0} are the e.m. form factors of the charged and neutral kaons, respectively.

The kaon e.m. form factors have been measured for both spacelike and timelike regions [10–13], where the processes $e^+ e^- \rightarrow K^+ K^-, K_L K_S$ provide the timelike data. The timelike $|F_{K^+}|$ and $|F_{K^0}|$ form factor data are given in Fig. 2 in the energy region $M_{K^+ K^-}, M_{K_L K_S} = 1-3$ GeV. The structure is complicated in the 1–2.1 GeV range, revealing both resonant as well as nonresonant contributions. A sharp $\phi(1020)$ peak is shown in the insets. The form factors drop quickly above 1.02 GeV, but a slower damping takes over for larger M_{KK} , and one must include ρ, ω, ϕ , and their higher resonances in modeling the form factors.

FIG. 2. Timelike $|F_{K^+}|$ (left) and $|F_{K^0}|$ (right) form factor data, where the inset is for the ϕ region. They are fitted by Eqs. (7),(8), respectively.

The asymptotic behavior in M_{KK} is characteristic of power-law falloff, and seems to satisfy the stringent asymptotic constraint [14] from perturbative QCD (PQCD)

$$F_K(t) \rightarrow \frac{1}{t} \left[\ln \left(\frac{t}{\Lambda^2} \right) \right]^{-1}, \quad (6)$$

where $t = q^2$ and $\Lambda \sim 0.3$ GeV is the QCD scale parameter. The $1/t$ power reflects the need for a hard gluon to redistribute large momentum transfer.

Since our aim is just to parametrize and fit the form factor data, we express the e.m. form factors by a phenomenological model that combines the resonant and PQCD terms

$$F_{K^+}(t) = \left(\sum_j \frac{c_j}{t - m_j^2 + i m_j \Gamma_j(t)} \right) \sqrt{C(t)} + \left(\frac{x_1}{t} + \frac{x_2}{t^2} \right) \times \left[\ln \left(\frac{t}{\Lambda^2} \right) \right]^{-1}, \quad (7)$$

$$F_{K^0}(t) = \sum_j \frac{(-)^{I_j} c_j}{t - m_j^2 + i m_j \Gamma_j(t)} + \left(\frac{y_1}{t} \right) \left[\ln \left(\frac{t}{\Lambda^2} \right) \right]^{-1}, \quad (8)$$

where one sums over appropriate meson poles and I_j is the isospin of the j th meson. We put very few asymptotic PQCD terms because the large t data are sparse. The Coulomb factor $C(t)$ [15] accounts for soft photon exchange which can enhance the cross section by a few percent for low q^2 and is needed for the fit. But, in obtaining F_1^{KK} from F_{K^+} via Eq. (5), we divide out this effect since there is no exchange of soft photons for $K^- K^0$.

B. Fitted kaon weak vector form factor

A major difference from our earlier studies of baryonic modes [3] is the presence of the resonance part, which turns out to be important. To be able to account for the rich structure in the experimental data in $m_{K^+ K^-}, m_{K_L K_S} \leq 2$ GeV, we take the eight vector mesons $\rho(770)$, $\omega(782)$, $\phi(1020)$, $\omega(1420)$, $\rho(1450)$, $\omega(1650)$, $\phi(1680)$, and $\rho(1700)$, with their masses m_j kept at the experimental values [16]. The widths $\Gamma_j(t)$ for $j = \rho, \omega, \phi$ are given by [11]

$$\Gamma_\rho(t) = \frac{m_\pi^3 \Gamma_\rho}{s m_\rho} \left(\frac{t - 4m_\pi^2}{m_\rho^2 - 4m_\pi^2} \right)^{3/2}, \quad \Gamma_\omega(t) = \Gamma_\omega, \\ \Gamma_\phi(t) = \frac{m_\phi^2 \Gamma_\phi}{2s} \left[\left(\frac{t - 4m_{K^+}^2}{m_\phi^2 - 4m_{K^+}^2} \right)^{3/2} + \left(\frac{t - 4m_{K^0}^2}{m_\phi^2 - 4m_{K^0}^2} \right)^{3/2} \right], \quad (9)$$

where $\Gamma_{\rho, \omega, \phi}$ are the full widths [16]. For the other higher-mass vector mesons, we simply take their experimental width values [16].

