Thermal relaxation in arrays of coupled ferromagnetic particles

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A multilevel master equation formalism is used to describe the relaxational dynamics of thermally relaxing ferromagnetic particles with dipolar interactions. For weak coupling the transition probabilities between distinct metastable configurations of the entire particle array are found perturbatively within the coherent rotation approximation. The hysteresis properties and the number of metastable configurations contributing to the magnetization reversal process are studied as functions of the array geometry. A change in the effective number of metastable states is found to be accompanied by a local maximum in the value of the switching field distribution function at coercivity.

I. INTRODUCTION

The phenomenon of thermally activated magnetization reversal in single-domain ferromagnetic particles has been intensively studied for many years and the appealingly simple double-well model (e.g., Refs. 1 and 2) found numerous applications in the theory of noninteracting particles. Theoretical studies of interacting particle ensembles were carried out by means of Monte Carlo simulations (e.g., Ref. 3) while analytic work has been, so far, confined to a cluster of two interacting particles.⁴⁻⁶ The approximation scheme most commonly used in treating thermally relaxing interacting systems is the mean field theory⁷ based on the two-level description inherent to the double-well model. We propose here a multilevel^{5,8} master equation approach to relaxational dynamics of interacting systems and apply it then to one- and two-dimensional particle arrays. The two-level theory outlined in Eq. (1) below is recovered in some special cases.

Let there be an array of identical, noninteracting uniaxial particles and let $\kappa_{i \rightarrow j} = \kappa_{ij}$ ($i \neq j = 1, 2$) be the rate of transitions from one metastable orientation of magnetization into the other. The system has two metastable states (levels) and the fraction n_i of particles found in the i th state then satisfies the probability-conserving master equation

$$\dot{n}_1 = -\kappa_{12}n_1 + \kappa_{21}n_2 = -\dot{n}_2; \quad (1)$$

$\dot{n}_i = dn_i/dt$, and thermal equilibrium corresponds to the zero eigenvalue of Eq. (1). If further the magnetization of the i th state is $\vec{M}_i$, then the mean magnetization becomes $\langle \vec{M} \rangle = \sum_i \vec{M}_i n_i$ and we define also the reduced magnetization $\vec{m} = \langle \vec{M} \rangle / M_s$ where M_s is the saturation magnetization.

In order to gain some insight into the thermodynamics of an interacting system we review first the exactly solvable⁴ problem of a pair of identical, uniaxial particles parallel to the applied field $\vec{H} = (0, 0, H)$. The energy of an individual particle is^{1,4}

$$E^{(i)} = KV \sin^2 \theta_i - M_s H V \cos \theta_i, \quad (2)$$

where K is the anisotropy constant, V the activation volume, and θ_i the angle spanned by the particle's easy axis and its magnetization vector $\vec{M}^{(i)}$. The nucleation field is $H_n = 2K/M_s$ and $h = H/H_n$ is the reduced applied field. The energy of magnetostatic coupling between the two particles has the form

$$E_{\text{int}} = r^{-3} \vec{M}^{(1)} \cdot \vec{M}^{(2)} - 3r^{-5} (\vec{M}^{(1)} \cdot \vec{r})(\vec{M}^{(2)} \cdot \vec{r}), \quad (3)$$

