Analysis and summarization of correlations in data cubes and its application in microarray data analysis

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Abstract. This paper presents a novel mechanism to analyze and summarize the statistical correlations among the attributes of a data cube. To perform the analysis and summarization, this paper proposes a new measure of statistical significance. The main reason for proposing the new measure of statistical significance is to have an essential closure property, which is exploited in the summarization stage of the data mining process. In addition to the closure property, the proposed measure of statistical significance has two other important properties. First, the proposed measure of statistical significance is more conservative than the well-known chi-square test in classical statistics and, therefore, inherits its statistical robustness. This paper does not simply employ the chi-square test due to lack of the desired closure property, which may lead to a precision problem in the summarization process. The second additional property is that, though the proposed measure of statistical significance is more conservative than the chi-square test, for most cases, the proposed measure yields a value that is almost equal to the z test, a conventional measurement of statistical significance based on the normal distribution. Based on the closure property addressed above, this paper develops an algorithm to summarize the results from performing statistical analysis in the data cube. Though the proposed measure of statistical significance avoids the precision problem due to having the closure property, its conservative nature may lead to a recall rate problem in the data mining process. On the other hand, if the chi-square test, which does not have the closure property, was employed, then the summarization process may suffer a precision problem. In this paper, we also show a possible application in bioinformatics. We applied the proposed mechanism on a microarray dataset, in order to identify groups of genes with similar expression patterns in subspaces of feature space.

Keywords: Correlation, data cube, statistical test, segmentation, summarization

1. Introduction

Data mining is a process aimed at discovering interesting data patterns hidden in large data sets [8, 12, 20]. Two conventional measures of interestingness or significance of data patterns are “support” and “confidence”, which are widely used in mining association rules algorithms [2–5]. However, as addressed in several articles [1, 10], “support” and “confidence” are not appropriate in some situations, mainly due

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to lack of statistical robustness. As a result, a number of alternative measures or tests have been proposed in recent years [1,6,9,10,13,16–18]. One of the measures that have been employed recently is the level of significance based on the chi-square test of independence in classical statistics [6,18]. The chi-square test is so well known and so widely used that its characteristics has been well studied and its robustness has been well proved.

Brin et al. exploited the chi-square test of independence to find correlation rules in the market basket data set [6]. The same idea has since been exploited in several research works [9,10,18]. Nevertheless, previous research works that exploited the chi-square test mainly concerned the market basket data set. One important data model that is widely employed in modern data warehousing and OLAP system is data cube [7,11,12]. Therefore, it is of interest to perform correlation analysis in data cubes.

This paper presents a novel mechanism to analyze and summarize the statistical correlation among the attributes of a data cube. Figure 1(a) shows a 2-dimensional data cube example for illustrating the main characteristics of the data mining process. The data set records the network transactions on National Taiwan University Campus during a period of 4 weeks. Figure 1(b) marks the cells in the data cube based on the measure of statistical significance employed in this paper. A cell is marked by “1” (“–1”), if the count in the cell is sufficiently large (small) to exceed a statistical threshold based on the measure of statistical significance.

Based on the primitive statistical analysis, the data mining process proposed in this paper proceeds to summarize the findings of statistical analysis. The summarization process exploits an essential closure property of the measure of statistical significance employed in this paper. The closure property guarantees that if the individual count in each cell of a subcube is statistically significant based on the measure, then the lumped sum of the counts in the subcube must also be statistically significant. The shaded area in Fig. 1(b) shows an example of the summarization process. The summarized result is that all students' dormitories have significantly high network transactions in the evening and midnight.

This paper does not simply employ the chi-square test due to lack of the desired closure property as shown in the 3-D data cube example provided in Fig. 2. This example alone proves that the chi-square test does not have the desired closure property in data cubes.

In this paper, we also show a possible application of the proposed mechanism to microarray data analysis. Wang et.al. proposed that two objects are similar if they exhibit a coherent pattern on a subset of dimensions [19]. In DNA microarray analysis, the expression levels of two genes may rise and fall synchronously in only a subset of environmental conditions. The correlation analysis proposed in this paper is able to identify a set of genes with similar expression patterns in the subspaces of feature space.

In the following part of this paper, Section 2 describes the proposed measure of statistical significance and presents a proof of its closure property. Section 3 elaborates the summarization process. Section 4 discusses the experiments conducted to study the characteristics of the proposed data mining mechanism. Section 5 shows an application of analyzing gene expression profiles. Section 6 presents the concluding remarks of this paper.

2. The statistical analysis of correlation

The statistical analysis process employed in this paper borrows some ideas from the chi-square test for independence. In the chi-square test, it is first assumed that the attributes that correspond to different dimensions of the contingency table are completely independent. Then, statistics analysis is applied to determine how likely the outcome of the experiment will distribute as in the contingency table under the

	Early Morning 3am~7am	Morning 8am~12pm	Afternoon 1pm~5pm	Evening 6pm~9pm	Midnight 10pm~2am
COLLEGE OF ELECTRICAL ENGINEERING AND COMPUTER SCIENCE	135477	673555	881915	682771	421976
COLLEGE OF ENGINEERING	203370	1313483	1594946	1101980	620188
COLLEGE OF SCIENCE	165602	895710	1028130	663038	370613
COLLEGE OF AGRICULTURE	147298	757774	868433	480345	379431
COLLEGE OF MANAGEMENT	116577	269939	317198	163136	105287
COLLEGE OF MEDICINE AND PUBLIC HEALTH	549810	2011089	1957590	1137670	866356
COLLEGE OF LIBERAL ARTS	12925	176390	202568	63075	45442
COLLEGE OF LAW AND SOCIAL SCIENCES	7083	284083	351940	132700	49150
ADMINISTRATION	1469	136128	105681	41303	9620
COMPUTER CENTER	170553	774370	1060452	555856	350399
OTHER CENTERS	29866	444519	433194	189210	58955
LIBRARY	1960	180219	161480	92509	20338
DORMITARY OF GRADUATE STUDENTS	164117	260686	333022	453445	817079
DORMITARY OF MALE UNDERGRADUATE STUDENTS	900031	1098114	1381261	1591107	2637444
DORMITARY OF FEMALE UNDERGRADUATE STUDENTS	193472	382212	547290	685176	1049192

(a) A 2-D data cube example.