We should stress that our phenomenological model is devised to account for e.m. form factor data in the region of our interest. No attempt is made for deeper theoretical understanding, nor for data outside the t region of interest. We therefore do not need all the ϕ -region data even though more precise measurements have been obtained, as one would need a more sophisticated treatment of $\Gamma_\phi(t)$ (including a $\phi \rightarrow 3\pi$ phase space term) [13] which would complicate our parametrization. Actually, our neglect of such data causes no harm since ϕ is an isoscalar. While it is needed to account for F_{K^+} and F_{K^0} , its effect cancels in the weak form factor $F_1^{KK}(q^2)$, as required by isospin symmetry.

The coefficients c_j are treated as free real parameters although by some physical conditions they are not all independent. In the vector meson dominance (VMD) framework, c_ρ, c_ω, c_ϕ are expressed as $c_\rho = g_{\rho\gamma} g_{\rho KK}$, $c_\omega = g_{\omega\gamma} g_{\omega KK}$, and $c_\phi = g_{\phi\gamma} g_{\phi KK}$. Using experimental values of the electronic widths [16] for the coupling constants $g_{V\gamma}$ and assuming SU(3) relations with ideal mixing, namely, $g_{\rho KK} = g_{\phi KK} / \sqrt{2} = g_{\omega KK}$, we have [11]

$$c_\rho : c_\omega : c_\phi = 1 : \frac{1}{3} : 1. \quad (10)$$

For other higher-mass vector mesons, c_j are free from the above constraint.

We take Eqs. (7) and (8) to make a phenomenological fit to the experimental data of the charged [10,11] and neutral

FIG. 3. The kaon weak vector form factor $F_1^{KK}(t)$.

kaon form factors [12,13]. The best fit values are obtained by finding the minimum of the χ^2 function, $\chi^2 \equiv \chi_+^2 + \chi_0^2$, where

$$\chi_{+,0}^2 \equiv \sum_i \frac{[z_i - |F_{K^+,0}(t_i)|]^2}{\sigma_i^2}, \quad (11)$$

where the z_i 's are the measured absolute values of the time-like kaon e.m. form factors and the σ_i 's are the error bars. Note that we fit the absolute values of the kaon form factors since these are what can be obtained by experiment. Also, due to the common resonance part in both F_{K^+} and F_{K^0} , we search for the minimum of χ^2 as a combination of these two form factors.

We find the best fit values (in units of GeV^2)

$$\begin{aligned} c_\rho &= 0.363, & c_{\rho(1450)} &= 7.98 \times 10^{-3}, & c_{\rho(1700)} &= 1.71 \times 10^{-3}, \\ c_{\omega(1420)} &= -7.64 \times 10^{-2}, & c_{\omega(1650)} &= -0.116, \\ c_{\phi(1680)} &= -2.0 \times 10^{-2}, \end{aligned} \quad (12)$$

and

$$x_1 = 3.26 \text{ GeV}^2, \quad x_2 = -5.02 \text{ GeV}^4, \quad y_1 = -0.47 \text{ GeV}^2. \quad (13)$$

Note that the ratios of c_ϕ, c_ω with c_ρ are already fixed by Eq. (10), and the best fit value for c_ρ is consistent with Ref. [11]. As can be seen from Fig. 2, the fit is reasonable (χ^2 per number of degrees of freedom is $194/130 \sim 1.5$) for both low and high energies.

