Field-dependent prefactor of the thermal relaxation rate in single-domain magnetic particles

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Within a model of the coherent rotation, we assume underdamped dynamics of magnetization in single-domain ferromagnetic particles and calculate the prefactor $f_0(h)$ of the thermal decay rate as a function of the applied field h . Four different models of dissipative coupling are considered. As opposed to the previously reported temperature dependence of the prefactor, the field dependence is remarkably independent of the chosen coupling mode. The prefactor $f_0(h)$ drops off sharply as h approaches the nucleation field and significantly enhances the coercivity whose model calculation, together with that of the switching-field distribution, is carried out.

I. INTRODUCTION

In considerations involving the thermal decay rate κ of magnetic systems it has become customary to treat its prefactor f_0 as a constant, estimated in micromagnets to be on the order of e^{25} Hz. The theory of thermal relaxation in micromagnets is, however, well established and allows one to calculate the prefactor in more detail. It was launched by Brown¹ who wrote down Gilbert's equation in stochastic form, derived its Fokker-Planck operator, and calculated the relaxation rate for strictly axially symmetric model of coherent rotation of magnetization in a uniaxial ferromagnetic particle. The problem was later reviewed by Klik and Gunther^{2,3} who found the axially symmetric model of Brown to be degenerate; this is discussed in Sec. II, where we present our model, and in the conclusion to Sec. III. They showed that results for a nondegenerate system of arbitrary point symmetry coincide with findings of the general theory of thermal relaxation (see, e.g., Ref. 4) even though the magnetic system is driven by vector multiplicative noise rather than by the usually considered scalar additive noise. The present work employs this general theory to calculate the field- (h) dependent prefactor of thermal decay rate within the model of coherent rotation.

Stochastic models are necessarily phenomenological, drawing, however, on microscopic calculations to establish the nature of the noise field, its intensity and the mode of its coupling to the damped system. The physics of dissipative effects in micromagnetics has been recently intensively studied, in particular, at very low temperatures, in conjunction with macroscopic quantum phenomena yet their nature remains rather obscure: Magnetoelastic dissipation has been shown to be very weak⁵ and the recently proposed damping by nuclear spins⁶ is an unlikely candidate for room-temperature effect driving thermal relaxation. The prefactor of thermal decay rate, however, cannot be calculated unless the dissipative mechanism is known and in view of the doubts surrounding the dominant dissipative mechanism we adopt the following strategy: We assume underdamped magnetization dynamics and use a recently developed axiomatic method⁷ of constructing stochastic systems driven by multiplicative

noise to study, apart from Gilbert's equation¹⁻³, yet three other conceivable models of dissipative coupling. Reassuringly, the prefactor $f_0(h)$, calculated for all four models in Section 3, turns out to be essentially model independent.

This program has been carried out recently⁸ in investigations of the prefactor temperature (T) dependence. Neglecting field effects it was found that in the underdamped limit $f_0(T) \propto T^{-1}(1-T/T_C)^\nu$ with a specific value of the exponent $\nu \geq 2$ corresponding to each damping model. At temperatures well below the Curie point T_C its influence is negligible. In contrast, the function $f_0(h)$ studied here greatly suppresses the rate of thermal relaxation at large applied fields. In Sec. IV, we discuss the concomitant enhancement of coercivity together with some other possible measurable effects due to the nonconstant prefactor.

Thermal decay rate⁴ in the underdamped limit is

$$\kappa = \frac{\omega_0 \Delta E}{2\pi T} e^{-Q/T}. \quad (1)$$

Here Q is barrier height in the exponential (Arrhenius) factor and we set $k_B = 1$ throughout. Further, ω_0 is the undamped frequency at the bottom of the metastable well (energy E_0) and ΔE is the energy loss per period of the almost periodic underdamped motion at the saddle-point energy E_1 . Whereas $Q = E_1 - E_0$ and ω_0 are determined by conservative dynamics the energy loss ΔE must be calculated from the dissipative equations of motion. The formalism of Ref. 7 which we employ in their derivation requires that they be parametrized by means of the canonical pair $\phi \in (0, 2\pi)$ and $P \in (-P_0, P_0)$:

$$\gamma_0^{-1} \mathcal{M} = \mathbf{S} = [(P_0^2 - P^2)^{1/2} \cos \phi, (P_0^2 - P^2)^{1/2} \sin \phi, P]. \quad (2)$$

Note that the two points $P = \pm P_0$ are not included in the phase space. The gyromagnetic ratio is γ_0 and $\mathcal{M} = V\mathbf{M}$ is the magnetic moment of a particle with volume V and magnetization $\mathbf{M}$. By assumption, $|\mathbf{M}| = M_s$ and $|\mathbf{S}| = P_0$ at all times.