	Early Morning 3am~7am	Morning 8am~12pm	Afternoon 1pm~5pm	Evening 6pm~9pm	Midnight 10pm~2am
COLLEGE OF ELECTRICAL ENGINEERING AND COMPUTER SCIENCE	-1	-1	1	1	-1
COLLEGE OF ENGINEERING	-1	1	1	1	-1
COLLEGE OF SCIENCE	-1	1	1	1	-1
COLLEGE OF AGRICULTURE	-1	1	1	-1	-1
COLLEGE OF MANAGEMENT	1	1	1	-1	-1
COLLEGE OF MEDICINE AND PUBLIC HEALTH	1	1	1	-1	-1
COLLEGE OF LIBERAL ARTS	-1	1	1	-1	-1
COLLEGE OF LAW AND SOCIAL SCIENCES	-1	1	1	-1	-1
ADMINISTRATION	-1	1	1	-1	-1
COMPUTER CENTER	-1	1	1	-1	-1
OTHER CENTERS	-1	1	1	-1	-1
LIBRARY	-1	1	1	0	-1
DORMITARY OF GRADUATE STUDENTS	1	-1	-1	1	1
DORMITARY OF MALE UNDERGRADUATE STUDENTS	1	-1	-1	1	1
DORMITARY OF FEMALE UNDERGRADUATE STUDENTS	-1	-1	-1	1	1

(b) Data cube cells whose counts exceed the threshold of statistical significance.

Fig. 1. A 2-dimensional example for illustrating the main characteristics of the mining process.

assumption of independence. If it is highly unlikely in the statistical sense, then it is said that the initial assumption of independence does not hold, i.e. these attributes are correlated or dependent.

The statistical analysis proposed in this paper also assumes that the attributes that correspond to different dimensions of the data cube are independent. Then, a measure of statistical significance is applied to evaluate the significance of the count in each cell of the data cube under the assumption of independence. Let $A_1, A_2, \dots, A_M$ denote the M attributes corresponding to different dimension of the data cube, and N denote the total count of objects in the data cube. Assume that attribute A_k has L_k possible values, indexed by $1, 2, \dots, L_k$ and all the attributes are uncorrelated or independent. Figure 3 shows an example of a 3-dimensional data cube.

For each cell in the data cube, we can define a binomial random variable [14] corresponding to the

	C1	C2	C3
R1	2010	50	90
R2	197993	3947	7910
	C1'	C2'	C3'
R1'	5990	110	230
R2'	594007	11893	23770

The chi-square statistic for the cell indexed by R1 and C1 is

$$\begin{aligned}
& (2010-2000)^2 / 2000 + (140-120)^2 / 120 + (197993-198000)^2 / 198000 + (11857-11880)^2 / 11880 \\
& + (5990-6000)^2 / 6000 + (340-360)^2 / 360 + (594007-594000)^2 / 594000 + (35663-35640)^2 / 35640 \\
& = 4.571 > 2.706
\end{aligned}$$

, where 2.706 is the chi-square critical value used as threshold.

The chi-square statistic for the cell indexed by R1 and C2 is

$$\begin{aligned}
& (50-40)^2 / 40 + (2100-2080)^2 / 2080 + (3947-3960)^2 / 3960 + (205903-205920)^2 / 205920 \\
& + (110-120)^2 / 120 + (6220-6240)^2 / 6240 + (11893-11880)^2 / 11880 + (617777-617760)^2 / 617760 \\
& = 3.649 > 2.706
\end{aligned}$$

The chi-square statistic for the subcube containing the two cells above is

$$\begin{aligned}
& (2060-2040)^2 / 2040 + (90-80)^2 / 80 + (201940-201960)^2 / 201960 + (7910-7920)^2 / 7920 \\
& + (6100-6120)^2 / 6120 + (230-240)^2 / 240 + (605900-605880)^2 / 605880 + (23770-23760)^2 / 23760 \\
& = 1.948 < 2.706
\end{aligned}$$

Fig. 2. An example demonstrates that the chi-square test does not have the desired closure property in data cubes.

number of objects, out of the total of N objects, that satisfy this particular attribute combination. Let $V(k_1, k_2, \dots, k_M)$ denote the random variable that corresponds to the cell indexed by $(k_1, k_2, \dots, k_M)$ and $C(k_1, k_2, \dots, k_M)$ denote the actual count in the data cube, which is regarded as the experiment outcome. Just like in the chi-square test, the probability that a single object takes attribute combination $(A_1 = k_1, A_2 = k_2, \dots, A_M = k_M)$ is computed by $\prod_{i=1}^M \text{Prob}(A_i = k_i)$, where

$$\text{Prob}(A_i = k_i) = \frac{\text{summation of the counts of all the cells with } A_i = k_i}{N: \text{total count in the data cube}}.$$