Using Eq. (5), we give the kaon weak vector form factor F_1^{KK} in Fig. 3, with the overall factor $\sqrt{C(t)}$ in F_{K^+} removed. Contributions from poles of $I=0$ mesons cancel out, and one is left with only the $\rho(770)$, $\rho(1450)$, $\rho(1700)$, and PQCD contributions. Since $c_{\rho(1450)}, c_{\rho(1700)}$ are small, F_1^{KK} is modeled by the $\rho(770)$ and PQCD parts and hence is quite smooth. Under the factorization assumption, this could be the reason behind the absence of structures in the $K^- K^0$ pair spectrum for the $\bar{B}^0 \rightarrow D^{(*)+} K^- K^0$ decays [1].

C. Ansatz for $B \rightarrow K^- K^0$ transition form factors

In addition to the current-produced matrix element, we also need the transition matrix element $\langle K^- K^0 | (\bar{d}b)_{V-A} | B^- \rangle$ for $B^- \rightarrow D^{(*)0} K^- K^0$ decay amplitudes. Two out of four possible transition form factors are

eliminated by the experimental observation of $J^P = 1^-$ for the kaon pair [1]. There are no experimental data on the remaining form factors, denoted $w_-(q^2)$ and $h(q^2)$ in Eq. (4).

It is interesting to study the asymptotic behavior of these form factors from QCD counting rules. To produce a kaon pair with large invariant mass from a decaying B meson, at least two hard gluon exchanges are needed: one creating the $s\bar{s}$ pair in $K^- K^0$, the other kicking the spectator to catch up with the energetic s quark to form the K meson. This gives rise to a $1/t^2$ asymptotic behavior, where t is the $K^- K^0$ invariant mass squared.

Resonant contributions such as from an intermediate ρ pole should in principle be considered for the $B^- \rightarrow K^- K^0$ transition. This seems to be supported by the dominant ρ contribution in $F_1^{KK}(q^2)$ as we have just discussed. However, we would need further poles such as $\rho(1450), \rho(1700)$ to account for the $1/t^2$ asymptotic behavior implied by QCD counting rules. Our experience with F_1^{KK} does not help since these resonances are unimportant there, while we lack other independent experimental information. But there is as yet no clear sign of resonances in the kaon pair spectrum. Because of this, we shall use a very simple parametrization solely motivated from QCD counting rules, i.e.,

$$w_-(t) = \frac{c_w}{t^2}, \quad h(t) = \frac{c_h}{t^2}, \quad (14)$$

where $c_{w,h}$ are free parameters to be fitted by data.

IV. RESULTS

We use the central values of the effective coefficients $a_1^{\text{BSW}} = 0.91 \pm 0.08 \pm 0.07$ ($0.86 \pm 0.21 \pm 0.07$) and $a_2^{\text{BSW}} = 0.56 \pm 0.31$ (0.47 ± 0.41) extracted from [17] $\bar{B}^0 \rightarrow D^{(*)+} \rho^-$ and $B^- \rightarrow D^{(*)0} \rho^-$, respectively. Similarly, we use $a_1^{\text{MS}} = 0.935$ (0.803) and $a_2^{\text{MS}} = 0.5$ (0.553) for the MS form factor case. The CKM matrix elements V_{ud} and V_{cb} are taken to be 0.975 and 0.039, respectively.

It is useful to give first an outline of our results. Under factorization, the theoretical input for the current-produced $D^+ K^- K^0, D^{*+} K^- K^0$ modes are all determined, such as F_1^{KK} from e.m. data and a_1 from the $\bar{B}^0 \rightarrow D^+ \rho^-, D^{*+} \rho^-$ rates. The calculated rates turn out to be in good agreement with the experimental results [1]. The decay spectra are predictions, which can be checked soon. For the $D^{(*)0} K^- K^0$ case, since we have to fit the unknown transition form factors from the corresponding decay rates, our results are not as solid as in the purely current-produced case. But it is interesting that the resulting decay spectra agree well with experimental results [1]. We comment on $D^{(*)} K^- K^{*0}$ modes before we end this section.