II. THE MODEL

The energy loss ΔE of Eq. (1) is to be calculated by integration along a contour of critical (saddle-point) energy whose geometry is, as a rule, rather complicated. To keep things simple we shall consider a uniaxial particle with hexagonal symmetry in the basal plane (such as found in Ba ferrite systems) in applied field h parallel to its principal axis. Its magnetic energy in terms of the direction cosines α_i is^{3,9}

$$H(\alpha_i) = K^{(0)}(1 - \alpha_3^2) + K^{(1)}(1 - \alpha_3^2)^2 + K^{(2)}\alpha_1\alpha_2[\alpha_1^4 - (\frac{10}{3})\alpha_1^2\alpha_2^2 + \alpha_2^4] + B\alpha_3, \quad (3)$$

where $B = -VM_s h$ is an applied field and the scaled anisotropy constants $K^{(i)}$ are assumed to be positive and satisfying the inequality $K^{(0)} > K^{(1)} > K^{(2)}$. In the basal plane $H(\alpha_i)$ has six local minima (saddles of the energy surface) in the directions $[1, 1, 0]$, $[-\sin\pi/12, \cos\pi/12, 0]$, and $[\cos\pi/12, -\sin\pi/12, 0]$. We rotate the system (3) by $\pi/4$ about the three-axis, so that one of the six saddles lies in the 1-3 plane, and introduce then the parametrization (2):

$$H(\phi, p) = K^{(0)}(1 - p^2) + K^{(1)}(1 - p^2)^2 - \tilde{K}^{(2)}(1 - p^2)^3 \cos 6\phi + Bp, \quad (4)$$

$\tilde{K}^{(2)} = K^{(2)}/6$, and the scaled momentum $p = P/P_0 \in (-1, 1)$ corresponds to $\cos\theta$ of parametrization in spherical coordinates.² For the model calculation of the following section we use the rather arbitrarily chosen values $K^{(1)} = K^{(0)}/5$ and $K^{(2)} = K^{(0)}/10$. Inspection of Eq. (4) then shows that the influence of this small transverse anisotropy on the barrier height Q (measured along the line $\phi = 0$) is negligible. We shall see in the next section that a like statement is not true for the energy loss ΔE .

Problems arise^{2,3} in calculation of the prefactor $f_0 = \omega_0 \Delta E / (2\pi T)$ if one attempts to find the well frequency $\omega_0^2 = H_{pp}^{(0)} H_{\phi\phi}^{(0)} - [H_{p\phi}^{(0)}]^2$ (here $H_{pp}^{(0)} = \partial^2 H / \partial P^2$ at the metastable minimum, etc.) from the Hamiltonian (4) since this function has local minima at $P = \pm P_0$ (edges of the open interval) but not extrema: $\lim_{P \rightarrow \pm P_0} H_p(\phi, P) \neq 0$ as can readily be seen from Fig. 1. This situation has been encountered before;^{2,3} the difficulty is removed by rotating the metastable minimum to an arbitrary point away from the singular point $P = P_0$ of the parametrization (2). At any such point $H_p^{(0)} = H_{\phi}^{(0)} = 0$ and $\omega_0 P_0 = 2K^{(0)} - B$ below the nucleation field^{3,10} $B_N = 2K^{(0)}$.

Three cross sections (along the line $\phi = 0$) through the unrotated energy surfaces $H(\phi, p)$ for fields $B = 0$ and $\pm K^{(0)}$ are shown in Fig. 1. We are concerned here with thermal relaxation of the metastable state at $p = 1$ (spin up), its decay rate to the spin-down state at $p = -1$ is obviously enhanced if $B > 0$ (reversing applied field $h < 0$) but we shall also be considering magnetization reversal at the unfavorable bias $B < 0$.