The expected value and standard deviation of $V(k_1, k_2, \dots, k_M)$, denoted by $\mu(k_1, k_2, \dots, k_M)$ and $\sigma(k_1, k_2, \dots, k_M)$, respectively, are equal to

$$\mu(k_1, k_2, \dots, k_M) = N \cdot \prod_{i=1}^M \text{Prob}(A_i = k_i) \text{ and}$$

Fig. 3. An example of 3-D data cubes.

$$\sigma(k_1, k_2, \dots, k_M) = \sqrt{N \cdot \left(\prod_{i=1}^M \text{Prob}(A_i = k_i) \right) \left(1 - \prod_{i=1}^M \text{Prob}(A_i = k_i) \right)}.$$

The next step is to apply statistics analysis to determine whether the count in each cell is significant in the statistics sense or not. One conventional approach is to apply the z test [14]. That is, the count at cell indexed by $(k_1, k_2, \dots, k_M)$, i.e, $C(k_1, k_2, \dots, k_M)$, is said to be statistically significant, or attribute combination $(A_1 = k_1, A_2 = k_2, \dots, A_M = k_M)$ is said to be correlated, if

$$\frac{\left| \frac{C(k_1, k_2, \dots, k_M)}{N} - \prod_{i=1}^M \text{Prob}(A_i = k_i) \right|}{\sqrt{\frac{1}{N} \cdot \prod_{i=1}^M \text{Prob}(A_i = k_i) \cdot \left(1 - \prod_{i=1}^M \text{Prob}(A_i = k_i) \right)}} \geq z_{\alpha/2}$$

or

$$\frac{\left(C(k_1, k_2, \dots, k_M) - N \cdot \prod_{i=1}^M \text{Prob}(A_i = k_i) \right)^2}{N \cdot \prod_{i=1}^M \text{Prob}(A_i = k_i) \cdot \left(1 - \prod_{i=1}^M \text{Prob}(A_i = k_i) \right)} \geq z_{\alpha/2}^2,$$

where $z_{\alpha/2}$ is the threshold of statistical significance for the two-sided z test.

In this paper, in order to have a closure property, we employ a slightly more conservative measure and say that $C(k_1, k_2, \dots, k_M)$ is statistically significant, or attribute value combination $(A_1 = k_1, A_2 = k_2, \dots, A_M = k_M)$ is correlated, if

$$\begin{aligned} S(k_1, k_2, \dots, k_M) &= \frac{\left[C(k_1, k_2, \dots, k_M) - N \cdot \prod_{i=1}^M \text{Prob}(A_i = k_i) \right]^2}{N \cdot \prod_{i=1}^M \text{Prob}(A_i = k_i)} \\ &= \frac{(C(k_1, k_2, \dots, k_M) - \mu(k_1, k_2, \dots, k_M))^2}{\mu(k_1, k_2, \dots, k_M)} \geq z_{\alpha/2}^2. \end{aligned}$$

The significance measure employed in this paper is more conservative than the conventional z test, because

$$N \cdot \prod_{i=1}^M \text{Prob}(A_i = k_i) \geq N \cdot \left(\prod_{i=1}^M \text{Prob}(A_i = k_i) \right) \left(1 - \prod_{i=1}^M \text{Prob}(A_i = k_i) \right).$$

However, the difference is marginal for the applications that this paper concerns, because a data cube typically contains a large number of cells and therefore we generally have $\prod_{i=1}^M \text{Prob}(A_i = k_i) \ll 1$.

As a result, in general we have $\sqrt{S(k_1, k_2, \dots, k_M)} \cong \left| \frac{C(k_1, k_2, \dots, k_M) - \mu(k_1, k_2, \dots, k_M)}{\sigma(k_1, k_2, \dots, k_M)} \right|$.

The measure of statistic significance employed in this paper has an important closure property, which will be exploited in the summarization process.

Theorem 1. Let $S(h_1, h_2, \dots, h_M)$ and $S(k_1, k_2, \dots, k_M)$ be the significance measurements of the two cells indexed by $(h_1, h_2, \dots, h_M)$ and $(k_1, k_2, \dots, k_M)$, respectively. If $S(h_1, h_2, \dots, h_M) \geq \theta$, $S(k_1, k_2, \dots, k_M) \geq \theta$, $C(h_1, h_2, \dots, h_M) > \mu(h_1, h_2, \dots, h_M)$, and $C(k_1, k_2, \dots, k_M) > \mu(k_1, k_2, \dots, k_M)$, then

$$\frac{[(C(h_1, h_2, \dots, h_M) + C(k_1, k_2, \dots, k_M)) - (\mu(h_1, h_2, \dots, h_M) + \mu(k_1, k_2, \dots, k_M))]^2}{\mu(h_1, h_2, \dots, h_M) + \mu(k_1, k_2, \dots, k_M)} \geq \theta.$$

Similarly, if $S(h_1, h_2, \dots, h_M) \geq \theta$, $S(k_1, k_2, \dots, k_M) \geq \theta$, $C(h_1, h_2, \dots, h_M) < \mu(h_1, h_2, \dots, h_M)$, and $C(k_1, k_2, \dots, k_M) < \mu(k_1, k_2, \dots, k_M)$, then

$$\frac{[(C(h_1, h_2, \dots, h_M) + C(k_1, k_2, \dots, k_M)) - (\mu(h_1, h_2, \dots, h_M) + \mu(k_1, k_2, \dots, k_M))]^2}{\mu(h_1, h_2, \dots, h_M) + \mu(k_1, k_2, \dots, k_M)} \geq \theta.$$

