Magnetic-field-induced phase transitions in a Si/SiGe hole system

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Abstract

We have performed low-temperature transport experiments on a Si/SiGe hole system. The measured transverse and longitudinal conductivities σ_{xy} and σ_{xx} allow us to study the magnetic-field-induced transitions in this system. In particular, we present the study of temperature-driven flow lines in the “anomalous Hall insulator” regime near a Landau level filling factor $\nu = 1.5$. The “anomalous” temperature-driven flow lines could be due to the unusual energy level scheme in a Si/SiGe hole system. Moreover, for $3 < \nu < 5$, there is a temperature-independent point in $\rho_{xx}(B)$, $\rho_{xy}(B)$, $\sigma_{xx}(B)$, and $\sigma_{xy}(B)$ which corresponds to a boundary of the quantum phase transition.

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Investigation of the phase transitions in the quantum Hall effect is an important research topic in the two-dimensional hole gas (2DHG) or two-dimensional electron gas (2DEG) [1–10]. According to the global phase diagram (GPD) suggested by Kivelson, Lee and Zhang [1] a 2DEG or 2DHG is an insulator when highly disordered. At low disorder, there are a series of quantum Hall states and magnetic-field-induced transitions at various magnetic field B . The GPD also gives us the selection rule for such magnetic-field-induced phase transitions in the quantum Hall effect. For example, there are no insulators between neighbouring integer quantum Hall states. The GPD, however, is a zero temperature picture with infinite sample size. At a finite temperature T , the magnetic-field-induced transitions are

governed by the scaling theory [1,4,9–13]. The width in B of each transition region between two quantum Hall states is proportional to T^κ , and shrinks to a single metallic point as $T \rightarrow 0$ where κ is the critical exponent.

Early theoretical work on the integer quantum Hall effect describes a two-parameter scaling [12] in which both the dissipative conductance σ_{xx} and the Hall conductance σ_{xy} vary with sample length L . Note that the renormalisation-group function can be illustrated by flow lines [12]. After successive length-scale transformations the flow lines are directed towards fixed points $(\sigma_{xx}, \sigma_{xy}) = (0, n)$ (in the units of e^2/h) in which $n = 0, 1, 2, \dots$ is the Landau level index. In addition to these “localisation fixed points” which describe the localisation of the electron wave functions near the Fermi energy, there are “intermediate-coupling fixed points” on $\sigma_{xy} = n + \frac{1}{2}$ which are related to transitions between quantum Hall states. Note that this theory was developed for $T = 0$ and the length-scale

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transformations are accomplished by varying the sample length L . It has been shown that, in practice, the effective sample size can be varied by changing the temperature [9]. The studies of the “temperature-driven flow lines” in GaAs systems and more recently in Ge/SiGe systems [11] support the two parameter scaling theory of Pruisken [12]. In most cases, the observed quantum Hall states under unresolved spin-splitting are of even Landau level filling factors [2–4,9–11]. It was found that the critical exponent κ depends on whether the spin-splitting is resolved or not [10]. Spin-splitting also has effects on the $(\sigma_{xy} - \sigma_{xx})$ diagrams [9–15]. It was shown that the positions of the intermediate-coupling fixed points and the T -driven flow lines depend on the ratio of the spin-splitting to the Landau-level spacing [15].

Recently, there has been a great deal of interest in SiGe hole gases. For example, at a Landau level filling factor $\nu = 1.5$, an insulating phase [5–9] observed in such a system is not fully understood at present. It is called a “Hall insulator” since although σ_{xx} approaches infinity, the Hall resistivity remains finite at approximately $h/(2e^2)$. This magneto-driven quantum Hall transition was not expected within the GPD. In this paper, we present low-temperature transport measurements on a Si/SiGe hole gas. In particular, we report a study of the T -driven flow lines in the “Hall insulator” regime at around $\nu = 1.5$. At higher Landau level filling factors, $3 < \nu < 5$, there is a temperature-independent point in $\rho_{xx}(B)$ and $\rho_{xy}(B)$ which corresponds to a boundary of the quantum phase transition.