FIG. 1. Three cross sections through the energy surface $H(\phi, p)$ along the ϕ axis. The applied field is $B = 0$ and $\pm K^{(0)}$. The sections lie in a plane formed by the two minima and the saddle point. Flux proceeds over the barrier from right to left for all fields.

III. EQUATIONS OF MOTION AND THE PREFACTOR

So far we were dealing with reversible dynamics only, the irreversible, damped dynamics of magnetization under rotation of unison must be determined now. Since at present there exists no agreement as to the correct form of these phenomenological equations we shall carry out the prefactor calculation for the four dissipative models considered already in the study⁸ of the function $f_0(T)$ encompassing thus a wide range of possible dissipative couplings.

Our first model is furnished by the usual Gilbert's equation in which the damping term is proportional to $\mathbf{M} \times \mathbf{M}_t$ ($X_t = dX/dt$ for any X). In the parametrization (2) this equation assumes the convenient form⁷

$$\dot{\phi}_t = H_p + \eta_g^{(1)} \mathbf{S}_p \cdot \mathbf{S}_t, \quad (5)$$

$$\dot{P}_t = -H_\phi - \eta_g^{(1)} \mathbf{S}_\phi \cdot \mathbf{S}_t, \quad (6)$$

$\mathbf{S}_p = \partial \mathbf{S} / \partial P$, etc., which becomes Hamiltonian at zero dissipation strength, $\eta_g^{(1)} = 0$. From here the energy loss rate dH/dt follows:

$$H_t = -\eta_g^{(1)} [H_p \mathbf{S}_\phi - H_\phi \mathbf{S}_p] \cdot \mathbf{S}_t. \quad (7)$$

Note that $\eta_g^{(1)}$ is not dimensionless. Using the relations $\mathbf{S}_p \cdot \mathbf{S}_p = P_0^2 \rho^{-1}$, $\mathbf{S}_\phi \cdot \mathbf{S}_p = 0$, and $\mathbf{S}_\phi \cdot \mathbf{S}_\phi = \rho$ with $\rho = P_0^2 - P^2$, we then obtain²

$$\Delta E_g^{(1)} = \eta_g^{(1)} \oint_{\mathcal{C}_1} \{ H_p (P_0^2 - P^2) d\phi - H_\phi P_0^2 (P_0^2 - P^2)^{-1} dP \}. \quad (8)$$

$\mathcal{C}_1(\phi, P|B)$ is a contour of the saddle-point energy $E_1(B)$. Its geometry in the phase space (ϕ, P) will be discussed after all four of our dissipative models have been present-

ed. The system (5) and (6) corresponds to memoryless, ohmic damping. However, recent results⁵ indicate that magnetoelastic damping of magnetization is superohmic so that the first time derivatives in the above system should be replaced (in the memoryless approximation) by the third ones:¹¹ $\mathbf{S}_t \rightarrow \mathbf{S}_{ttt}$ and $\eta_g^{(1)} \rightarrow -\eta_g^{(3)}$ in Eqs. (5)–(7). The scalar products in the so-modified Eq. (7) now become

$$H_P \mathbf{S}_\phi \cdot \mathbf{S}_{ttt} = -H_P [3P_0^2 \rho^{-1} P_t^2 \phi_t + 3P(P_t \phi_t)_t + \rho(\phi_t^3 - \phi_{ttt})], \quad (9)$$

$$H_\phi \mathbf{S}_P \cdot \mathbf{S}_{ttt} = H_\phi [3P_0^2 P \rho^{-3} (PP_t^3 + \rho P_t P_{tt}) + \rho^{-1} (P_0^2 P_{ttt} - 3P^2 P_t \phi_t^2) + 3P \phi_t \phi_{tt}], \quad (10)$$

and the corresponding energy loss $\Delta E_g^{(3)}$ is obtained, to the first order of $\eta_g^{(3)}$, by expressing the time derivatives of ϕ and P as partial derivatives of $H(\phi, P)$ using Hamilton's equations of motion. Thus, e.g., $P_t^2 \phi_t \rightarrow H_\phi^2 d\phi$, $(P_t \phi_t)_t \rightarrow -H_{PP} H_\phi dP - H_{\phi\phi} H_P d\phi$, etc.