Proof. We will only prove the first case. The proof of the second case is similar. Since $S(h_1, h_2, \dots, h_M) \geq \theta$ and $S(k_1, k_2, \dots, k_M) \geq \theta$, we have $[C(h_1, h_2, \dots, h_M) - \mu(h_1, h_2, \dots, h_M)]^2 \geq \theta \cdot \mu(h_1, h_2, \dots, h_M)$ and $[C(k_1, k_2, \dots, k_M) - \mu(k_1, k_2, \dots, k_M)]^2 \geq \theta \cdot \mu(k_1, k_2, \dots, k_M)$. This implies that

$$\begin{aligned} & \frac{[(C(h_1, h_2, \dots, h_M) + C(k_1, k_2, \dots, k_M)) - (\mu(h_1, h_2, \dots, h_M) + \mu(k_1, k_2, \dots, k_M))]^2}{\mu(h_1, h_2, \dots, h_M) + \mu(k_1, k_2, \dots, k_M)} \\ &= \frac{[(C(h_1, h_2, \dots, h_M) - \mu(h_1, h_2, \dots, h_M)) + (C(k_1, k_2, \dots, k_M) - \mu(k_1, k_2, \dots, k_M))]^2}{\mu(h_1, h_2, \dots, h_M) + \mu(k_1, k_2, \dots, k_M)} \\ &\geq \frac{(C(h_1, h_2, \dots, h_M) - \mu(h_1, h_2, \dots, h_M))^2 + (C(k_1, k_2, \dots, k_M) - \mu(k_1, k_2, \dots, k_M))^2}{\mu(h_1, h_2, \dots, h_M) + \mu(k_1, k_2, \dots, k_M)} \\ &\geq \frac{\mu(h_1, h_2, \dots, h_M) \cdot \theta + \mu(k_1, k_2, \dots, k_M) \cdot \theta}{\mu(h_1, h_2, \dots, h_M) + \mu(k_1, k_2, \dots, k_M)} = \theta. \end{aligned}$$

Actually, there are two situations that make $S(k_1, k_2, \dots, k_M)$ pass the threshold. One case is $C(k_1, k_2, \dots, k_M) \gg \mu(k_1, k_2, \dots, k_M)$, which means that the attribute combination $(A_1 = k_1, A_2 = k_2, \dots, A_M = k_M)$ is positively correlated. Another case is $\mu(k_1, k_2, \dots, k_M) \gg C(k_1, k_2, \dots, k_M)$, which means that the attribute combination $(A_1 = k_1, A_2 = k_2, \dots, A_M = k_M)$ is negatively correlated. Accordingly, we will use a “+1” to mark a cell that is significant due to the former case, a “−1” to mark a cell that is significant due to the later case, and a “0” to mark a cell that is not significant. Figure 1(b) shows the marked cells for the example given in Fig. 1(a). In Fig. 1(b), the threshold used is $(1.645)^2 = 2.706$, where 1.645 is the threshold of the two-sided z test for 0.1 significance.

3. Summarization of the statistical analysis results

With the cells in the data cube marked according to their significance measurements, the next issue of interest is how to summarize the results from statistics analysis. Theorem 1 implies that if all the cells in a subcube are marked by “+1”(or “−1”), then the total count of the subcube also passes the significance criterion. We therefore define the following terms.

Definition 1. A positively(negatively) correlated subcube is a subcube in which all cells are marked by “+1” (“−1”).

Definition 2. A maximum positively (negatively) correlated subcube is a positively (negatively) correlated subcube that is not a subcube of another larger positively (negatively) subcube.

Basically, it is desired to find out all maximum positively correlated subcubes and maximum negatively correlated subcubes. To implement the summarization process, we developed a recursive algorithm based on the conventional algorithm for mining association rules [12]. Let us first use a 3-dimensional data cube to illustrate the basic idea. Assume that it is desired to find all the maximum correlated subcubes of size $l \times m \times n$ or larger in a 3-dimensional data cube as shown in Fig. 3. If we let the A_3 correspond to the dimension of transactions in the market basket problem, then the problem is transformed to finding all 2-dimensional subcubes of size $l \times m$ or larger on the each of the A_1 - A_2 planes defined by different A_3 values and checking each of such subcubes for having support n or larger on the A_3 dimension. The task of finding all 2-dimensional subcubes of size $l \times m$ can be carried out by recursively calling the function of finding maximum subcubes. Figure 4 shows the pseudo-code of the recursive algorithm for general multidimensional data cubes. Figure 5 lists all the maximum positively correlated subcubes of size 2×2 or larger in Fig. 1(b).

4. Experiments

This section discusses two experiments conducted to study the characteristics of the correlation analysis and summarization mechanism proposed in this paper. The first experiment is to demonstrate a high dimensional data cube case, since the example used in previous discussion has only two dimensions. The second experiment is to illustrate the so-called recall rate problem of the proposed mechanism due to its conservative nature.

The data set used in the first experiment is a network transaction log collected on the National Taiwan University campus over a period of 4 weeks. Each cell in the data cube records the number of network transactions under a particular attribute combination. The counts in the 3-D data cube are listed in Appendix A. The experiment is to mine the correlations among the three attribute dimensions. Figure 6 lists all the maximum positively correlated subcubes of size $2 \times 2 \times 2$ or larger. In this experiment, the threshold used is 2.706, a chi-square critical value.