The Hall bar device used in this work was made from a high-quality Si/Si_{0.88}Ge_{0.12} heterostructure [8]. In our structure, holes are confined in two dimensions in a SiGe layer ~ 40 nm thick, sandwiched between the Si substrate and the cap layer. The carrier concentration of the 2DHG is $3.3 \times 10^{11} \text{ cm}^{-2}$ with a mobility of $\sim 6000 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$. The experiments were performed in a top-loading He³ cryostat. Four-terminal magnetoresistivities were measured using standard phase-sensitive lock-in techniques.

Fig. 1(a) shows the four-terminal longitudinal and transverse magnetoresistivity ρ_{xx} and ρ_{xy} at various temperatures T . At zero magnetic field, there is a small decrease in ρ_{xx} as T is lowered, characteristic of the metallic phase [16,17] close to the metal–insulator transition. An interesting feature is that there is a point A1 which is T -independent around a critical field $B = 3.1$ T for both ρ_{xx} and ρ_{xy} for $3 < \nu < 5$. We suggest that the temperature-independent point corresponds

Fig. 1. Four-terminal resistivity measurements of $\rho_{xx}(B)$ and $\rho_{xy}(B)$ at $T = 0.30, 0.60, 0.97$, and 1.2 K. A1, B1, and B2 are temperature-independent points in $\rho_{xx}(B)$. Near A2, ρ_{xx} has a weak temperature dependence. (b) Longitudinal conductivity $\sigma_{xx}(B)$ and $\sigma_{xy}(B)$ at various temperatures $T = 0.30, 0.60, 0.97$, and 1.2 K.

to a quantum phase transition boundary and note that similar behaviour was observed in a higher quality SiGe hole system [18]. As shown in Fig. 1(a), an anomalous Hall insulator near $\nu = 1.5$ is observed. The two T -independent points in ρ_{xx} near $B_{B1} = 8.1$ T and $B_{B2} = 9.8$ T are the boundaries of the “Hall insulator”. Note that the distance in B between these two critical points is the “width” of the anomalous Hall insulator regime. Therefore, the transition region between the $\nu = 1$ and 2 quantum Hall states will not shrink to a single point, which is different from the usual systems with spin-splitting much smaller than the Landau-level spacing [9–11,14]. Converting ρ_{xx} and ρ_{xy} into σ_{xx} and σ_{xy} , we plot the corresponding σ_{xx} and σ_{xy} in Fig. 1(b). We can see that a large ρ_{xx} centred around $\nu = 1.5$ causes σ_{xy} to drop well below its quantised value $\nu e^2/h$ for $1 < \nu < 3$.

Fig. 2(a) shows the $\sigma_{xy} - \sigma_{xx}$ diagram for $1 < \nu < 3$ at $T = 0.3$ K. As $\nu \rightarrow 1$, $(\sigma_{xy}, \sigma_{xx}) \rightarrow (e^2/h, 0)$ because

Fig. 2. σ_{xx} (σ_{xy}) at $T=0.3$ K. There is a point Q where σ_{xx} (σ_{xy}) crosses itself. (b)–(e) Temperature-driven flow lines between $1 < \nu < 3$. The arrows indicate the directions of the flow lines (from high- T to low- T) and the full squares correspond to intermediate-coupling fixed points. Note that the dotted line represents σ_{xx} (σ_{xy}) at $T=0.3$ K.

of the appearance of the quantum Hall state $\nu = 3$. In addition, there is a point Q at which σ_{xy} (σ_{xx}) crosses itself. The T -driven flow lines are complicated in our system thus we divide our results into four parts as shown in Fig. 2(b)–(e). It is evident that $(\sigma_{xy}, \sigma_{xx}) \rightarrow (0, 0)$ for $\nu \rightarrow 1$, as illustrated by the T -driven flow lines as shown in Fig. 2(b). Figs. 2(c) and (d) show the T -driven flow diagrams near the two critical points $B_{B1} = 8.1$ T and $B_{B2} = 9.8$ T, respectively. It is evident that $(\sigma_{xy}, \sigma_{xx}) \rightarrow (0, 0)$ as $T \rightarrow 0$ for the “Hall insulator” since ρ_{xx} approaches infinity whereas ρ_{xy} remains finite. We can clearly see that the directions of the flow lines in the “Hall insulator” are different from those near a quantum Hall state. B_{B2} , labelled as full squares, is, in fact, the boundary of the