A peculiar difference, best exemplified by harmonic potential, exists between ohmic and superohmic damping. For ohmic damping, $x_{tt} + \eta^{(1)} x_t + V'(x) = 0$, the energy loss rate $H_t = -\eta^{(1)} x_t^2 \leq 0$ for arbitrary motion. From the superohmic equation $-\eta^{(3)} x_{ttt} + x_{tt} + V'(x) = 0$, however, one obtains $H_t = \eta^{(3)} x_t x_{ttt}$ and this is negative as expected for $x = \cos \omega t$, i.e., if $V'(x) = \omega^2 x$, but positive for $x = \cosh \omega t$ corresponding to $V'(x) = -\omega^2 x$. Thus, while the integrand of $\Delta E_g^{(1)}$ is positive definite everywhere along the contour $\mathcal{C}_1$ the integrand of $\Delta E_g^{(3)}$ is not and, as the above example suggests, it changes sign approximately at the same points as the Hessian of the Hamiltonian,

$\text{Hess}(H) = H_{\phi\phi} H_{PP} - [H_{\phi P}]^2$. The mean value of the Hessian along the closed contours $\mathcal{C}_1$ is positive and $\Delta E_g^{(3)} > 0$ accordingly. It is unclear (to us) what physical significance this odd phenomenon has but the energy balance between a system and its superohmically coupled bath certainly merits further attention.

Gilbert's equation of motion follows⁷ from isotropic, linear coupling of the magnetization vector $\mathbf{M}$ to a heat bath. If, however, one assumes that dissipation is due to magnetoelastic interactions then the bath coupling should be (at least) quadratic in the components M_i . The modified ohmic equations of motion are obtained⁷ by the formal replacement $\mathbf{S} \rightarrow \mathbf{f}$, $\eta_g^{(1)} \rightarrow \eta_f^{(1)}$ in Eqs. (5)–(7); we use the subscript f to distinguish this coupling mode from the linear, Gilbert case. The components of the coupling vector $\mathbf{f}$ are suitable polynomials in S_i . This procedure avoids complicated (and pointless) tensorial dissipative constants and for a model calculation we have chosen the vector

$$\mathbf{f} = [S_1^2 + S_3^2, S_2^2 + S_3^2, S_3^2] \quad (11)$$

representing simplified magnetoelastic interactions in a uniaxial crystal.¹² In the modified Eq. (7) for the loss rate H_t there is now

$$H_P \mathbf{f}_\phi \cdot \mathbf{f}_t = H_P \rho [P \sin 4\phi P_t + 2\rho \sin^2 2\phi \phi_t], \quad (12)$$

$$H_\phi \mathbf{f}_P \cdot \mathbf{f}_t = H_\phi P [4P \sigma P_t + \rho \sin 4\phi \phi_t] \quad (13)$$

where $\sigma = 1 + \sin^4 \phi + \cos^4 \phi$ for brevity. Finally, the modified superohmic equations with quadratic coupling are obtained by the replacements $\mathbf{S} \rightarrow \mathbf{f}$, $\mathbf{S}_t \rightarrow \mathbf{f}_{ttt}$, and $\eta_g^{(1)} \rightarrow -\eta_f^{(3)}$ in Eqs. (5)–(7). The choice (11) of the vector $\mathbf{f}$ yields the two components of the loss rate H_t in the form

$$H_P \mathbf{f}_\phi \cdot \mathbf{f}_{ttt} = H_P \{ \rho \sin 4\phi [3P_t(P_{tt} - 2P\phi_t^2) + PP_{ttt} + 6\rho \phi_t \phi_{tt}] - 2\rho \sin^2 2\phi [6P_t^2 \phi_t + 6P(P_t \phi_t)_t + \rho(2\phi_t^3 - \phi_{ttt})] \}, \quad (14)$$

$$H_\phi \mathbf{f}_P \cdot \mathbf{f}_{ttt} = H_\phi \{ 4P \sigma (3P_t P_{tt} + PP_{ttt}) + 12P \cos^2 2\phi (\rho \phi_t \phi_{tt} - 2PP_t \phi_t^2) - P \sin 4\phi [6P_t^2 \phi_t + 6P(P_t \phi_t)_t + \rho(4\phi_t^3 - \phi_{ttt})] \}. \quad (15)$$