where $\vec{r} = r(\sin \beta, 0, \cos \beta)$ is the radius vector joining the two particles and β is known as the bond angle. The total energy of the two particle cluster is then $E = E^{(1)} + E^{(2)} + E_{\text{int}}$ and we write $\mathcal{E} = E/KV$. Chen *et al.*⁴ found, for arbitrary coupling strength $\rho = M_s^2 V / (2Kr^3)$, the activation energies of this system for $\beta = 0$ (ferromagnetic) and $\beta = \pi/2$ (antiferromagnetic coupling); we cite here only the weak coupling results relevant to the present work. At small coupling strength $\rho < 1/3$, in applied fields $|h| < h_n(\rho)$, $h_n = \sigma(\beta)[(1-3\rho)(1-\rho)]^{1/2}$ with $\sigma(0) = 1$ and $\sigma(\pi/2) = [(1-3\rho)/(1+3\rho)]^{1/2}$, there exist^{4,5} four metastable configurations and four saddle points schematically depicted in Fig. 1. The four metastable configurations correspond to the state $\uparrow\uparrow$ (labeled as 1), to the physically equivalent states $\downarrow\uparrow$ and $\uparrow\downarrow$ (2₁ and 2₂), and to $\downarrow\downarrow$ (3) with energies $\mathcal{E}_1 = -\varepsilon - 4h$, $\mathcal{E}_{2_i} = \varepsilon$, and $\mathcal{E}_3 = -\varepsilon + 4h$; $\varepsilon = 4\rho$ if $\beta = 0$ and $\varepsilon = -2\rho$ if $\beta = \pi/2$. By symmetry only two of the four saddle point energies are distinct, with energies $\mathcal{E}^{(\alpha)} = 1 \pm 3\rho^2 + h^2 \mp 2h h_n$, where the upper (lower) sign at $3\rho^2$ corresponds to $\beta = 0$ ($\pi/2$) while the upper (lower) sign at $2h h_n$ refers to the saddle $\alpha = a$ (b).

We shall now assume that there exists a large array of the two-particle clusters. Following Pfeiffer's multilevel formalism⁸ we describe⁵ the evolution of the array in terms of the joint probabilities of finding both particles, at the same time, with a given metastable orientation ($\uparrow$ or $\downarrow$) of magnetization. Writing $n_2 = n_{2_1} + n_{2_2}$ it is possible (see Fig. 1) to reduce the four flux equations into three:

FIG. 1. A two-particle cluster: schematic representation of the four metastable minima and of the saddle points separating them. The minima 2_1 and 2_2 are identical and the scheme is equivalent to the graphical representation of the three-level master equation (6) shown on the right. Below is a graphical representation of a master equation for a chain of three identical particles with bond angles $\beta = \pi/2$. The single-particle relaxation rates κ_i , $i \in \langle 1, 10 \rangle$, are defined by the orientation of the nearest neighbors. The states $\downarrow\uparrow\uparrow$ and $\downarrow\downarrow\uparrow$ are twice degenerate, the same as the intermediate state 2 of the two-particle chain.

$$\dot{n}_1 = -2\kappa_{12}^{(a)} n_1 + \kappa_{21}^{(a)} n_2, \quad (4)$$

$$\dot{n}_2 = 2\kappa_{12}^{(a)} n_1 - (\kappa_{21}^{(a)} + \kappa_{23}^{(b)}) n_2 + 2\kappa_{32}^{(b)} n_3,$$

and $\dot{n}_3 = -\dot{n}_1 - \dot{n}_2$ by conservation of probability. The flux rate from i into j through the saddle α is $\kappa_{ij}^{(\alpha)} = f_0 \exp(-qQ_i^{(\alpha)})$, $q = KV/T$, where T is temperature ($k_B = 1$), f_0 is the prefactor, and the reduced barrier heights $Q_i^{(\alpha)} = \mathcal{E}^{(\alpha)} - \mathcal{E}_i$. There is $\lim_{\rho \rightarrow 0} Q_i^{(\alpha)}(h, \rho) = (1 \pm h)^2$ so that the isolated particle result⁴ is recovered for the Arrhenius factor of the rates $\kappa_{ij}^{(\alpha)}$. In the limit⁹ of moderate to strong damping $f_0 = \lambda(2\pi)^{-1}(\mathcal{H}_m/|\mathcal{H}_s|)^{1/2}$ where the frequency of the unstable mode λ follows from the dissipative equations of motion¹⁰ which decouple as $\rho \rightarrow 0$, yielding λ for an isolated particle. The Hessians of the total energy at the metastable minimum, $\mathcal{H}_m$, and at the saddle point, $\mathcal{H}_s$, decompose in the limit $\rho \rightarrow 0$ as $\mathcal{H}_m \rightarrow \mathcal{H}_m^{(1)}\mathcal{H}_m^{(2)}$ and $\mathcal{H}_s \rightarrow \mathcal{H}_s^{(1)}\mathcal{H}_s^{(2)}$ and the single-particle result is then recovered for f_0 . The prefactor is also known in the underdamped limit,^{9,10} but its decomposition properties are not clear and we use here the approximation^{10,11} $f_0 = e^{25}$ Hz throughout.