In the second experiment, the courses enrollment data Shih Chien University is used. Figure 7(a) shows the 2-dimensional data cube. The horizontal attributes dimension corresponds to the 10 categories of general-background courses offered to undergraduate students, while the vertical attributes dimension corresponds to the departments in the college. Each cell in the data cube records the number of students from a particular department who took a particular category of courses. Figure 7(b) shows the measurements of statistical significance of the cells. In Fig. 7(b), the appended positive sign means that the attribute combination corresponding to the cell is positively correlated and the appended negative

Procedure finding_maximum_subcubes ($M, s[1:M], d$)

Input

- M : the dimension of the data cube;
- $s[1:M]$: subcube size array;
- d : M^{th} -dimensional value;

Output

- C_M : the set of M -dimensional subcubes

Begin

if ($M > 2$) **then**

for $i = 1$ **to** L_M **begin**

$C_{M-1} = \text{finding_maximum_subcubes}(M - 1, s[1 : M - 1], i);$

for each member $c \in C_{M-1}$ **begin**

if (c satisfies support $s[M]$) **then**

 add $c \times \{c_1, c_2, \dots, c_k\}$ to C_M ; // $c_1, c_2, \dots, c_k$ are the supports of c in the M^{th} dimension, and k is the number of the support of c in the M^{th} dimension.

endif

end

end

else

 apply conventional mining association rule algorithm to find frequent itemsets with $s[1]$ or more items and support no lower than $s[2]$;

 return C_2

endif

 return C_M

End

Fig. 4. The pseudo-code for finding maximum correlated subcubes.

sign means that the attribute combination corresponding to the cell is negatively correlated. The so-called recall rate problem occurs at the cell indexed by R4 and C4. The value of statistical significance in the cell based on the proposed measure does not exceed 2.076, the chi-square critical value used as threshold. However, as Fig. 7(c) reveals, the chi-square statistic corresponding to the cell does exceed the threshold, i.e. attributes R4 and C4 are correlated based on the chi-square test. The reason why the recall rate problem occurs is that the measure of statistical significance proposed in this paper is more conservative than the chi-square statistic. The proposed measure has a nice closure property, but the tradeoff is that it suffers the recall rate problem. On the other hand, if an alternative measure of significance, such as the chi-square test, that does not has the closure property were employed, then the summarization process might faultily include a subcube, in which the lump sum of the counts does not pass the threshold of

	C1: Early Morning 3am~7am	C2: Morning 8am~12pm	C3: Afternoon 1pm~5pm	C4: Evening 6pm~9pm	C5: Midnight 10pm~2am
R1: COLLEGE OF ELECTRICAL ENGINEERING AND COMPUTER SCIENCE	-1	-1	1	1	-1
R2: COLLEGE OF ENGINEERING	-1	1	1	1	-1
R3: COLLEGE OF SCIENCE	-1	1	1	1	-1
R4: COLLEGE OF AGRICULTURE	-1	1	1	-1	-1
R5: COLLEGE OF MANAGEMENT	1	1	1	-1	-1
R6: COLLEGE OF MEDICINE AND PUBLIC HEALTH	1	1	1	-1	-1
R7: COLLEGE OF LIBERAL ARTS	-1	1	1	-1	-1
R8: COLLEGE OF LAW AND SOCIAL SCIENCES	-1	1	1	-1	-1
R9: ADMINISTRATION	-1	1	1	-1	-1
R10: COMPUTER CENTER	-1	1	1	-1	-1
R11: OTHER CENTERS	-1	1	1	-1	-1
R12: LIBRARY	-1	1	1	0	-1
R13: DORMITARY OF GRADUATE STUDENTS	1	-1	-1	1	1
R14: DORMITARY OF MALE UNDERGRADUATE STUDENTS	1	-1	-1	1	1
R15: DORMITARY OF FEMALE UNDERGRADUATE STUDENTS	-1	-1	-1	1	1

Dimension of the subcubes	Maximum positively correlated subcubes
2×3	{R5,R6}×{C1,C2,C3} {R13,R14}×{C1,C4,C5}
3×2	{R1,R2,R3}×{C3,C4} {R13,R14,R15}{C4,C5}
11×2	{R2,R3,R4,R5,R6,R7,R8,R9,R10,R11,R12}×{C2,C3}

Fig. 5. The maximum positively correlated subcubes in the example shown in Fig. 1.

statistical significance. This is a precision problem.

5. Application

We applied the proposed correlation analysis to a microarray dataset, HL-60 [15]. HL-60, a human myeloid cancer cell line, can be induced to differentiate into benign noncancerous macrophages. The differentiation of HL-60 can be mediated not only by the cell surface receptors but also by numerous soluble and membraneanchored mediators. HL-60 cells are also induced to differentiate into mature cells by exogenous chemicals such TPA in particular. Therefore, chemical induction of HL-60 cell differentiation into macrophages is a good model to study the roles of genes and proteins in cell differentiation [15]. The biologist who created the dataset selected 1697 genes from the original raw data generated by the microarray experiment. The expression values of these 1697 genes are significantly different when compared to a reference experiment. Each gene is described by a profile, which contains 8 expression values measured at the following time points respectively: 0.5-hr, 1-hr, 3-hr, 6-hr, 9-hr, 12-hr, 24-hr, and 48-hr. This dataset can be viewed as a 2-dimensional datacube, on which we are going to analyze the correlation between some genes and some particular time points. Figure 8 shows two subcubes derived by the proposed algorithm. In this experiment, the threshold used is 3.841. In Fig. 8(a),