The newly modified loss functions $\Delta E_f^{(1)}$ and $\Delta E_f^{(3)}$ then follow from Eqs. (12) and (13) and (14) and (15) via the procedure described at Eqs. (9) and (10) and the variation of their integrands along $\mathcal{C}_1$ is same as discussed above for the integrals $\Delta E_g^{(i)}$.

Some caution is necessary in doing the line integral $\oint_{\mathcal{C}_1}$ if (as in the present case) the contour $\mathcal{C}_1$ encircles the excluded singular points $P = \pm P_0$. No ambiguity is possible for the Hamiltonian (4), however, since all the four integrands (8), (9) and (10), (12) and (13), and (14) and (15) are regular and vanish identically at $P = \pm P_0$.

In the notation of Eqs. (3) and (4) the energetically favorable flux over the barrier goes from P_0 to $-P_0$ for positive field bias. In the next section we calculate the frequency-dependent coercivity and to this end we must also consider the less favorable negative bias since cer-

tainly the system is relaxing (albeit perhaps slowly) towards thermal equilibrium even before the field polarization is fully reversed. The same as in Fig. 1 we show therefore in Fig. 2 the contours $\mathcal{C}_1(\phi, P|B)$ both for positive and negative field bias. It is seen that as $B \rightarrow \pm B_N$ the contour $\mathcal{C}_1$ contracts to a point ($\pm P_0$) and therefore $\lim_{B \rightarrow \pm B_N} \Delta E = 0$ since our four integrands are all finite (they vanish) at $P = \pm P_0$. Integration along these contours then yields the loss functions $\Delta E_g^{(i)}(B)$ and $\Delta E_f^{(i)}(B)$ plotted in Fig. 3. A most conspicuous feature is the great uniformity of their functional dependence on B , a rather surprising finding in view of the significant differences between the ohmic and superohmic integrands. The only observable difference (we deem the overall scale irrelevant) is the somewhat steeper slope of the ohmic loss functions as compared to the superohmic ones. The field

FIG. 2. Contours of critical energy $\mathcal{E}_1(\phi, p|B)$ for selected values of the applied field: $B/K^{(0)} = 1.75, 1.5, \dots, -1.75$. For $B > 0$ the contours lie entirely in the $p > 0$ region, the cusp of the $B = 0$ contour is at the point $(0, 0)$. Shown is the segment $(-\pi/6, \pi/6) \times (-1, 1)$ of the scaled phase space. Note that the lines $p = \pm 1$ correspond to single points (poles) on a unit sphere.

dependence of the prefactor is dominated, apparently, by the contraction of the integration contour while the detailed dynamics proper are of far lesser importance. The functions ΔE have a maximum at small negative fields which corresponds to magnetization reversal at the unfavorable bias. The variation of all four integrands, namely, is greatest at the line $P = 0$ and the negative fields at which these maxima occur correspond to contours $\mathcal{E}_1(\phi, P|B)$ which deviate little from this region, cf.

FIG. 3. The scaled energy loss functions: $\Delta E_g^{(1)}/(K^{(0)}P_0\eta_g^{(1)})$, $\Delta E_g^{(3)}/([K^{(0)}]^3P_0^{-1}\eta_g^{(3)})$, $\Delta E_f^{(1)}/(K^{(0)}P_0^3\eta_f^{(1)})$, and $\Delta E_f^{(3)}/([K^{(0)}]^3P_0\eta_f^{(3)})$. Solid lines correspond to ohmic ($i = 1$), dashed lines to superohmic ($i = 3$) damping. Data points: * marks linear and $\circ$ quadratic dissipative coupling.

Fig. 2. The field dependence of the prefactor, i.e., the function $\omega_0\Delta E(B) = 2\pi T f_0(B)$, is finally presented in Fig. 4.