Returning now to Eq. (4) we define the number of “up” (“down”) particles as $n_\uparrow = n_1 + n_2/2$ ($n_\downarrow = n_3 + n_2/2$). If $\rho \rightarrow 0$, then $\kappa_{12}^{(a)} = \kappa_{23}^{(b)} \rightarrow \kappa_{12}$ and $\kappa_{21}^{(a)} = \kappa_{32}^{(b)} \rightarrow \kappa_{21}$ and Eq. (4) yields the master equation (1) for an isolated particle with $1 = \uparrow$ and $2 = \downarrow$. It is not, however, possible to write down a closed differential set for the probabilities $n_\uparrow$ and $n_\downarrow$ for any $\rho > 0$. Nonetheless, our previous analysis⁵ shows that if $\rho > 0$ and $\beta = 0$ (ferromagnetic coupling) then $n_2 \ll 1$ at all times and Eq. (4) effectively reduces to a two-level theory interpreted in terms of the two *joint* probabilities n_1 and n_3 . A mean field approximation⁷ is

in this case applicable, in contrast to an antiferromagnetically coupled system ($\beta = \pi/2$) which requires the full three-level description.

Unless otherwise specified we choose in our model calculations the values $q = 42$ (the lifetime of an isolated particle in zero field is then about 10 months) and $f = 10^1$ Hz for the sweep rate of the applied field $h(t) = \cos 2\pi ft$.

II. WEAK COUPLING LIMIT

A generalization of the multilevel formalism to arrays composed of a large number of identical clusters of N interacting uniaxial particles is quite straightforward, provided that the local energy minima and saddle points of the N -particle cluster can be found. The two-particle model is instructive since inspection of the barrier heights $Q_i^{(\alpha)}(\rho)$ reveals that to *first order* in the coupling strength ρ they may be calculated by assuming that one particle remains frozen in its metastable position throughout the reversal of the partner particle. At larger ρ this is no longer true since even in a single-particle reversal both particles initially deviate from the local minimum but only one overcomes the barrier while the other reverts to its initial position.⁴

The weak coupling scheme of single-particle reversals is readily extended to more complex systems. For simplicity we shall assume that all constituent particles are identical and that they are deposited in the x - z plane with easy axes parallel to the applied field $\vec{H} = (0, 0, H)$. The variables $p = \cos \theta$ and ϕ parametrize the magnetization vector of the one reversing particle which we place into the origin. All its neighbors are frozen in their unperturbed metastable positions with $\tilde{p}_m = \cos \theta_m = \pm 1$. A cluster of N uniaxial particles has up to $\mathcal{N} \leq 2^N$ distinct metastable configurations which are uniquely determined by the $\mathcal{N}$ sets of N numbers $\{\tilde{p}_m\}$. To first order in ρ the energy of a particle reversing in a given configuration becomes, by virtue of Eqs. (2) and (3),

$$\mathcal{E} = 1 - p^2 - 2hp + \mathcal{E}_{\text{int}}, \quad (5)$$

$$\mathcal{E}_{\text{int}} = 2 \sum_m \rho_m \tilde{p}_m \{p(1 - 3 \cos^2 \beta_m) - 3(1 - p^2)^{1/2} \cos \phi \cos \beta_m \sin \beta_m\}, \quad (6)$$

where the summation is over all frozen particles at $\vec{r}_m = r_m(\sin \beta_m, 0, \cos \beta_m)$ and $\rho_m = M_s^2 V / (2K r_m^3)$. Interactions introduce anisotropy into the hard plane of the reversing particle and the function $\mathcal{E}(p, \phi)$ of Eq. (5) has an extremum at $(p^{(s)}, \phi^{(s)})$. From the condition $\partial \mathcal{E} / \partial \phi = 0$ it follows that $\sin \phi^{(s)} = 0$ and the extremum is a saddle if $\partial^2 \mathcal{E} / \partial \phi^2 > 0$, i.e., if

