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Coercivity and nucleation field of hollow ferromagnetic particles

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Abstract

Calculations of nucleation fields and critical radii of three possible magnetization reversal modes for an infinite ferromagnetic hollow cylinder with exchange and uniaxial anisotropy are made using Brown's equation. It shows that the curling mode is the favored mode for large radius and the nucleation fields for curling and buckling modes decrease for increasing outer radius and the ratio of inner to outer radius. The transition between these modes is very sensitive to the ratio of inner to outer radius.

Keywords: Coercivity; Ferromagnetic particles; Magnetization reversal modes; Nucleation fields

1. Introduction

It has been well-known for a long time, by virtue of the method of micromagnetics [1,2], that there are three possible nucleation modes of magnetization reversal when a reverse field acts on an initially saturated ferromagnetic infinite solid cylinder. Among these, the curling mode is the most favorable for large radius, the mode of rotation in unison takes place at small radius, and the buckling mode can exist only in a narrow range of radius between curling and rotation in unison, thus, usually does not need to be considered.

Although these results come from the rigorous treatment using Brown's equation, there are two reasons to extend the analysis to the case of a hollow cylinder. First, the fact known as 'Brown's paradox' [3] that the nucleation field predicted by the theory is higher than the practical value may be explained with an imperfect cylinder. The hollow cylinder can be recognized as a ferromagnetic particle with some nonmagnetic impurities inside. Therefore, to investigate the borderlines between these modes for hollow cases becomes more practical. Secondly, in the description of the curling mode for a solid cylinder is imperfect [4]. According to the solution of the linearized Brown's equation, the

curling mode is the cylindrically symmetric mode. The deviation of magnetization is zero at the center of the cylinder and increases with increasing radius. It was, therefore, believed that the magnetization near the cylinder surface reverses first at the nucleation field. However, the amplitude of this deviation is an undetermined constant from the linearized Brown's equation and turns out to be zero from the full nonlinear Brown's equation [4]. This result indicates that instability occurs at the nucleation field of the linearized Brown's equation but that the curling mode of the full nonlinear equation cannot propagate because of its null amplitude. This paradox results from an inappropriate boundary condition chosen for the center of a solid cylinder [i.e., $\theta(r=0)=0$], which is a line singularity in micromagnetic calculations with cylindric symmetry. This singularity not only restricts the possible magnetization configurations but also the number of possible magnetization reversal processes. Therefore, the curling mode with nonzero amplitude can never occur during the reversal process in a solid cylinder [4]. Nevertheless, such nonzero amplitude solutions exist in the hollow cylinder case. So the hollow cylinder model can remove this line singularity and keep the micromagnetic theory self-consistent.

In this paper, because of the above two reasons, we analyze the behaviors of curling, buckling and rotation in unison modes for a hollow cylinder.

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Fig. 1. An infinite hollow cylinder of inner radius a and outer radius b . The easy axis is along the z -axis.

2. Theory

2.1. Rotation in unison

Fig. 1 shows an infinite hollow cylinder of inner radius a and outer radius b . We assume the material has saturation magnetization M_s , exchange stiffness constant A , the first-order anisotropy constant K , and the applied field H_a is along the z -axis.

For uniform magnetization the exchange energy is absent and the magnetic free energy per unit length is a sum of the anisotropy, demagnetization and Zeeman energy terms:

$$E = \pi(b^2 - g^2)(\mathcal{E}_K + \mathcal{E}_M + \mathcal{E}_H) \quad (1)$$

The anisotropy energy density

$$\mathcal{E}_K = K \sin^2 \theta$$

the demagnetization energy density

$$\mathcal{E}_M = \pi M_s^2 \sin^2 \theta$$

and the Zeeman energy

$$\mathcal{E}_H = -M_s H_a \cos \theta$$

where θ is the angle between the magnetization and z -axis.