A. $\bar{B}^0 \rightarrow D^{(*)+} K^- K^0$

From Eqs. (2) and (3), the $\bar{B}^0 \rightarrow D^+ K^- K^0$ decay amplitude is given by

TABLE I. $\mathcal{B}(\bar{B} \rightarrow D^{(*)} K^- K^0)$ in units of 10^{-4} , where the upper and lower limits come from scanning the $\chi^2_{\min} + 1$ region for the kaon weak form factor F_1^{KK} .

	MS	BSW	Experiment [1]
$\bar{B}^0 \rightarrow D^+ K^- K^0$	$1.67^{+0.24}_{-0.21}$	$1.54^{+0.22}_{-0.20}$	$1.6 \pm 0.8 \pm 0.3$ < 3.1 (90% C.L.)
$\bar{B}^0 \rightarrow D^{*+} K^- K^0$	$2.8^{+0.30}_{-0.36}$	$3.05^{+0.32}_{-0.39}$	$2.0 \pm 1.5 \pm 0.4$ < 4.7 (90% C.L.)

$$\mathcal{A}(D^+ K^- K^0) = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* a_1 (p_B + p_D) \cdot (p_2 - p_1) \times F_1^{BD}(q^2) F_1^{KK}(q^2), \quad (15)$$

where p_1, p_2 stand for the momenta of K^- and K^0 , respectively. Factorization implies that the amplitude involves only the known weak kaon form factor F_1^{KK} and the $B \rightarrow D$ form factor F_1^{BD} . For $\bar{B}^0 \rightarrow D^{*+} K^- K^0$, the decay amplitude is

$$\begin{aligned} \mathcal{A}(D^{*+} K^- K^0) = & \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* a_1 F_1^{KK}(q^2) \\ & \times \left\{ \frac{2V(q^2)}{m_B + m_{D^*}} \epsilon_{\mu\nu\alpha\beta} (p_2 - p_1)^\mu \epsilon_{D^*}^{*\nu} p_B^\alpha p_{D^*}^\beta \right. \\ & + i \left[(m_B + m_{D^*}) A_1(q^2) \epsilon_{D^*}^* \cdot (p_2 - p_1) \right. \\ & \left. \left. - \frac{A_2(q^2) \epsilon_{D^*}^* \cdot q}{m_B + m_{D^*}} (p_B + p_{D^*}) \cdot (p_2 - p_1) \right] \right\}, \quad (16) \end{aligned}$$

which involves the $B \rightarrow D^*$ transition form factors $V, A_{1,2}$. There are no tunable parameters for these two modes; hence they provide a good test for the factorization approach. The $p_2 - p_1$ factor implies that the $K^- K^0$ pair must be in a P -wave state, which is just what the Belle experiment observes [1].

We show in Table I the calculated rates of these two modes. Both turn out to be at the 10^{-4} level. The upper and lower limits are from scanning the $\chi^2_{\min} + 1$ region for the maximum and minimum of the kaon weak form factor F_1^{KK} . We find good agreement between factorization and experimental results, especially for the $D^+ K^- K^0$ case. This provides *evidence* that the factorization approach works.

It is instructive to understand the contribution to the $D^{(*)+} K^- K^0$ rates from the resonant and nonresonant parts of F_1^{KK} , where the latter refers to the $x_{1,2}$ and y_1 terms in Eqs. (7) and (8). As previously noted, the ρ contribution dominates the resonant part. We find that the resonant part contributes 40% (43%) of the $D^{(*)+} K^- K^0$ rate, while the nonresonant part contributes 13% (15%). Constructive inter-

FIG. 4. The $K^- K^0$ mass spectrum for $\bar{B}^0 \rightarrow D^+ K^- K^0$ (lower) and $D^{*+} K^- K^0$ (upper), where solid (dashed) line stands for using the MS (BSW) hadronic form factors.

ference between the two is needed to give rates close to experimental results, and hence is indeed important.