Dimension of the subcubes	Maximum positively correlated subcubes
2×11×2	{A6,A7}×{B1,B2,B3,B4,B5,B6,B7,B8,B9,B10,B11}×{C3,C4}
4×10×2	{A4,A5,A6,A7}×{B2,B3,B4,B5,B6,B7,B8,B9,B10,B11}×{C3,C4}
5×8×2	{A3,A4,A5,A6,A7}×{B2,B3,B4,B5,B7,B8,B9,B11}×{C3,C4}
5×3×2	{A1,A4,A5,A6,A7}×{B3,B4,B6}×{C3,C4}, {A1,A3,A4,A6,A7}×{B3,B4,B12}×{C3,C4}
3×3×2	{A3,A4,A5}×{B13,B14,B15}×{C1,C5}
2×3×2	{A1,A4}×{B4,B5,B6}×{C2,C3}, {A3,A7}×{B2,B5,B6}×{C2,C3}
6×2×2	{A1,A2,A3,A4,A5,A6}×{B5,B6}×{C2,C3}, {A1,A3,A4,A5,A6,A7}×{B3,B4}×{C3,C4}
5×2×2	{A1,A2,A5,A6,A7}×{B13,B14}×{C1,C2}, {A1,A2,A5,A6,A7}×{B13,B14}×{C1,C2}
4×2×2	{A1,A2,A3,A4}×{B14,B15}×{C1,C5}, {A2,A3,A4,A5}×{B13,B15}×{C1,C5}, {A4,A5,A6,A7}×{B5,B6}×{C2,C4}
3×2×2	{A3,A4,A5}×{B2,B3}×{C3,C5}, {A3,A6,A7}×{B1,B2}×{C3,C5}, {A3,A4,A5}×{B2,B3}×{C4,C5}
2×2×2	{A3,A7}×{B2,B10}×{C3,C5}, {A4,A7}×{B2,B10}×{C4,C5}, {A3,A7}×{B1,B2}×{C4,C5}
4×2×3	{A4,A5,A6,A7}×{B5,B6}×{C2,C3,C4}
3×2×3	{A3,A4,A5}×{B2, B4}×{C3,C4,C5 }
2×2×3	{A2,A3}×{B13, B14}×{C1,C2,C5 }, {A1,A3}×{B14, B15}×{C1,C2,C5 }, {A1,A4}×{B4, B6}×{C2,C3,C4 }, {A3,A7}×{B2, B5}×{C2,C3,C4 }

Fig. 6. Maximum positively correlated subcubes of size larger than $2 \times 2 \times 2$ mined in the first experiment.

four genes are identified as having similar expression pattern at time points, 3-hr and 6-hr, while in Fig. 8(b), six genes are identified as with similar expression pattern at time points, 6-hr and 9-hr. The results of correlation analysis are useful in guiding the biologists to design further experiments, in order to identify significant relationships between interested genes.

6. Conclusion

Data cube is a data model that is widely employed in modern data warehousing and OLAP system. Therefore, it is of interest to develop a data mining process for data cubes. This paper presents a mechanism to analyze and summarize the statistical correlations among the attributes of a data cube. To perform the analysis and summarization, this paper employs a new measure of statistical significance. The main reason for proposing the new measure of statistical significance is to have a desired closure property. Based on the closure property, this paper develops an algorithm to summarize the results from performing statistical analysis in the data cube.

Though the proposed measure of statistical significance has a nice closure property, its conservative nature may lead to a recall rate problem. That is, it may happens that the count in a cell of the data cube is significant by the classic z test, but is classified as insignificant by the proposed measure. However, as elaborated in the paper, for the applications that this paper concerns, the difference between the measure proposed in this paper and the z test is marginal. On the other hand, if an alternative measure of significance, such as the chi-square test, that does not have the closure property were employed, then the summarization process might faultily include a subcube in which the lumped sum of the counts does not pass the threshold of statistical significance. That is, a precision problem results.

	C1 Linguistic	C2 Art	C3 Literature	C4 Psychology and social studies	C5 Natural sciences	C6 Laws and politics	C7 Educational	C8 Medical	C9 Applied science	C10 Economy
R1:International Business	176	244	76	361	21	84	51	336	147	284
R2:Accounting	58	198	86	246	31	44	48	360	173	146
R3:Insurance	68	196	91	252	57	116	51	348	134	154
R4:Business Administration	82	181	132	298	64	90	22	236	137	176
R5:Applied Foreign Language	105	238	100	272	14	55	28	195	80	127
R6:Information Management	70	103	24	110	37	25	13	149	60	74

(a) A 2-D data cube.

	C1 Linguistic	C2 Art	C3 Literature	C4 Psychology and social studies	C5 Natural sciences	C6 Laws and politics	C7 Educational	C8 Medical	C9 Applied science	C10 Economy
R1:International Business	20.733 (+)	1.047 (-)	12.961 (-)	0.749 (+)	17.138 (-)	0.859 (-)	0.217 (+)	2.311 (-)	1.799 (-)	22.305 (+)
R2:Accounting	16.487 (-)	0.138 (-)	0.114 (-)	2.143 (-)	1.740 (+)	11.327 (-)	3.073 (+)	20.770 (+)	16.023 (+)	3.035 (-)
R3:Insurance	12.256 (-)	1.637 (-)	0.104 (-)	3.865 (-)	5.893 (+)	20.530 (+)	3.443 (+)	7.882 (+)	0.010 (-)	3.231 (-)
R4:Business Administration	3.251 (-)	3.431 (-)	18.719 (+)	1.982 (+)	14.419 (+)	3.496 (+)	6.815 (-)	10.524 (-)	0.314 (+)	0.107 (+)
R5:Applied Foreign Language	4.478 (+)	21.098 (+)	6.343 (+)	5.835 (+)	12.045 (-)	1.109 (-)	0.650 (-)	11.887 (-)	9.200 (-)	2.784 (-)
R6:Information Management	11.503 (+)	0.347 (-)	8.208 (-)	2.843 (-)	17.733 (+)	2.723 (-)	1.322 (-)	1.240 (+)	0.027 (-)	0.538 (-)

(b) The measurements of statistical significance in the second experiment.