$$\cos \phi^{(s)} \sum_m \rho_m \tilde{p}_m \cos \beta_m \sin \beta_m > 0,$$

$|\cos \phi^{(s)}| = 1$. Substituting this saddle point condition into Eq. (6) we finally obtain

$$\mathcal{E}_{\text{int}} = 2p \sum_m \rho_m \tilde{p}_m (1 - 3 \cos^2 \beta_m) - 3(1 - p^2)^{1/2} \left| \sum_m \rho_m \tilde{p}_m \sin 2\beta_m \right|, \quad (7)$$

whence the barrier heights $Q_i^{(\alpha)}$ may be calculated.

To every metastable configuration we now assign the reduced magnetic moment $\mu_i = \sum_{m=1}^n \tilde{p}_m$, $|\mu_i| \leq N$, and the occupation probability n_i and generalize the master equations (1) and (4) to the form $\dot{n}_i = -\sum_{j=1}^N R_{ij} n_j$. The transition rates matrix R is a function of the $N \times N$ barrier heights (local fields) defined by Eqs. (5) and (7); it is, in general, asymmetric, singular (zero eigenvalue by conservation of probability) but sparse. This follows from the exclusion of simultaneous multiparticle reversals which demands that $R_{ij} \neq 0$ only if $i = j$ or if $|\mu_i - \mu_j| = 2$.

The local field approximation outlined above has been employed in Monte Carlo simulations³ involving a large number of randomly switching interacting particles; we stress again that it holds only in the weak coupling limit where the rotational coherence between neighboring particles (asymmetric fanning⁴) may be neglected. Equations (5) and (7) thus underestimate the relaxation rates $\kappa_{ij}^{(\alpha)}$ as may be seen from Fig. 2 where the exact⁵ reduced coercivity $h_c(\rho) = H_c(\rho)/H_n$ of the two-particle cluster is always lower (unless $\rho = 0$) than the weak coupling result.

FIG. 2. The exact reduced coercivity $h_c(\rho)$ of a two-particle chain (dashed line, after Ref. 5) compared with the weak coupling results for $N = 2, 3, \dots, 7$ particle chains (labeled solid lines). For bond angle $\beta = 0$ the weak coupling result is independent of $N \geq 2$; for bond angle $\beta = \pi/2$ (we plot these curves with $\rho \rightarrow -\rho$) the distinction between odd and even N vanishes at sufficiently small ρ . The $N = 6$ curve is almost identical to the $N = 4$ one.

III. PARTICLE CHAIN

We now apply Eqs. (5) and (7) to a chain of N identical uniaxial particles parallel to $\vec{H} = (0, 0, H)$. For simplicity we consider only two values of the bond angle, $\beta = 0$ and $\beta = \pi/2$, and nearest neighbor interactions. There exist then six relaxation rates $\kappa_1 - \kappa_6$ for single-particle reversals within the chain and four rates $\kappa_7 - \kappa_{10}$ for reversals of the edge particles which have only one neighbor. The rates κ_i are depicted in Fig. 1 where we present a graphical representation of the matrix R for three-particle chain $N = 6$ metastable configurations (nodes). Analogous graphs exist for arbitrary N . Only single-particle reversals are allowed and each node of the graph is therefore connected by N relaxation channels to other nodes since every particle of the N -particle cluster has a nonzero (albeit perhaps small) reversal probability in each metastable state. For large N we use a simple combinatorial program which writes down the FORTRAN code for $R = R(\kappa_i)$. The resultant differential system is stiff and we solve it numerically using the backward Euler method.¹²