The $K^- K^0$ mass spectra of the $D^{(*)+} K^- K^0$ modes are shown in Fig. 4. Both peak close to threshold, which is due to the near-threshold behavior of the F_1^{KK} form factor (see Fig. 3). There is no other clear structure, other than the $B \rightarrow D^*$ form factor effect at larger q^2 . Because of lower $D^{(*)+}$ reconstruction efficiencies, the spectra have yet to be measured experimentally [1], but our predicted spectrum can be checked soon with more data.

B. $B^- \rightarrow D^{(*)0} K^- K^0$

From Eqs. (2), (3) and (4), we have the amplitudes

$$\begin{aligned} \mathcal{A}(D^0 K^- K^0) = & \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* [a_1 (p_B + p_D) \cdot (p_2 - p_1) F_1^{BD}(q^2) \\ & \times F_1^{KK}(q^2) - a_2 f_{D^{*0}}(q^2) p_B \cdot (p_2 - p_1)], \quad (17) \end{aligned}$$

$$\begin{aligned} \mathcal{A}(D^{*0} K^- K^0) = & \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* \left\{ a_1 F_1^{KK}(q^2) \left[\frac{2V(q^2)}{m_B + m_{D^*}} \epsilon_{\mu\nu\alpha\beta} \epsilon_{D^*}^{*\nu} \right. \right. \\ & \times p_B^\alpha p_{D^*}^\beta (p_2 - p_1)^\mu + i \left(A_1(q^2) (m_B + m_{D^*}) \epsilon_{D^*}^* \cdot \right. \\ & \cdot (p_2 - p_1) - \frac{A_2(q^2)}{m_B + m_{D^*}} \epsilon_{D^*}^* \cdot q (p_B + p_{D^*}) \cdot (p_2 - p_1) \left. \right) \\ & \left. \left. + a_2 f_{D^*} m_{D^*} (i w - (q^2) \epsilon_{D^*}^* \cdot (p_2 - p_1) \right. \right. \\ & \left. \left. + h(q^2) \epsilon_{\mu\nu\alpha\beta} \epsilon_{D^*}^{*\mu} p_B^\nu (p_2 + p_1)^\alpha (p_2 - p_1)^\beta \right) \right\}. \quad (18) \end{aligned}$$

The a_1 and a_2 terms correspond to the current-produced and transition parts, respectively. There are only two form factors

TABLE II. Fitted values of transition form factor parameters c_w and c_h , in units of GeV^3 , by using the central values of $D^{(*)0}K^-K^0$ rates.

	$c_w^{\text{MS(BSW)}}$	$c_h^{\text{MS(BSW)}}$	$\mathcal{B}(10^{-4})$ [1]
$B^- \rightarrow D^0 K^- K^0$	$-35.4 (-33.0)$	—	$5.5 \pm 1.4 \pm 0.8$
$B^- \rightarrow D^{*0} K^- K^0$	$-35.4 (-33.6)$	$11.3 (13.1)$ or $-16.1 (-18.5)$	$5.2 \pm 2.7 \pm 1.2$

$w_-(q^2)$ and $h(q^2)$ in the transition matrix element because K^-K^0 is seen only in the 1^- state. For the $D^0K^-K^0$ case, only $w_-(q^2)$ contributes.

Equation (17) involves only one free parameter c_w in $w_-(t) = c_w/t^2$, which can be obtained by fitting the central value of the observed rate $\mathcal{B}(D^0K^-K^0) = (5.5 \pm 1.4 \pm 0.8) \times 10^{-4}$ [1]. We find $c_w^{\text{MS(BSW)}} = -35.4 (-33.0) \text{ GeV}^3$ and $109.2 (97.4) \text{ GeV}^3$, depending on constructive or destructive interference between the current-produced and transition amplitudes, respectively. If we take $h=0$ for now, Eq. (18) would give $\mathcal{B}^{\text{MS(BSW)}} = 1.27 (1.45) \times 10^{-4}$ and $35.62 (24.97) \times 10^{-4}$. The latter result seems too large compared with the experimental result of $\mathcal{B}(D^{*0}K^-K^0) = (5.2 \pm 2.7 \pm 1.2) \times 10^{-4}$, and hence disfavors the destructive $F_1^{KK} - \omega_-$ interference case unless a fine-tuned $h(t)$ term is used. To obtain the central value of the experimental $D^{*0}K^-K^0$ rate, we find $c_h^{\text{MS(BSW)}} = 11.3 (13.1) \text{ GeV}^3$ or $-16.1 (-18.5) \text{ GeV}^3$. We summarize in Table II the relevant parameters and the rates measured by experiment.

