Cyclotron Resonance in Quasi-Two-Dimensional Electron Systems in the Quantized Hall Region

By

YANG-FANG CHEN and JI-LIN SHEN

With the advent of high-quality samples, it has been possible to measure the prominent new features that reflect the two-dimensional or the quasi-two-dimensional character of electrons in semiconductor heterojunctions and superlattices such as the quantum-Hall effect /1/, fractional quantum Hall effect /2/, classical Wagner crystallization /3/, and Anderson localization /3/. From the very beginning cyclotron resonance has proven to be a powerful tool for the investigation of two-dimensional electron systems /4/ and is often used to determine the quantities such as effective mass and various scattering mechanisms. A large number of recent cyclotron resonance experiments on the above systems has been reported /4 to 33/. A variety of anomalous behaviors in the cyclotron resonance such as line narrowing or splitting, positional shifts, frequency dependent cyclotron mass, Landau-level filling factor dependent linewidth, and temperature dependent linewidth and effective mass, etc., has been observed either in Si-MOS structures or in compound semiconductor heterostructures. On the theoretical side, there have been numerous calculations /34/ of the effects on screening, and the interactions due to different types of scattering mechanisms such as acoustic phonon, longitudinal and transverse optical phonons, interface phonon, impurity, and polaron etc. Even various explanations for the observed effects were given, but none of them proved to unambiguously stand the experimental evidence. For instance, at temperatures below 50 K, the dominant mechanism of linewidth has been attributed to the scattering by either impurities in the well /30/ or acoustic phonon /34/. In low-density samples, the oscillation on the linewidth as a function of the Landau-level filling factor has been attributed to either the dependence of impurity screening on the occupation of Landau levels /27/ or the effect of level crossing /29/.

To extract effective mass and linewidth from the cyclotron resonance spectra, most authors used the classical Drude model /27, 29, 33/. In this note we wish to point out that the cyclotron resonance spectra can be greatly modified by the quantized conductivity of the two-dimensional electrons observed in the quantized

1) Taipei 10764, Taiwan.
Hall effect. The previous analyses of the cyclotron resonance spectra may be flawed, and the large amount of the experimental results has to be re-examined.

Let us consider a high-mobility quasi-two-dimensional electron system in a dc magnetic field along the z direction perpendicular to the layer. The electrons are regarded as two-dimensional, since they are confined within the layer of thickness d and move freely along the xy plane. In order to observe the cyclotron resonance, the incident electromagnetic wave with respect to the applied dc magnetic field has to form the Faraday geometry, in which the electromagnetic wave propagates along the direction of the dc magnetic field. The formulation of the wave propagation and its relation to the dielectric response function of the electrons in the layer can be obtained by using Maxwell's equations

\[ \nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 , \tag{1} \]
\[ \nabla \times \vec{H} - \frac{\partial \vec{D}}{\partial t} = \vec{J} , \tag{2} \]

and the constitutive equations

\[ \vec{D} = \varepsilon \vec{E} , \tag{3} \]
\[ \vec{J} = \sigma \cdot \vec{E} , \tag{4} \]
\[ \vec{B} = \mu \vec{H} , \tag{5} \]

where, as usual, \( \vec{E} \) is the electric field, \( \vec{D} \) is the electric displacement vector, \( \vec{H} \) is the magnetic field, \( \vec{B} \) is the magnetic induction, \( \vec{J} \) is the conduction current density, \( \sigma \) is the conductivity tensor, \( \varepsilon \) is the lattice permittivity, and \( \mu \) is the permeability. Combining the above equations, we can have

\[ \nabla \times (\nabla \times \vec{E}) + \mu \sigma - \frac{\partial \vec{E}}{\partial t} + \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0 . \tag{6} \]

If we neglect the magnetic properties of the medium and assume that all fields vary as

\[ \vec{E} = \vec{E}_0 \exp(i(\vec{k}z - \omega t)) , \tag{7} \]

where \( \vec{k} \) is the wave vector in the medium, and \( \omega \) is the frequency of the wave, it leads to

\[ \vec{k} \times (\vec{k} \times \vec{E}) + \left( \frac{\omega}{c} \right)^2 \vec{k} \cdot \vec{E} = 0 , \tag{8} \]

where

\[ \vec{K} = k \vec{I} + i \frac{\omega}{c} \vec{E}_0 , \tag{9} \]
where \( \mathbf{T} \) is a unit tensor and \( k_1 = \epsilon/\varepsilon_0 \). From (8), it can be shown that for any given direction of propagation there are two polarizations of electromagnetic waves which can propagate independently in the medium. In the case of Faraday geometry, the two solutions for \( k \) are

\[
k^2_{\pm} = \omega^2/c^2 K_{\pm}^{(3D)},
\]

where

\[
K_{\pm}^{(3D)} = K_{xx}^{(3D)} \pm i K_{xy}^{(3D)}.
\]

The modes associated with the two roots are purely transverse and circularly polarized, such that

\[
\hat{E}_\pm = E_0(x \pm iy) \exp(i(k_x z - \omega t)).
\]

By the quantized Hall effect, the two-dimensional conductivity tensor takes the form

\[
\sigma^{(2D)} = \begin{bmatrix}
\sigma_{xx} & \sigma_{xy} \\
-\sigma_{xy} & \sigma_{yy}
\end{bmatrix}
\]

with \( \sigma_{xx} = \sigma_{yy} = 0 \), and \( \sigma_{xy} = ne^2/h \) in a quantized Hall region. Here the Landau filling factor \( n \) is an integer for the integer quantum Hall effect or a rational fraction for the fractional quantum Hall effect. Considering the finite thickness \( d \) of the layer, the three-dimensional conductivity tensor is related to the two-dimensional conductivity tensor by the following expression:

\[
\sigma^{(3D)} = \sigma^{(2D)}d.
\]