	C4	~C4
R4	298	1120
~R4	1241	5275

$$\chi^2=2.891$$

(c) The cell at which the recall rate problem occurs.

Fig. 7. An experiment on a 2-dimensional data cube to illustrate the so-called recall rate problem of the proposed mechanism due to its conservative nature.

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(a) Four genes have similar behaviors at the time points, 3-hr and 6-hr.

(b) Six genes have similar behaviors at the time points, 6-hr and 9-hr.

Fig. 8. Two subcubes that identify two groups of genes with similar expression behaviors on the specified time points.

Appendix A

A1: Saturday
A2: Sunday
A3: Monday
A4: Tuesday
A5: Wednesday
A6: Thursday
A7: Friday

C1: Midnight (10pm~2am)
C2: Early morning (3am~7am)
C3: Morning (8am~12pm)
C4: Afternoon (1pm~5pm)
C5: Evening (6pm~9pm)

B1: COLLEGE OF ELECTRICAL ENGINEERING AND COMPUTER SCIENCE
B2: COLLEGE OF ENGINEERING
B3: COLLEGE OF SCIENCE
B4: COLLEGE OF AGRICULTURE
B5: COLLEGE OF MANAGEMENT
B6: COLLEGE OF MEDICINE AND PUBLIC HEALTH
B7: COLLEGE OF LIBERAL ARTS
B8: COLLEGE OF LAW AND SOCIAL SCIENCES
B9: ADMINISTRATION
B10: COMPUTER CENTER
B11: OTHER CENTERS
B12: LIBRARY
B13: DORMITARY OF GRADUATE STUDENTS
B14: DORMITARY OF MALE UNDERGRADUATE STUDENTS
B15: DORMITARY OF FEMALE UNDERGRADUATE STUDENTS

(a) Attribute indices of the 3-D data cube

Saturday					
	C1	C2	C3	C4	C5
B1	111646	11317	70782	93015	51889
B2	70703	20309	154233	258454	136797
B3	33901	16065	103029	120678	74350
B4	64689	36492	114657	102943	47563
B5	16412	17706	36642	27491	18478
B6	119077	83742	289254	251231	117701
B7	6045	308	14967	14841	8554
B8	5299	69	19431	21408	13268
B9	3953	270	13390	4988	8005
B10	56480	32480	98298	101150	39453
B11	6165	3119	52497	26601	13104
B12	5515	426	34186	23649	8046
B13	118707	25011	49957	65744	53263
B14	330760	155653	196951	258862	219926
B15	144538	33437	65685	92015	88874

Sunday					
	C1	C2	C3	C4	C5
B1	55773	10424	49413	147849	108099
B2	102074	10713	84757	195817	147368
B3	39388	13930	38407	76182	59132
B4	31796	10528	43731	78532	44237
B5	14699	16817	16183	24624	15853
B6	140999	73572	112946	173858	157720
B7	10895	1854	6995	14451	5676
B8	7059	321	11122	16744	12340
B9	4276	184	3240	5482	454
B10	42710	29649	60759	53901	33541
B11	6347	3887	12870	20018	3844
B12	39	44	5370	10166	863
B13	98811	26136	39839	73985	63449
B14	355626	122935	177543	294633	235663
B15	126736	21595	61789	106034	97346

Monday					
	C1	C2	C3	C4	C5
B1	40794	9906	102050	109396	90006
B2	109620	61021	215304	196588	151966
B3	57276	26790	124939	148371	103654
B4	41933	17255	102136	123190	70326
B5	12909	16677	35027	47575	21721
B6	121665	69343	273055	253318	154806
B7	3490	1026	28296	35503	9300
B8	6939	3205	61119	48287	21860
B9	676	216	22344	20344	9817
B10	48928	15766	97225	146378	82513
B11	6566	6805	67548	69070	27344
B12	414	151	22115	24393	11666
B13	114816	18697	31938	36553	75468
B14	401349	132539	123690	149115	249349
B15	167181	40723	46102	55270	96807

Tuesday					
	C1	C2	C3	C4	C5
B1	54667	30136	117193	107626	82355
B2	107460	16407	205663	224036	177474
B3	45423	20307	157492	157888	108729
B4	44071	35297	138452	144829	78369
B5	19124	17918	45681	48264	26979
B6	126062	77123	351238	325840	174630
B7	3806	1647	32649	28235	13482
B8	6644	2071	47754	68570	26433
B9	48	157	25076	19620	7845
B10	48502	20936	123205	194752	123268
B11	11182	4135	84636	80841	34058
B12	1325	481	28464	21345	11410
B13	118133	17264	28654	34214	79253
B14	437777	145739	166556	169407	268977
B15	172433	27627	52594	70562	130340

Wednesday					
	C1	C2	C3	C4	C5
B1	47115	12906	84859	97541	65657
B2	73272	24768	214851	238361	177878
B3	85522	16597	156042	148937	113768
B4	69863	13758	109249	121326	66084
B5	16074	17456	39920	46387	27687
B6	122036	73242	331344	291828	184212
B7	6815	1253	27408	34274	12068
B8	5368	468	38572	48670	19794
B9	34	83	23592	16771	3474
B10	59764	29525	135980	154170	81342
B11	12743	4611	67727	63384	33972
B12	1430	236	17747	19132	10165
B13	132963	28908	45473	38517	67933
B14	414513	117110	141644	149411	208899
B15	170582	27525	51702	67620	98622