Both $D^0K^-K^0$ and $D^{*0}K^-K^0$ rates are used as input and hence are not predictions. However, we can still make predictions on the decay spectra. By taking the fitted values of $c_w^{\text{MS(BSW)}}$, we plot the differential decay rate for the $B^- \rightarrow D^0K^-K^0$ mode in Fig. 5 and compare with the experimental data. The agreement is good. We see that the data themselves show a maximum near the K^-K^0 threshold, which can be naturally explained by our model, where threshold enhancement is a genuine result from the form factors in both current-produced and transition processes. If threshold enhancement is even more pronounced, as data seem to suggest some structure around $m_{K^-K^0} \sim 1.4 \text{ GeV}$, then perhaps the simple approximation of $\omega_-(t), h(t) \propto 1/t^2$ has to be reexamined.

The mass spectrum for $B^- \rightarrow D^{*0}K^-K^0$ mode is plotted in Fig. 6 with c_h as given in Table II. This indicates the effect of the additional $h(t)$ term in the transition amplitude. Like

FIG. 5. $B^- \rightarrow D^0K^-K^0$ spectrum, where solid (dashed) line is for the MS (BSW) model, and the data are from Ref. [1].

$B^- \rightarrow D^0K^-K^0$, one also has threshold enhancement, which again is a genuine result of both the kaon weak form factor and the $B^- \rightarrow K^-K^0$ transition form factor. We see clearly that the $w_-(t)$ and $h(t)$ form factors contribute much in the low K^-K^0 mass range.

C. $B \rightarrow D^{(*)}K^-K^{*0}$

Before we end this section, we comment on some features of the $\bar{B} \rightarrow D^{(*)}K^-K^{*0}$ modes under the factorization picture. Since there are no independent data such as e.m. form factors that one could use, we do not go into the details.

The Belle experiment has already made some interesting observations [1]: (i) although K^-K^{*0} could have $J^P = 0^-, 1^-, 1^+$ quantum numbers, the data prefer the $J^P = 1^+$ case; (ii) the rates seem to be dominated by the a_1 resonance, but the fitted $a_1 \rightarrow K^-K^{*0}$ rate is 2–5 times larger than the CLEO result [18]. The data therefore suggest that the amplitude contains both the $a_1 \rightarrow K^-K^{*0}$ resonance plus a non-resonant part. The situation is quite similar to the $D^{(*)+}K^-K^0$ case, where the kaon pair mainly comes from a ρ pole plus a QCD motivated contribution. The ρ pole alone only gives about 40% of the rate, while constructive interference with the QCD part is quite important. If the same picture holds in the $D^{(*)+}K^-K^{*0}$ case, the discrepancy with the CLEO result $a_1 \rightarrow K^-K^{*0}$ rate may be resolved.

On the other hand, the K^-K^{*0} pair is produced by an axial current, and no e.m. data could be used as in the K^-K^0 case. However, further theoretical tools, such as Weinberg sum rules [19], may be useful to transfer information on vector current form factors to the axial current form factor. One can in turn extract axial current form factors from the

FIG. 6. The K^-K^0 mass spectrum for $\bar{B}^0 \rightarrow D^{*0}K^-K^0$, where solid, dot-dashed, dashed, and dotted lines are for MS model with $c_h = 11.3, -16.1 \text{ GeV}^3$ and BSW model with $c_h = 13.1, -18.5 \text{ GeV}^3$, respectively.

