INTERFEROMETRIC MEASUREMENTS OF ANNULUS-FILTERED POLARIZATIONS PATTERNS IN THE CMB

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Due to the finite-size beam, interferometry observations of the CMB polarization tend to measure a mixture of E and B modes arising from uv -space smearing. We propose an interferometry observation strategy that can disentangle the polarization E and B modes in a clean manner. This strategy employs a similar principle as that of the single-dish observation proposed earlier, but requiring additional linear combinations of signals measured from different baselines. In this work, we consider a hexagonally packed interferometry array as an example to demonstrate the utility and efficiency of the proposed method. The measurement from a unit cell of equilateral triangular baselines in interferometry measurement is found to be comparable to that from a single dish in single-dish measurements.

Keywords: cosmic microwave background radiation; polarization

1. Introduction

Recently, DASI⁶, WMAP⁵, and Boomerang⁸ have successfully detected the polarization of cosmic microwave background (CMB), and several future experiments (QUaD, POLARBeaR, SPTpol, CMBpol and PLANCK) propose to measure the B mode polarization. The primordial B mode polarization is believed to be caused by primordial gravitational waves and provides information of inflation energy scale¹¹. On the other hand, the secondary B mode polarization induced by gravitational lensing of CMB will give us another probe of large scale density distribution⁴. However, the weak intensity of B mode polarization makes it difficult to be measured cleanly in the presence of E mode. Contamination by E mode arises from the limitation of practical observation, such as incomplete sky coverage, finite resolution, and the foreground effects; the way to solve E - B separation problem for real-space maps is also discussed in Ref. 14, Ref. 12, Ref. 2, Ref. 7 and Ref. 1.

For interferometric measurements, the E and B modes of CMB polarization can be represented as the projections of the Fourier components of the Stokes

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parameters¹³, Q and U , onto different directions with respect to the baseline $\mathbf{k}$. Hence, interferometry which derives amplitude and phase in k -space directly seems able to extract E and B modes straightforwardly. However, the finite dish size, especially in the case of a close-packed array, optimized for the sensitivity to CMB, makes the baseline $\mathbf{k}$, and thus the directional projections, ill-defined, thereby mixing up E and B modes in the measurements.

In this work, we show that clean separation of E and B modes can actually be achieved with interferometric measurements by scanning along a sky annulus. A similar method has been proposed for single-dish measurement in our previous work². Different from the conventional interferometry observations of CMB polarization^{13,9,10}, which have afore-mentioned difficulties, our proposed strategy incorporates the advantage of real-space measurements that warrants separation of E and B modes in practice.

2. Observation strategy and simulation

The polarization field can be expressed in the form of second order derivatives of 2 scalar fields f and g , from which the E and B tensor modes are generated respectively. For convenience, it is written in polar coordinates as

$$\begin{aligned} \mathbf{P} = Q + iU &= 4 \frac{\partial^2}{\partial \bar{w}^2} (f + ig) \\ &= e^{2i\phi} \left[r \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} \right) - \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} \right] + \frac{2i}{r} \frac{\partial}{\partial \phi} \left(\frac{\partial}{\partial r} - \frac{1}{r} \right) (f + ig). \end{aligned} \quad (1)$$

The map generated by interferometric array is constructed by the visibility (i.e., Fourier transformation) of the beam-weighted map ($\mathbf{P}$) integrated over the selected sky. In previous work², we have suggested that the E and B modes can be cleanly separated in a real-space map if one adopts an annulus filter function. In practice, it employs scanning in a sky circle where the polarization axes also rotate synchronously. The radius R_q of the scanning circle as well as the primary beam size D are free parameters to be optimized according to the l -range of interest and the responsiveness of measurements. Such an idea can be carried over to the interferometric measurement. For simplicity of the following analysis, we assume a Gaussian beam.