“Hall insulator” as well as the intermediate-coupling fixed point. It is worth mentioning that near B_{B1} , ρ_{xy} slowly varies at different temperatures. Thus, although B_{B1} corresponds to the boundary of the “Hall insulator” regime, the intermediate-coupling fixed point is estimated to be at around $B = 8.2$ T, ~ 0.1 T higher than B_{B1} . To the right of this fixed point, the flow lines all show a “kink”. In contrast, to the left of the fixed point, the lines flow directly $(0, 0)$ as $T \rightarrow 0$. The main finding is that, in contrast to a *single* intermediate-coupling fixed point between two neighbouring quantum Hall states, there are two previously predicted coupling points in a SiGe hole system. The reason for this is the existence of the “anomalous Hall insulator” near $\nu = 1.5$. Our new

Fig. 3. Temperature-driven flow lines between $3 < \nu < 5$. The arrows indicate the directions of the flow lines (from high- T to low- T) and the full squares correspond to an intermediate-coupling fixed point. The dotted line represents σ_{xx} (σ_{xy}) at $T = 0.3$ K.

experimental results on T -driven flow lines, together with the pioneering work on the “Hall insulator” [5–9] urge further investigations in order to understand the physical origin of this “anomalous insulating state”. Fig. 2(e) shows the T -driven flow lines for $2 < \nu < 3$. At around A2, the directions of the T -driven flow lines seem ambiguous.

Finally, we turn our attention to low-field results. Fig. 3 shows experimental results for σ_{xy} and σ_{xx} plotted as T -driven flow lines from $T = 0.3$ to 1.648 K. Each line corresponds to a fixed B . As shown in Fig. 3, we can see an intermediate-coupling fixed point labelled as squares. In fact, this intermediate-coupling fixed point shown in Fig. 3 corresponds to A1 (T -independent σ_{xy} (T) and σ_{xx} (T)) as shown in Fig. 1(a) and (b). To the left of this point A1, which corresponds to a higher magnetic field, the flow lines are directed toward $(0, 4.5e^2/h)$, since the Hall conductance plateau at $\nu = 5$ is not well-quantised. In contrast to the semicircle-like feature seen previously [9,11], to the left of this intermediate-coupling fixed point, the T -driven flow lines are directed towards $\sigma_{xy} = 2.0e^2/h$ and eventually converge at $\sigma_{xy} = 3.0e^2/h$. This effect is believed to be due to strong Landau level mixing as described by the model of Ando [19]. The intermediate-coupling fixed point is close to $\sigma_{xy} = 3.25e^2/h$, not the theoretical value of $\sigma_{xy} = 3.5e^2/h$ [12]. We note that early experimental results on a GaAs electron system [9] did show a similar behaviour.

In conclusion, we have presented low-temperature magnetotransport results on a Si/SiGe hole system. For Landau level filling factors $3 < \nu < 5$, there is a T -independent point in ρ_{xx} and ρ_{xy} , which corresponds to a boundary of a quantum phase transition. Due to the existence of the “Hall insulator” around $\nu = 1.5$, the T -driven flow line study shows that there are *two* intermediate-coupling fixed points, in contrast to the single point previously predicted and observed in a GaAs sample. We speculate that the observed unusual T -driven flow lines could be due to the unusual energy level scheme in which the Landau level splitting is comparable to the Zeeman splitting in a Si/SiGe hole system. Our new results, together with previous work on the “Hall insulator” at $\nu = 1.5$, urge further theoretical and experimental investigations to understand the underlying physics of this “anomalous insulating state” observed in a Si/SiGe hole gas.

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