Letter to the Editor

Duration of magnetic aftereffect

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An initially saturated array of uniaxial particles with log-normal distribution (parameter σ) of activation volumes V relaxes in reversing field h at temperature T . We find that the time t_c at which its net magnetization vanishes is $\kappa(\langle V \rangle, h) \cdot t_c(h, \sigma) = \ln 2 \cdot \exp[\gamma(\sigma)Q(h)/T]$ where $\kappa(\langle V \rangle)$ is decay rate associated with the most probable volume and $Q(h)$ is barrier height. The function $\gamma(\sigma) > 0$ if the particle magnetic moment $\mathcal{M}$ is proportional to V and $\gamma(\sigma) < 0$ if it is not. In comparison, the magnetic viscosity $S(t_c)$ is insensitive to the relation between $\mathcal{M}$ and V .

In magnetic aftereffect measurements one observes the net magnetization of a system composed, as a rule, of a large number of individually relaxing components. The distribution of relaxation rates [1,2] within the system causes the net magnetization $\langle M(t) \rangle$ decay logarithmically over a time interval [3] on which $dS/dt \approx 0$ where $S(t) = -\partial \langle M(t) \rangle / \partial \ln t$. The temperature and field dependence of the magnetic viscosity S (taken to be the time independent maximum value of $S(t)$) have attracted considerable attention [4,5] and it is rather surprising that the closely related problem of the magnetic aftereffect duration has apparently never been systematically studied.

We consider an initially saturated system, $\langle M(0) \rangle = M_s$, relaxing thermally at temperature T in a sufficiently large reversing field $-h < 0$ so that

$$\lim_{t \rightarrow \infty} \langle M(t) \rangle = M_{eq}(-h, T) \approx -M_s.$$

Our aim is to find the time t_c at which the net magnetization becomes zero and to this end we solve numerically the equation $\langle M(t_c) \rangle = 0$.

Let first (other models are considered below) the relaxing system be an array of single domain

uniaxial particles whose activation volumes V have to log-normal distribution

$$P(V) = (\sigma\sqrt{2\pi}V)^{-1} \exp\left[-(2\sigma^2)^{-1} \ln^2(V/V_0)\right]. \quad (1)$$

The two quantities of interest are the most probable volume $V_p = V_0 e^{-\sigma^2}$ and the average volume $\langle V \rangle = V_0 e^{\sigma^2/2} > V_p$; for a pure ($\sigma = 0$) sample, obviously, $\langle V \rangle = V_p = V_0$. The fraction of particles with volumes $V < \langle V \rangle$ is

$$n(\sigma) = \int_0^{\langle V \rangle} dV P(V) = \frac{1}{2} \left[1 + \operatorname{Erf}\left(\frac{\sigma}{2\sqrt{2}}\right) \right] \quad (2)$$

where $\operatorname{Erf}(x)$ is the error function. Obviously $\lim_{\sigma \rightarrow \infty} n(\sigma) = 1$ since $\lim_{\sigma \rightarrow \infty} \langle V \rangle = \infty$. For the thermal decay rate of a particle with activation volume V in applied reversing field $-b = -h/h_n$ (h_n is the nucleation field) we write [3,6]

$$\kappa(V, b) = f_0 \exp\left[-\frac{V}{V_p} \frac{Q}{T} (1-b)^2\right] \quad (3)$$

with $k_b \equiv 1$ and we choose $f_0 = e^{25}$ Hz. Q is therefore the barrier height of a particle with the most probable volume V_p in zero field. For brevity we shall write $QT^{-1}(1-b)^2 \equiv \tilde{Q}(b)$; $\tilde{Q}(0)$ is con-

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veniently calculated from Eq. (3) by specifying the lifetime $\kappa^{-1}(V_p, 0)$.

Having defined the sample composition and the relaxation properties of its individual components there remains yet to determine the relation between the activation volume V and the magnetic moment $\mathcal{M}$ of the particle. This is a very difficult problem [7] quite beyond the scope of the present work and for this reason we consider separately two models: In the first one the magnetic moment $\mathcal{M}$ of a particle is assumed to be proportional to V and we write $\mathcal{M} = M_s V$ while in the second model all particles are assumed to have equal magnetic moments independent of their activation volume. Within the above approximations the crossing time $t_c = t_c(b, \sigma)$ is then given, for $b > 0$, as a solution of the equation

$$\begin{aligned} M_s^{-1} \langle M(t_c) \rangle \\ = 2 \langle V \exp[-\kappa(V, b) \cdot t_c(b, \sigma)] \rangle \langle V \rangle^{-1} - 1 \\ = 0 \end{aligned} \quad (4)$$