$\bar{B}^0 \rightarrow D^{(*)+}K^-K^{*0}$ data, which may not be obtained by other means.

V. DISCUSSION AND CONCLUSION

In this paper, we use the factorization approach to study three-body $B \rightarrow D^{(*)}K^-K^0$ decays. There are two mechanisms of kaon pair production, namely, current produced and transition. The $D^+K^-K^0$ and $D^{*+}K^-K^0$ modes involve only current-produced contributions. Under factorization, the kaon pair can only be generated through the weak vector current, which can be related to the e.m. current through isospin. These modes provide good means to test factorization, and the result is encouraging. The $D^+K^-K^0$ and $D^{*+}K^-K^0$ rates in the factorization approach are in good agreement with data.

The $B^- \rightarrow D^{(*)0}K^-K^0$ decays also receive a transition contribution. The forms of these transition form factors are determined through QCD counting rules, and we fix the parameters by using the measured $D^0K^-K^0$ and $D^{*0}K^-K^0$ decay rates. The predicted mass spectra of the $B^- \rightarrow D^{(*)0}K^-K^0$ modes agree well with data and exhibit threshold enhancement as do the $\bar{B}^0 \rightarrow D^{(*)+}K^-K^0$ cases. The $D^0K^-K^0$ spectrum shows a peak around $m_{K^-K^0} \sim 1.2$ GeV, while the $D^{*0}K^-K^0$ spectrum has a peak around $m_{K^-K^0} \sim 1.5$ GeV. This is due to the different $1/m_{K^-K^0}^2$ behavior of current-produced and transition form factors and can be understood from QCD counting rules. To be specific, QCD counting rules require the transition form factors to have a faster damping in the $m_{K^-K^0}$ spectrum than the current-produced ones.

Despite the success in describing the mass spectrum of the $D^0K^-K^0$ mode, our treatment of the $B^- \rightarrow K^-K^0$ transition form factors may be oversimplified. Assuming the asymptotic form required by PQCD may be too strong an assumption, and might have overenhanced the contribution from the near-threshold region. More careful study of other possibilities, such as using pole models for transitions via resonances, would be helpful in clarifying the underlying dynamics of $B^- \rightarrow D^{(*)0}K^-K^0$ transitions.

It is interesting to compare with the treatment of B^0

$\rightarrow D^{*+}p\bar{n}$ [3], $\rho^-(\pi^-)p\bar{n}$ [20], and $B^- \rightarrow p\bar{p}K^-$ modes [6]. The success in explaining the rates of both $D^{*+}p\bar{n}$ and $p\bar{p}K^-$ modes, and the mass spectrum of the latter, encouraged us to apply the factorization approach further to the present cases. However, due to the lack of independent information on axial form factors, the difficulty of measuring neutrons, and the presence of a large number of operators in the $p\bar{p}K^-$ case, none of these modes provides a good testing ground for the factorization approach. The present $D^{(*)}K^-K^0$ cases turn out to be a good place for such a test. In particular, the $D^+K^-K^0$ and $D^{*+}K^-K^0$ modes are free from any undetermined parameters in the factorization approach. The good agreement with data provides support for the idea of factorization plus usage of isospin-related form factor data. Further support comes from the agreement of the $D^0K^-K^0$ spectra with data.

Even a hand-waving physical argument may be welcome to explain why a simple factorization approach works so well in these potentially complicated three-body decays. Before we end this paper, we would like to offer one such argument. The QCD counting rules constrain the kaon pair in the light pair mass region. With a small invariant mass, the two kaons move collinearly and energetically. This is certainly a conducive situation for the kaon pair to decouple from the recoil $D^{(*)}$ meson, and hence “factorize.” This heuristic picture therefore resembles the case of B decay to two mesons [21]. It should be noted that PQCD may not be the only explanation of factorization [22,23]. The success of factorization in $\bar{B} \rightarrow D^{(*)}K^-K^0$ decays urges a serious study of the underlying mechanism.

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