There remains yet to consider the influence of transverse anisotropy on thermal decay rate and to briefly state the relation of Eq. (1) to Brown's formula¹ derived for axially symmetric systems. In the present case this corresponds to setting $K^{(2)} = 0$ in the Hamiltonian (4). Then obviously $H_\phi \equiv 0$ and also $H_P = 0$ everywhere along $\mathcal{E}_1$ (but only at the saddle points if $K^{(2)} \neq 0$) so that $\Delta E \equiv 0$ and relaxation rate vanishes identically for all models considered. This is obviously wrong, the trouble is that the derivation of Eq. (1) explicitly assumes²⁻⁴ the presence of saddle points absent in systems of full axial symmetry. The limit $K^{(2)} \rightarrow 0$ therefore does not exist within the theory of Eq. (1) which applies to systems qualitatively different from those described by Brown's formula. Indeed, at present there exists no analytic method that could bridge these two in the asymptotic limit $Q/T \gg 1$ (but see Ref. 13 for numerical calculations). The matter is confounded by the fact that Brown's formula cannot be rederived^{2,3} if the easy axis of the strictly axially symmetric system is rotated away from the singular direction inherent to any parametrization of a spherical surface in two variables. Be that as it may, a decrease in $K^{(2)}$ is accompanied (as long as there is a saddle) by a rather slow decrease in the peak value of ΔE . The values of H_P and H_ϕ , namely, go to zero but at the same time the small B trajectories $\mathcal{E}_1$ stay almost entirely within the favorable region near $P = 0$. The coefficients of magnetic anisotropy are temperature dependent,⁸

$$K^{(n)}T \propto [M_s(T)/M_s(0)]^{(2n+2)(2n+3)/2}$$

in the notation of Eqs. (3) and (4) and $M_s(T) \propto (1 - T/T_C)^{1/3}$, so that the high-order transverse anisotropy is preferentially suppressed by increasing temperature. As $T \rightarrow T_C$ the prefactor $f_0(T)$ therefore van-

FIG. 4. The scaled functions ΔE shown in Fig. 3 multiplied by the scaled well frequency $\omega_0 P_0 / K^{(0)} = 2 - B/K^{(0)}$.

ishes faster than predicted by the estimates of Ref. 8 discussed in the Introduction.

IV. HYSTERESIS EFFECTS

According to the results of the preceding section the function $f_0(B, T)$ decreases rapidly as $B \rightarrow \pm B_N$ or $T \rightarrow T_C$ and the corresponding switching rate is slower than predicted by theory with a constant prefactor. Significant differences between the here proposed and customary ($f_0 = \text{const}$) models should therefore occur in processes in which thermal relaxation takes place over a large interval of applied fields, such as in hysteresis¹⁴⁻¹⁶ where the external field varies, or in investigations of magnetic viscosity¹⁷ where h is held constant but the nucleation fields within the sample vary according to some distribution. By the same token, the T dependence of the prefactor should be taken into account whenever observations are carried out over a large temperature interval, such as in measurements of the thermomagnetic curve.^{15,18} However, as opposed to the essentially model-independent variation of the prefactor with h one must contend in this case with the unknown dissipative mechanism determining the exponential factor ν in $f_0(T)$.

The prefactor is an integral part of thermal decay rate dominated by the Arrhenius factor and it is rather unlikely that it could be measured separately. We wish to demonstrate here the qualitative effects of decay rate suppression by a model calculation of the frequency-dependent coercivity $B_c(\omega)$. An estimate within the purely irreversible switching regime¹⁵ is obtained from Sharrock's formula¹⁶

$$\omega = \mathcal{A} \kappa(B_c) = \mathcal{A} f_0(B_c, T) \exp[-Q(B_c)/T],$$

where $\mathcal{A}$ is a constant of the order of unity. Both $f_0(B)$ and $Q(B)$ decrease with B enhancing preferentially high coercivity. In this simple formula one may approximately set $Q(B) \approx K^{(0)}(1 - B/B_N)^2$ and

$$f_0(B, T) \propto (B_N - B)(B_N^2 - B^2)^2 \cdot T^{-1}(1 - T/T_C)^\nu, \quad (16)$$