The visibility of interferometry of baseline k with annulus filtering could be expressed as:

$$\begin{aligned} \Delta T(k; R_q, D) &= e^{-ikR_q \cos \phi_{k0}} \int_{\phi_{k'} > 0} e^{-(k^2 + k'^2) D^2 / 2} \\ &\quad \sum_{n=-\infty}^{\infty} I_n(kk' D^2) J_{n+2}(k' R_q) e^{in(\pi/2 - \phi_{k0})} (k')^2 (Re[f_{\mathbf{k}'}] + i Re[g_{\mathbf{k}'}]) \frac{d^2 \mathbf{k}'}{2\pi^2}, \end{aligned} \quad (2)$$

where J s and I s are (modified) Bessel functions. Following the convention of eqn (1), the real/imaginary part of the measurement corresponds to the Stokes Q/U respectively. Obviously, the exponential term in eqn (2) mixes up f and g , the

sources of E and B modes, in the measured Q and U . Nevertheless the mixing can be disentangled by linearly combining the data measured by any pair of baselines that satisfy the "separation condition", $\phi_{kb} = (2n + 1)\pi - \phi_{ka}$ (ϕ_k is the phase angle of the baseline). The resulting visibility is

$$\begin{aligned} \Delta T_{\pm}(k; R_q, D) &= \frac{1}{\sqrt{2}}(e^{ikR_q \cos \phi_{ka}} \Delta T_a(k; R_q, D) \pm e^{ikR_q \cos \phi_{kb}} \Delta T_b(k; R_q, D)) \\ &= \sqrt{2} \int_{\phi_{k'} > 0} e^{-(k^2 + k'^2)D^2/2} \\ &\quad \sum_{n=-\infty}^{\infty} I_n(kk'D^2) J_{n+2}(k'R_q) G_{\pm}(\phi_{ka}) k'^2 (Re[f_{\mathbf{k}'}] + i Re[g_{\mathbf{k}'}]) \frac{d^2 \mathbf{k}'}{2\pi^2}, \quad (3) \end{aligned}$$

where $G_+(\phi_{ka}) \equiv \cos(n(\phi_{ka} + \pi/2))$ and $G_-(\phi_{ka}) \equiv i \sin(n(\phi_{ka} + \pi/2))$. In addition, the square variance of $\Delta T_{\pm}(k; R_q, D)$ can be calculated in terms of the conventional angular power spectrum $l(l+1)C_l/2\pi$.

Fortunately, the baselines of the most compact configuration, hexagonal packed, all satisfy the *separation condition*. These equilateral triangle baselines can be categorized into 2 configurations. The α -configuration has baselines subtending from the radial direction with angles $\phi_{k0} = 0, \pi/3, 2\pi/3, \pi, 4\pi/3, 5\pi/3$, and the β -configuration with angles $\phi_{k0} = \pi/6, \pi/2, 5\pi/6, 3\pi/2, 7\pi/6, 11\pi/6$.

The squared variance of primordial CMB polarization in Λ CDM cosmology is plotted against the annulus ring radius R_q and beam size D in Fig 1. As a reference, the result of single dish measurement is also plotted with some beam sizes.

Fig. 1. On the left hand side, it is the surface brightness (ΔT_{rms}^2 in K) of E-mode and B-mode polarization of single dish measurement. Smaller beam size, i.e., larger dish, has better sensitivity. On the right hand side, the contour is the surface brightness (ΔT_{rms}^2 in K) of E-mode and B-mode polarization with α -configuration and β -configuration of an interferometry. The optimized dish size is small, 8cm, in 90GHz.

3. Discussion

We demonstrate that interferometry measurements with the scanning annulus observing strategy can be clean in separating E and B modes and involving no statistical methods for E - B separation employed conventionally¹². Even pixelization of data can be bypassed here, in that we measure the continuously integrated surface brightness on a sky annulus. Because of the simplicity of this strategy, the quantity derived above can be used for an indication of B mode detection.

Comparing the two figures, the responsivity to the primordial B mode polarization for one single dish (the $k=0$ case) is 6 times higher than that for one baseline. When the dish number N is large, the performance ratio between the N -single-dish measurements and the N -dish interferometry measurement of close packed configuration drops to two. An experiment targeting at the CMB B -mode can be conducted on a turning table that houses many ~ 10 cm dishes. In such a setup, one can perform both interferometry measurement and (many) single-dish measurement at the same time. Though the small dish size is not optimal in sensitivity for single-dish measurement, it helps suppress E modes much more than B modes and serves to further avoid accidental E -mode leakage². In view of recent advents in the integrated MMIC technology for CMB detectors³, such a CMB B -mode experiment is expected to be realizable in the foreseeable future.

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