The diagonal three-dimensional conductivity \( \sigma_{zz} \) makes no contribution to the effective dielectric response function \( k_\pm \). Therefore, in the quantized Hall region, \( K_\pm \) are given by

\[
K_\pm = k_1 \frac{1}{\omega_0^2} \frac{ne^2}{\hbar d}
\]

\[
= k_1 \frac{n\omega_0}{\omega},
\]

where \( \omega_0 = e^2/\hbar \varepsilon_0 \). For typical quasi-two-dimensional electron systems, the layer thickness \( d \) is of order 10 nm, so that \( \omega_0 \) is of order \( 10^{14} \) s\(^{-1}\). To include the cyclotron resonance into the dielectric response functions, we treat this transition
as a classical oscillator having a resonance frequency \( \omega_c = eB_0/m^* \), where \( B_0 \) is the magnitude of the dc magnetic field and \( m^* \) is the effective mass, an oscillator strength \( f_c \), and a collision frequency \( v_c \). The effective dielectric response functions now can be written as

\[
K_\pm = k_\pm \left(1 + \frac{n\omega_0}{k_1 \omega} - \frac{f_c \omega^2}{c \omega_0} \right),
\]

where \( \omega_p = \sqrt{N_e e^2 / k_\parallel m^*} \), \( N_e \) is the carrier concentration. If we write the wave number in complex form,

\[
k_\pm = \alpha_\pm + i\beta_\pm,
\]

where \( \alpha \) and \( \beta \) are the real and imaginary parts of the wave number, respectively. Substituting (16) and (17) into (10), we obtain

\[
\alpha_\pm^2 - \beta_\pm^2 = \frac{\omega^2}{c^2} k_1 \left(1 + \frac{n\omega_0}{k_1 \omega} - \frac{f_c \omega^2}{c \omega_0} \right) \left(\frac{\omega + \omega_c}{(\omega + \omega_c)^2 - \nu_c^2} \right),
\]

\[
2\alpha\beta = \frac{\omega^2}{c^2} k_1 \left(\frac{f_c \omega^2}{c \omega_0} \nu_c \right) \left(\frac{\omega + \omega_c}{(\omega + \omega_c)^2 - \nu_c^2} \right).
\]

From (18) and (19), \( \alpha \) and \( \beta \) can be solved.

In order to see clearly the influence of the quantized conductivity of quasi-two-dimensional electrons on the cyclotron resonance spectra, let us consider the situation that the resonance intensity is weak. This condition can be satisfied if the two-dimensional concentration \( N_e \) is less than \( 10^{10} \) cm\(^{-2} \), the wavelength of the far-infrared radiation is 100 \( \mu \)m (3 \( \times \) 10\(^{13} \) Hz), and two-dimensional electrons are in a GaAs well. Under these conditions, \( K_\pm \) are simply given by

\[
K_\pm \approx k_\parallel + \frac{n\omega_0}{\omega}.
\]

The polarization that is associated with the cyclotron resonance is designated by the subscript "+" in our notation. We call this mode the cyclotron resonance active (CRA) polarization. Thus, we can see clearly that the quantum effect of the two-dimensional electrons can greatly modify the optical constants and hence the cyclotron spectra.

Experimentally the quantity of the measurement is the attenuation of light intensity. The transmission through a thickness \( d \) of the medium assuming \( \beta < \alpha \) is
\[ T = \frac{(1 - R^2)e^{-\alpha d}}{1 - R^2e^{-2\alpha d}} , \]  

(21)

where \( R \) is the reflectivity. If \( R \) is small, \( T \) is approximately given by

\[ T \approx e^{-\alpha d} . \]  

(22)

From (20), the absorption coefficient \( \alpha \) for the CRA polarization is

\[ \alpha_+ \approx \frac{\omega}{c} \sqrt{\frac{n\omega_o}{\omega} - k_1} . \]  

(23)

Thus, we expect that the integrated absorption recorded from the cyclotron resonance spectra in the quantized Hall region should decrease with increasing magnetic field because of the Landau level filling factor \( n \). Indeed, this prediction is consistent with the recent observation by Manasreh et al. /33/.

Previous analysis of the cyclotron resonance spectra was based on the classical Drude model /17, 19, 33/. In this model, the conductivity is given by

\[ \sigma_+ = \sigma_0 \left[ 1 + i\tau_{\text{CR}}(\omega \pm eB/m^*) \right]^{-1} , \]  

(24)

where \( \sigma_0 = \frac{e^2}{m^*} \tau_{\text{CR}} \) is the average collision time. The authors did not take into account the quantum behavior of the two-dimensional electrons observed in the quantized Hall effect. We do not believe that this analysis is appropriate. In this paper we point out that the quantized conductivity of the two-dimensional electron systems placed in a dc magnetic field can greatly modify the cyclotron resonance spectra. Thus, reinterpretation of the cyclotron resonance data is needed. In view of the ability to obtain the important physical parameters from the cyclotron resonance and the large amount of research on two-dimensional electron systems, this analysis should be useful and timely.

This work was supported in part by the National Science Council of the Republic of China.

References

K108

Physica status solidi (b) 170


(Received January 2, 1992)