Thursday					
	C1	C2	C3	C4	C5
B1	37456	26618	120350	150704	111170
B2	79465	17610	214848	243986	151507
B3	59018	56617	169928	198325	119299
B4	64256	17840	121702	149095	82544
B5	10499	17333	50298	60337	23802
B6	135178	97832	348000	358920	177033
B7	6300	3257	38040	39145	5830
B8	7349	686	50496	94668	25066
B9	130	160	25039	20723	4053
B10	42258	21911	144285	216720	93097
B11	9036	3208	81745	92946	37714
B12	962	450	34143	31650	18454
B13	119252	23480	32472	45406	60165
B14	370648	123958	156210	186306	218473
B15	143169	19766	50225	84903	97416

(b) The 3-D data cube (to be continued).

Friday

	C1	C2	C3	C4	C5
B1	74525	34170	128908	175784	173595
B2	77594	52542	223827	237704	158990
B3	50085	15296	145873	177749	84106
B4	62823	16128	127847	148518	91222
B5	15570	12670	46188	62520	28616
B6	101339	74956	305252	302595	171568
B7	8091	3580	28035	36119	8165
B8	10492	263	55589	53593	13939
B9	503	399	23447	17753	7655
B10	51757	20286	114618	193381	102642
B11	6916	4101	77496	80334	39174
B12	10653	172	38194	31145	31905
B13	114397	24621	32353	38603	53914
B14	326771	102097	135520	173527	189820
B15	124553	22799	54115	70886	75771

(b) continues

Saturday

	C1	C2	C3	C4	C5
B1	1	0	0	0	0
B2	0	0	0	1	1
B3	0	0	1	1	0
B4	0	1	1	1	0
B5	0	1	1	0	0
B6	0	1	1	1	0
B7	0	0	0	0	0
B8	0	0	0	0	0
B9	0	0	1	0	0
B10	0	1	1	0	0
B11	0	0	1	0	0
B12	0	0	1	1	0
B13	1	1	0	0	0
B14	1	1	0	0	1
B15	1	1	0	0	1

Sunday

	C1	C2	C3	C4	C5
B1	0	0	0	1	1
B2	0	0	0	1	1
B3	0	0	0	0	0
B4	0	0	0	0	0
B5	0	1	0	0	0
B6	0	1	0	0	1
B7	0	0	0	0	0
B8	0	0	0	0	0
B9	0	0	0	0	0
B10	0	1	0	0	0
B11	0	0	0	0	0
B12	0	0	0	0	0
B13	1	1	0	1	1
B14	1	1	0	1	1
B15	1	0	0	1	1

Monday

	C1	C2	C3	C4	C5
B1	0	0	1	0	1
B2	0	1	1	1	1
B3	0	0	1	1	1
B4	0	0	1	1	0
B5	0	1	1	1	0
B6	0	1	1	0	0
B7	0	0	1	1	0
B8	0	0	1	1	0
B9	0	0	1	1	1
B10	0	0	0	1	0
B11	0	0	1	1	0
B12	0	0	1	1	0
B13	1	0	0	0	1
B14	1	1	0	0	1
B15	1	1	0	0	1

Tuesday

	C1	C2	C3	C4	C5
B1	0	0	1	0	0
B2	0	0	1	1	1
B3	0	0	1	1	1
B4	0	1	1	1	0
B5	0	1	1	1	0
B6	0	1	1	1	0
B7	0	0	1	1	0
B8	0	0	1	1	0
B9	0	0	1	1	0
B10	0	0	1	1	1
B11	0	0	1	1	0
B12	0	0	1	1	0
B13	1	0	0	0	1
B14	1	1	0	0	1
B15	1	0	0	0	1

Wednesday

	C1	C2	C3	C4	C5
B1	0	0	0	0	0
B2	0	0	1	1	1
B3	0	0	1	1	1
B4	0	0	1	1	0
B5	0	1	1	1	0
B6	0	1	1	1	0
B7	0	0	1	1	0
B8	0	0	1	1	0
B9	0	0	1	1	0
B10	0	0	1	1	0
B11	0	0	1	1	0
B12	0	0	1	0	0
B13	1	1	0	0	1
B14	1	1	0	0	0
B15	1	0	0	0	1

Thursday

	C1	C2	C3	C4	C5
B1	0	0	1	1	1
B2	0	0	1	1	0
B3	0	0	1	1	1
B4	0	0	1	1	0
B5	0	1	1	1	0
B6	0	1	1	1	0
B7	0	0	1	1	0
B8	0	0	1	1	0
B9	0	0	1	1	0
B10	0	0	1	1	0
B11	0	0	1	1	0
B12	0	0	1	1	1
B13	1	1	0	0	0
B14	1	1	0	0	0
B15	1	0	0	0	1

Friday

	C1	C2	C3	C4	C5
B1	0	1	1	1	1
B2	0	1	1	1	1
B3	0	0	1	1	0
B4	0	0	1	1	1
B5	0	1	1	1	0
B6	0	1	1	1	0
B7	0	0	1	1	0
B8	0	0	1	1	0
B9	0	0	1	1	0
B10	0	0	1	1	1
B11	0	0	1	1	1
B12	0	0	1	1	1
B13	1	1	0	0	0
B14	1	1	0	0	0
B15	1	0	0	0	0

(c) Data cube cells that meet statistical significance threshold with positive correlation.

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