The relaxational properties of the system depend on N . According to Fig. 2 the coercivity of a ferromagnetically coupled chain ($\beta = 0$) is essentially independent of $N \geq 2$ since only the two saturated states, “all up” and “all down,” occur with nonzero probability. The properties of an antiferromagnetically coupled chain ($\beta = \pi/2$) are influenced by the stability of the $N - 2$ partially demagnetized intermediate states: The coercivity of a three-particle chain surprisingly *increases* with ρ and we find that the chain rapidly switches from the initial state $\uparrow\uparrow\uparrow$ into the relatively stable symmetric configuration $\uparrow\downarrow\uparrow$ with $\mu_i > 0$ and similarly also for larger, odd N . The $N = 4$ loop, on the other hand, is dominated by the presence of the stable, twice-degenerate ordered state $\uparrow\downarrow\uparrow\downarrow$ with $\mu_i = 0$. Figure 2 shows that for odd N and small ρ the functions $h_c(\rho)$ initially decrease with ρ , following the even N curves, until at some $\rho_c(N)$, $\rho_c(N + 2) > \rho_c(N)$, they deviate and start increasing. Models with a small *even* number of particles thus represent (qualitatively) extended systems better than models with small odd N .

IV. PLANAR ARRAYS

Long-range coupling reduces somewhat the stability of the intermediate states and, to a degree, removes the distinction between odd and even N . Its effects shall now be studied in two-dimensional lattice arrays of identical uniaxial particles whose easy axes lie in the lattice plane (the x - z plane), parallel to $\vec{H} = (0, 0, H)$. We consider here a square and a diamond lattice which may both be stretched by a factor ξ in the z direction. The distance a between nearest neighbors is identical in all unstretched ($\xi = 1$) lattices. In order to avoid very large differential systems we use here a simplifying periodicity assumption:¹³ We choose an elementary (square or diamond) cell with $N = 4$ particles and cover the entire plane by identical copies of a given metastable configuration of the elementary cell. This assumption defines the

neighborhood of every particle and by symmetry there are then $\mathcal{N} = 7$ and 9 distinct metastable configurations for the square and diamond lattices, respectively.

Let first $\xi = 1$ and let us consider the square lattice in the nearest neighbor approximation where every particle has two pairs of identical neighbors in the vertical (z) and horizontal (x) directions, respectively. Magnetostatic interactions favor in this case the formation of alternating vertical chains of “up” and “down” particles and this is reflected in the hysteresis curve (Fig. 3) dominated by the

FIG. 3. Top: hysteresis loops for a square lattice. Nearest (*, $\xi = 1$) and next nearest neighbor interaction ($\circ$, $\xi = 1$) models are compared, for $\rho = 0.015$, with the long-range model of Eq. (8) shown by unmarked curves: From left to right $\xi = 0.75, 1, 1.25, 1.5, 2$, and 3. For $\xi = 3$ we also show loops with $\rho = 0.02$ (+) and $\rho = 0.025$ ($\times$). Bottom: the probabilities ν_0 (short dashed line), ν_2 (short dashed line, inverted in the plot), and ν_4 (solid lines) corresponding to the six long-range hysteresis loops with $\rho = 0.015$ shown above. The curves ν_0 and ν_4 shift to the right with increasing ρ . The probabilities ν_2 , which are negligible for $\xi = 0.75$ and 1, are not shown; at higher values of ξ the functions ν_2 progressively depart from zero.

presence of a very stable demagnetized state. Note that an elementary cell with odd number of particles in the x direction does not admit this alternating chain structure and yields qualitatively different results. Inclusion of the four identical next nearest neighbors reduces the stability of the demagnetized configuration and stabilizes the saturated state (Fig. 3).

On the periodic lattice of elementary cells it is easy to take into account long-range interactions by carrying out the lattice sums¹⁴ of Eq. (7): Let the i th particle of the elementary cell, with $p_i = \cos \theta_i$, be reversing while all its neighbors are found in their unperturbed metastable positions $\tilde{p}^{(k)} = \pm 1$, $k \in \langle 1, 4 \rangle$. Its interaction energy becomes

$$\mathcal{E}_{\text{int}}^{(i)} = 2p_i \sum_{k=1}^4 A_{ik} \tilde{p}^{(k)} - 3(1 - p_i^2)^{1/2} \left| \sum_{k=1}^4 B_{ik} \tilde{p}^{(k)} \right|, \quad (8)$$

and the 4×4 elements $A_{ik} = \sum_m \rho_m (1 - 3 \cos^2 \beta_m) |_{\tilde{p}_m = \tilde{p}^{(k)}}$ and $B_{ik} = \sum_m \rho_m \sin 2\beta_m |_{\tilde{p}_m = \tilde{p}^{(k)}}$ are computed numerically. They correspond to coupling of the reversing (i)th particle to all frozen particles of the k th type. A single-particle reversal causes the elementary cell and, by assumption, also the entire array to go over to a new metastable configuration.