After using the reduced units defined as follows:

$$\varepsilon = a/b, \quad R_0 = \sqrt{A/M_s}, \quad S = b/R_0$$

$$h = H_a/4\pi M_s, \quad Q = K/2\pi M_s^2$$

the energy per unit length will become:

$$\bar{E} \equiv E/4\pi A S^2 = (1 - \varepsilon^2) \left[\frac{1}{2} \left(Q + \frac{1}{2} \right) \sin^2 \theta - h \cos \theta \right] \quad (2)$$

The nucleation field is defined as the value of applied field at which magnetization oriented along the saturation direction is no longer a local energy minimum state.

From this definition the nucleation field h_n can be obtained by solving the equation:

$$\delta^2 \bar{E}|_{\theta=0} = (1 - \varepsilon^2) \left[\left(Q + \frac{1}{2} \right) + h_n \right] = 0 \quad (3)$$

thus:

$$h_n = - \left(Q + \frac{1}{2} \right) \quad (4)$$

or in real units:

$$H_n = - \frac{2K}{M_s} - 2\pi M_s$$

This result is the same both for a solid particle and in the presence of a pore within a particle [5]. It should be noted that the nucleation field mentioned here means its absolute value because this field is usually in the opposite direction to magnetization.

2.2. Magnetization curling

This case has been discussed in detail [4]. Here we only give the results. The nucleation field is:

$$h_n = -Q - \frac{\mu^2(\varepsilon)}{2\pi S^2} \quad (5)$$

or in real units:

$$H_n = - \frac{2K}{M_s} - \frac{\mu^2(\varepsilon)/\pi}{S^2} 2\pi M_s$$

where μ satisfies

$$J'_1(\mu)N'_1(\mu\varepsilon) - J'_1(\mu\varepsilon)N'_1(\mu) = 0$$

In comparison with the solid case, the nucleation field depends sensitively on the radii ratio ε except for the outer radius S . However, when ε approaches zero, the value of μ is equal to 1.841, the same value obtained in the solid case [1,2]. Hence, the solid cylinder can be considered as the limiting case for the hollow cylinder.

2.3. Magnetization buckling

The magnetization pattern of the buckling mode is:

$$\mathbf{M} = M_s [\sin \theta \hat{x} + \cos \theta \hat{z}]$$

and θ has cosine dependence on z with period $2T$:

$$\theta = C \cos\left(\frac{\pi}{T}z\right)$$

where C is a small constant.

With a similar procedure as Frei et al. [1] used the nucleation field of a hollow cylinder can be obtained as:

$$h_n = -Q - \frac{l^2}{2\pi S^2} - \frac{1}{1 - \varepsilon^2} \mathcal{G}_\varepsilon(l) \quad (6)$$

where $l = \pi b/T$ is some particular period that maximizes h_n , and $\mathcal{G}_\varepsilon$ is given as:

Fig. 2. $\pi b/T$ for the buckling mechanism in an infinite hollow cylinder vs. its radius S .

$$\mathcal{G}_\varepsilon = \frac{I_1(l) - \varepsilon I_1(\varepsilon l)}{l \left[I_0(l) + \frac{K_0(l)}{K_1(l)} I_1(l) \right]} \quad (7)$$

Here $I(x)$, $K(x)$ are modified Bessel functions of the first and second kind, respectively.

Fig. 2 shows that when reduced outer radius S is small enough (~ 0.5) the period T will become very large and thus buckling reduces to rotation in unison. On the other hand, when S is very large, by asymptotic expansions of the Bessel functions

$$\begin{aligned} l(\varepsilon, S) &\simeq \left(\frac{\pi}{2} \right)^{1/3} (1 + \varepsilon)^{-1/3} S^{2/3} \\ &\simeq 1.16(1 + \varepsilon)^{-1/3} S^{2/3} \end{aligned} \quad (8)$$

When ε approaches zero, again we find that the result is identical to that in the solid cylinder case [2]. The nucleation field for a large value of S thus can be written as

$$\begin{aligned} h_n(\varepsilon, S) &\simeq -Q - \frac{3}{2} [4\pi(1 + \varepsilon)^2]^{-1/3} S^{-2/3} \\ &\simeq -Q - 0.645(1 + \varepsilon)^{-2/3} S^{-2/3} \end{aligned} \quad (9)$$