for the first model and of

$$\begin{aligned} M_s^{-1} \langle M(t_c) \rangle \\ = 2 \langle \exp[-\kappa(V, b) \cdot t_c(b, \sigma)] \rangle - 1 = 0 \end{aligned} \quad (5)$$

for the second one. If $\sigma = 0$ (pure sample) the two models coincide, $\kappa(V_0, b) t_c(b, 0) = \ln 2$, and for $S(t_c)$ we obtain in either case

$$\begin{aligned} \lim_{\sigma \rightarrow 0} S(t_c) &= - \lim_{\sigma \rightarrow 0} \frac{\partial \langle M(t) \rangle}{\partial \ln[\kappa(V_p) \cdot t]} \bigg|_{t=t_c} \\ &= M_s \ln 2. \end{aligned} \quad (6)$$

If $\sigma > 0$ (disordered sample) then the two models (4) and (5) yield similar [3] values of magnetic viscosity but, as is shown below, widely different results for the crossing time t_c .

In plotting the numerical solutions of Eqs. (4) and (5) we found it convenient to normalize the crossing time t_c by the relaxation rate

$$\kappa(\langle V \rangle, b) = f_0 \exp[-e^{3\sigma^2/2} \tilde{Q}(b)]$$

Fig. 1. The crossing time t_c as a solution of (a) Eq. (4) (note the logarithmic time scale) and of (b) Eq. (5). Sample composition: $\sigma = 0.05$ (lowermost curve in (a), uppermost curve in (b)), 0.1, 0.2, 0.3, 0.4, 0.5 and 0.6 (uppermost in (a) and lowermost in (b)). In all cases $\kappa^{-1}(V_p, 0) = 1$ year. The $\sigma = 0.1$ solution of Eq. (4) was omitted as it is too close to the $\sigma = 0.05$ one. The dashed horizontal line is the pure sample solution $\kappa(V_0, b) \cdot t_c(b, 0) = \ln 2$.

associated with the average volume $\langle V \rangle = e^{3\sigma^2/2} V_p$. Representative results of a model calculation with $\kappa^{-1}(V_p, 0) = 1$ year are shown in Fig. 1 and to highlight the great disparity in their respective scales we choose to plot the solutions of the two equations (4) and (5) separately. The upper plot shows the function $\ln[\kappa(\langle V \rangle, b) \cdot t_c(b, \sigma)]$ where t_c solves Eq. (4). The pure sample result

$$\ln[\kappa(V_0, b) \cdot t_c(b, 0)] = \ln(\ln 2)$$

is recovered for small σ and also for large reversing fields $b \sim 1$ at which the $(1-b)^2$ factor suppresses all σ -dependence. Analysis of data arrays with different values of $\kappa^{-1}(V_p, 0)$ (i.e. at different temperatures T) shows that

$$\kappa(\langle V \rangle, b) \cdot t_c(b, \sigma) = \ln 2 \cdot \exp[\gamma_1(\sigma) \tilde{Q}(b)] \quad (7)$$

and interpolation of these numerical data yields, on the interval $\sigma \in (0, 0.7)$, the function $\gamma_1(\sigma)$ as $\gamma_1(\sigma) \approx \sigma^{5/2} e^{\sigma^2/2}$. According to Eq. (7), therefore, t_c is in this case substantially larger than the crossing time associated with the average volume, $t_c(b, \sigma) > \ln 2 \cdot \kappa^{-1}(\langle V \rangle, b)$. The situation is exactly opposite for the model (5) where $t_c(b, \sigma) < \ln 2 \cdot \kappa^{-1}(\langle V \rangle, b)$. The two families of graphs are separated by the pure sample solution $t_c(b, 0) = \ln 2 \cdot \kappa^{-1}(V_0, b)$ and never intersect. The σ -dependence of the solution of Eq. (5) easily understood: By virtue of Eq. (2) the number of particles with lifetimes $\kappa^{-1}(V, b) \leq \kappa^{-1}(\langle V \rangle, b)$ increases with σ and the rapidly switching particles contribute, by assumption, the same magnetic moment to the net magnetization as particles with larger activation volumes (i.e. slower relaxation rates). We write in this case

$$\kappa(\langle V \rangle, b) \cdot t_c(b, \sigma) = \ln 2 \cdot \exp[-\gamma_2(\sigma) \tilde{Q}(b)] \quad (8)$$

where $\gamma_2(\sigma)$ is interpolated as $\gamma_2(\sigma) \approx \sigma^{9/4} e^{5\sigma^2/4}$.