For a square lattice the matrix $B \equiv 0$ and A is symmetric with respect to both the main and the side diagonals, with only four independent components. Comparing now the nearest and next nearest neighbor hysteresis loops of Fig. 3 with the $\xi = 1$, long-range result, we see that the stability of the saturated state and, consequently, also the coercivity are further enhanced while the stability of the demagnetized state is no longer significantly affected. In Fig. 4 we plot the reduced coercivity $h_c(\xi, \rho)$ and the value of the reduced switching field distribution¹ (SFD) at the coercive field $\chi_c(\xi, \rho) = \partial \langle m \rangle / \partial h |_{h=h_c}$. With increasing ξ the influence of ferromagnetic coupling in the vertical direction wanes, causing the saturated state to be less and the demagnetized state to be more stable. As expected, therefore, both h_c and χ_c drop off (as $\xi \rightarrow \infty$) towards an asymptotic value corresponding to a horizontal particle chain with lattice constant a and bond angle $\beta = \pi/2$. This is also quite apparent from Fig. 3 where we plot the probabilities $\nu_\mu = \sum_{i=1}^{\mathcal{N}} n_i |_{\mu_i = \pm \mu}$ that the system finds itself in any of the states whose reduced magnetic moment is $\mu_i = \pm \mu$: For $\xi \leq 1$ only the saturated states (stabilized by ferromagnetic coupling) and the ordered demagnetized state (alternating chains stabilized by antiferromagnetic coupling) occur with nonzero probabilities ν_4 and ν_0 , respectively. The strong correlation in the vertical direction is reduced for $\xi > 1$ where also $\nu_2 > 0$. There exist therefore two distinct relaxation regimes differing by the number of participating levels and Fig. 4 shows that the SFD function $\chi_c(\xi, \rho)$ has a local maximum in the transition region between them. A similar, even more pronounced local maximum found for the diamond lattice is discussed below.

Consider now the coercivity function $h_c(\xi, \rho)$. According to Fig. 4 the coercivity of an unstretched lattice ($\xi = 1$) increases with increasing ρ (decreasing lattice constant), contrary to the measurements carried

out by Smythe *et al.*¹⁵ on lithographically deposited arrays whose geometry (ideally) coincides with the present square lattice. The unstretched lattice model thus apparently overestimates the influence of ferromagnetic coupling and Fig. 4 shows that the function $h_c(\xi, \rho)$ has, in fact, a rather complicated behavior: At a given q the curves $h_c|_{\rho=\text{const}}$ all intersect at $\xi = \xi_0$ (here $\xi_0 \approx 1.6$ for $q = 42$) and the common value $h_c(\xi_0, \rho)$ is close to the coercivity of a decoupled array, $h_c(\xi_0, \rho) \approx h_c(\xi, 0)$. Coercivity grows with increasing ρ in a compressed lattice with $\xi < \xi_0$ but decreases with increasing ρ in a stretched lattice with $\xi > \xi_0$. At the point ξ_0 the majority of particle reversals at coercivity take place in a local field which is independent of the coupling strength ρ . The remaining ρ -dependent local fields influence the value of the switch-

ing field distribution and cause the intercept point ξ_0 to shift to lower values at higher temperatures.

Strong coupling endows, for any ξ , the hysteresis loops with long tails (see Fig. 3) which arise due to the decreasing relative (to the demagnetized state) stability of the partially demagnetized intermediate states, in particular beyond coercivity.