where $B_N = 2K^{(0)}(T)$ and ν is not known. Results from a more detailed loop calculation^{14,15} are shown in Fig. 5. The nonequilibrium magnetization (major hysteresis loop) of system (4) in a periodic applied field $B(t) = -B_N \cos \omega t$ was tabulated as a function of time on the frequency interval $10^0 - 10^8$ Hz. Since the numerical value of the dissipation constant is not available we adopted the prefactor normalization $f_0(0) = e^{25}$ Hz and wrote

$$\omega_0(0) \Delta E_g^{(1)}(0) f_0(B) = \omega_0(B) \Delta E_g^{(1)}(B) e^{25} \quad (17)$$

choosing the customary Gilbert-type coupling as representative of the four models discussed above. The value of the ratio $K^{(0)}/T \approx 42$ entering the Arrhenius factor was set by imposing the condition that the particle lifetime to 10 yr in zero field. Our loop calculation loses accuracy if large variations of magnetization take place near the nucleation field B_N and serious instabilities occur at $B > 0.95 \times B_N$. The approach of coercivity to

FIG. 5. Reduced coercivity $B_c(\omega)/B_N$ for a particle with lifetime $\kappa^{-1}(0) = 10$ yr (solid lines, $K^{(0)}/T \approx 42$) and for a stable particle with $\kappa^{-1}(0) = 10^{18}$ yr (*, $K^{(0)}/T \approx 82$). Solid lines show results based on $f_0 = e^{25}$ Hz, dashed lines show the enhanced coercivity based on Eq. (17).

B_N , shown in Fig. 5, must therefore be regarded as approximate only, but the dramatic effect of the prefactor (17) is plainly seen: The system follows the Stoner-Wohlfarth model [$B_c(\omega) \equiv B_N$] at frequencies much lower than predicted by theory with a field-independent prefactor.

Of considerable interest is also the switching field distribution^{15,19} (SFD) found by differentiating the major hysteresis loop $\hat{M}[B(t)]$ with respect to applied field, $\chi_\omega(B) \approx \Delta \hat{M}[B(t)]/\Delta B(t)$. Plots of these functions for selected sweep rates are shown in Fig. 6. The SFD peak

FIG. 6. The reduced switching field distribution $\chi_\omega(B) \times (B_N/M_s)$ for a particle with lifetime of $\kappa^{-1}(0) = 10$ yr. Results based on $f_0 = e^{25}$ Hz are shown by solid lines and on Eq. (17) by dashed lines. Sweep rates are 10^1 Hz (leftmost curve), 10^3 , 10^5 , and 10^7 Hz (rightmost curve).

becomes sharper as the sweep rate is lowered for the obvious reason that upon reaching a low enough barrier height $Q[B(t)]$ the system has enough time to appreciably relax while the applied field changes but little. The prefactor (17) suppresses relaxation and has therefore a twofold effect on SFD $\chi_\omega(B)$: Its peak shifts closer to the nucleation field and its width increases, significantly so in systems with very high coercivity. Measurements of $B_c(\omega)$ and $\chi_\omega(B)$ on samples well aligned with applied field (so as to suppress the peaks of transverse susceptibility²⁰ occurring naturally at large fields) should be able to reveal these large field deviations from predictions of theory with constant prefactor. From an experimental point of view it is perhaps easier to alter the field at which the majority of switching events occur (and, in so doing, to increase the coercivity range) by varying the temperature rather than the sweep rate. The evaluation of such data is then hampered by the insufficiently known temperature dependence of the prefactor (the exponent ν) but the large field deviations are still present.

The same as in Ref. 8 we wished also in this work to make a case for a nonconstant, in this case a field-dependent, prefactor $f_0(B)$ of thermal decay rate. We have shown, in particular, that such a prefactor significantly suppresses thermal relaxation at applied fields close to the nucleation field B_N . On a model calculation we demonstrated the corresponding coercivity enhancement and discussed then the feasibility of experimental observation of this and related effects due to the prefactor $f_0(B, T)$.

Note added in proof. For experimental measurements of the prefactor see P. D. E. Dickson, N. M. K. Reid, C. Hunt, H. D. Williams, M. El-Hilo, and K. O'Grady, J. Magn. Magn. Mater. **125**, 345 (1993).

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