3. Results and discussion

The dependences of the nucleation fields on the outer radius S for different modes and radii ratio ε are shown in Fig. 3. We neglect the effect of anisotropy in Fig. 3, which, however, only gives rise to a shift for a nonzero anisotropy. From the figure we see that for the curling and buckling modes, the larger ε is, the smaller the nucleation field is. On the contrary, the nucleation field of the rotation in unison mode depends neither on the size of the cylinder nor on the extent of the hollowness of the cylinder. This is because the only field inside the material which resists the applied field, the demagnetization field, is the constant $2\pi M_s$. However, for incoherent rotations, the applied field needs to overcome the exchange and/or the demagnetization field, which clearly depends on size and hollowness.

Fig. 3. Relations between the nucleation fields and outer radius S for different modes and different radii ratio ε .

It can be seen also from the figure that, for given ε and S , different modes have different nucleation fields. The physical system will choose the one with smallest H_n . Hence, the intersections of field curves play the role of critical radius, which divides S into three different ranges, and the acting mechanism for a given ε is determined by which range S lies in.

Fig. 4 shows the relations between critical radius and radii ratio ε . When S is larger than the critical radius between the curling and the buckling mode, the magnetization will choose the curling mode as its initial reversal mechanism. If S is smaller than this critical radius and greater than about 0.5, the buckling mode is the acting mechanism. The value 0.5 is approximately the critical radius between the buckling and the rotation in unison mode (Fig. 2). The rotation in unison mode exists when $S \simeq 0.5$. Note that the critical radius between the curling and the rotation in unison mode plays no role here. However, from Fig. 2 we see that in the narrow range in which the buckling mode acts, i.e., $0.5 \simeq S \simeq 1$, the period of variation of magnetization T is long enough to regard it approximately as the rotation in unison mode. Moreover, with increasing of hollowness of the cylinder, this range will narrow down still more. So the buckling has less chance to occur both in the solid and hollow cases.

Fig. 4. Relations between critical radius and radii ratio ε . The solid curve is the critical radius between the curling and buckling modes, and the dashed curve is the critical radius between the curling and rotation in unison modes.

By virtue of the model of the hollow cylinder instead of the solid cylinder, it has been shown that some results in the solid case, such as the nucleation field and the critical radius, can be reached again by letting $\varepsilon \rightarrow 0$. Nevertheless, the analysis of the curling mode of the hollow cylinder in the nonlinear regime shows a different result, even letting ε approach zero [4]. From this point of view, the model of the hollow cylinder is better than the solid cylinder model.

Besides, the hollow model is also more practical than the solid one because it can reduce the nucleation field which, as predicted by the solid model, is too large. Although it does not take into account the factors such as the finite length of the cylinder [6], thermal fluctuations [7] and the impossibility of saturating the ferromagnetic samples completely [8], from theoretical and practical points of view, the hollow cylinder model is a more satisfactory model than the solid cylinder model.

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References

- [1] E.H. Frei, S. Shtrikman and D. Treves, *Phys. Rev.*, **106** (1957) 446.
- [2] A. Aharoni and S. Shtrikman, *Phys. Rev.*, **109** (1958) 1522.
- [3] A. Aharoni, *Rev. Modern Phys.*, **34** (1962) 227.
- [4] C.R. Chang and C.M. Lee, *Phys. Rev. B*, **50** (1994) 6461.
- [5] C.R. Chang and J.P. Shyu, *J. Appl. Phys.*, **70** (1991) 7504.
- [6] Y. Ishii and M. Sato, *J. Appl. Phys.*, **65** (1989) 3146.
- [7] H.B. Braun, *Phys. Rev. Lett.*, **71** (1993) 3557.
- [8] A. Aharoni, *Phys. Rev. B*, **43** (1991) 8670.