For a diamond lattice the matrices A and B are again symmetric with respect to both diagonals; moreover, $A_{14} = A_{12}$ and the only nonzero elements of B are $B_{14} = -B_{12}$ and their symmetric partners. By virtue of Eq. (8) and the definition of $\tilde{p}^{(k)} = \pm 1$ the matrix B therefore acts only on the intermediate configurations. According to Fig. 4 the coercivity of a diamond lattice has essentially the same ξ dependence as that of a square lattice; only its asymptotic ($\xi \rightarrow \infty$) value is somewhat larger since the interparticle spacing in the resultant decoupled horizontal chains is $2^{1/2}a > a$. The critical intercept ξ_0 is here considerably higher than in the previous case, $\xi_0 \approx 3.2$. Interestingly enough, not only $h_c(\xi_0, \rho) \approx h_c(\xi, 0)$, but contrary to the square lattice, also $\chi_c(\xi_0, \rho) \approx \chi_c(\xi, 0)$, apparently because at coercivity the magnetization reversal is absolutely dominated (at $\xi = \xi_0$) by a single-particle reversal process which is independent of ρ . However, there are $\mathcal{N} \times N = 36$ local fields and $\xi_0 = \xi_0(T)$ as before.

Analyzing the $\langle m \rangle$ and ν_μ plots analogous to those shown in Fig. 3 we find that at small ξ the diamond lattice is distinguished by direct reversal between the two saturated states. There is then $\nu_4 \approx 1$ and the sojourn time in the intermediate configurations is negligible. The relative shift of neighboring vertical columns of particles precludes the formation of the stable demagnetized state encountered at the square lattice and the system thus effectively has only two levels. With increasing stretching factor ξ both partially and fully demagnetized configurations occur with nonzero probabilities and this transition between the two relaxation regimes is again accompanied by a local maximum of the switching field distribution function $\chi_c(\xi, \rho)$. Intuition suggests that the prominence of this local maximum is related to the fact that the transition here takes place between a two-level (small ξ) and a nine-level (large ξ) system. The less pronounced local maximum found for the square lattice is associated with a transition between a five-level system at small ξ and a seven-level system at large ξ . A local maximum of $\chi_c(q)$ was predicted (and subsequently observed) also for the transition between irreversible and reversible relaxation regimes in noninteracting particles¹ and it is interesting to encounter it anew, this time associated with a transition between relaxation regimes with different effective numbers of contributing metastable states.¹⁶

The tiling of the lattices by the chosen elementary cells is not unique¹⁷ and by direct calculation we verified that the properties of the array are independent of the chosen tiling mode.

V. SUMMARY

The relaxational dynamics of N interacting, thermally relaxing units are described here in terms of transition

FIG. 4. Top: the reduced coercivity h_c vs the stretching factor ξ for the square ($\square$) and diamond (unmarked) lattices at $q = 42$. Coupling strength $\rho = 0.015$ (short dashed line), 0.02 (solid line), and 0.025 (long dashed line). Also shown (*) is the square lattice coercivity at $q = 39$ and $\rho = 0.015$. Horizontal lines depict the coercivity of a decoupled system at given q . Bottom: the corresponding values of χ_c , markings same as above.

rates between the $\mathcal{N}$ metastable configurations of the entire system. We find these rates for arrays of weakly interacting ferromagnetic uniaxial particles and demonstrate that in the limit of zero magnetostatic coupling the master equation for the joint probabilities n_i may be reinterpreted as a simple two-level (double-well) model which was also recovered by other means in the limit of strong ferromagnetic coupling.

We varied the geometry of planar lattice arrays and compared the value χ_c of the switching field distribution function at coercivity with the number of metastable states whose occupation probabilities n_i remain negligible during one sweep of the driving field. It was found that the function χ_c attains a local maximum whenever the total number of nonempty levels changes. Similar be-

havior of the function χ_c is known¹ to be associated with a transition between irreversible and reversible relaxation regimes and our recent work¹⁸ shows that the initial dc susceptibility also attains its maximum value (at a given temperature and measurement time) in the transition region where the number of contributing metastable states changes.

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