The concept of activation volume in thermal relaxation is encountered, with different [6] field dependence of the barrier height, also in models of domain wall unpinning and nucleation. In these

Fig. 2. The value of magnetic viscosity $S(t_c)$ calculated from Eq. (4) (full lines) and Eq. (5) (dashed lines). Sample composition: $\sigma = 0.05$ (uppermost curves), 0.2, 0.4 and 0.6 (lowermost curves). In all cases $\kappa^{-1}(V_p, 0) = 1$ year. The dashed horizontal line is the pure sample value $S(t_c) = M_s \ln 2$.

models changes of the sample magnetization are due to domain wall motion from a metastable pinning site to a stable one. If the distance between the pinning sites (i.e. the distance the wall travels) is independent of their activation volumes then the model (5) holds, in the contrary case a modified form of Eq. (4) obtains. We have tested a simple modification of Eq. (4) with $V \rightarrow V^{1/3}$ and found, not surprisingly, that the resultant crossing time t_c satisfies the condition $\kappa(\langle V \rangle, b) \cdot t_c(b, \sigma) > \ln 2$ in accordance with Eq. (7) (with different $\gamma_1(\sigma)$, of course).

As opposed to the function $t_c(b)$ the values of the derivatives $S(t_c) \leq M_s \ln 2$ calculated from the two models (4) and (5) are virtually indistinguishable (see Fig. 2). For the first model there follows [3] from Eq. (4) that at the (scaled) time $\tau = \kappa(V_p, b) \cdot t$ the magnetic viscosity

$$S(\tau) \sim \int_0^\infty dx g_1 \tau g_2 e^{-\tau g_2}$$

where

$$g_1(x) = \exp[-(2\sigma^2)^{-1} \ln^2 x]$$

and

$$g_2(x) = \exp\left[-\tilde{Q}(b)(x e^{\sigma^2} - 1)\right].$$

The function $g_1(x)$ has a maximum at the point $x_1 = 1$ while the remaining factor $\tau g_2 e^{-\tau g_2}$ of the integrand has a movable peak at the point

$$x_2(t) = e^{\sigma^2/2} + e^{-\sigma^2} \tilde{Q}^{-1}(b) \ln[\kappa(\langle V \rangle, b) \cdot t]. \quad (9)$$

Logarithmic decay $dS/dt \approx 0$ is observed while the movable peak at $x_2(t)$ passes through the stationary peak at $x_1 = 1$ and at this time the function $S(t)$ attains its maximum value [3]. For small σ the immobile peak of g_1 is very narrow and so is the interval of passage. Similarly, large T and b (small $\tilde{Q}$) accelerate, by virtue of Eq. (9), the shift of $x_2(t)$ to the right and reduce the time of passage even at large σ for which the stationary peak $g_1(x)$ is very broad. Using the interpolation of $\gamma_1(\sigma)$ in Eqs. (7) and (9) we obtain the approximate expression

$$x_2[t_c(b, \sigma)] \approx e^{\sigma^2/2} + \sigma^{5/2} e^{-\sigma^2/2} + e^{-\sigma^2} \tilde{Q}^{-1}(b) \ln(\ln 2) \quad (10)$$

for the position of the mobile peak at the crossing time t_c . It shows that at $t = t_c$ logarithmic decay is *not* observed only at very large reversing fields b (approximately for $b > 0.75$ for the sample data of Fig. 1). In these fields $x_2(t_c)$ is to the left of the stationary peak at $x_1 = 1$ and logarithmic decay has not yet set in. Analogous situation obtains for the model (5) in which $g_1(x) \rightarrow x^{-1}g_1(x)$ and $x_1 \rightarrow x_1(\sigma) = e^{-\sigma^2/2} < 1$. According to Eqs. (8) and (9) the position of the movable

peak at the crossing time t_c is approximately given by

$$x_2[t_c(b, \sigma)] \approx e^{\sigma^2/2} - \sigma^{9/4} e^{\sigma^2/4} + e^{-\sigma^2} \tilde{Q}^{-1}(b) \ln(\ln 2) \quad (11)$$

and a significant shift of $x_2(t_c)$ to the left of $x_1(\sigma)$ is again observed only at large reversing fields.

The ratio of magnetic viscosity S and irreversible susceptibility χ_{irr} , known as the coefficient of magnetic viscosity S_v [2,3], is approximately sample independent and has therefore the significance of a material constant. The crossing time t_c certainly lacks this property but the drastic qualitative difference between Eqs. (7) and (8) recommends it as a tool in studies of the relation between magnetic moment and activation volume.

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