Transmission Network Fault Location Observability With Minimal PMU Placement

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Abstract—This paper presents a concept of fault-location observability and a new fault-location scheme for transmission networks based on synchronized phasor measurement units (PMUs). Using the proposed scheme, minimal PMUs are installed in existing power transmission networks so that the fault, if it occurs, can be located correctly in the network. The scheme combines the fault-location algorithm and the fault-side selector. Extensive simulation results verify the proposed scheme.

Index Terms—Fault location, phasor measurement unit (PMU), transmission network.

I. INTRODUCTION

ACCURATE estimate of the fault location is critical to inspection, maintenance, and repair of transmission lines [1]–[8]. Several two-terminal fault-location algorithms based on phasor measurement units (PMUs) techniques have been proposed [9]–[13] recently. These PMU-based algorithms propose the calculation of fault location using synchronized voltage and current phasors. While they can achieve high accuracy in fault location, they are limited to locate faults in a transmission network installed by PMUs on every bus. Considering the installation cost of PMUs, it is important to investigate the placement scheme of the PMUs at minimal locations on the network in the sense that the fault-location observability can be achieved over the entire network. By borrowing from the concept of network state observability, which means that the entire steady-state bus voltage phasors can be estimated using the installed meter measurements, we define the fault-location observability as that if a fault occurs on the network, then it can be located exactly using the installed digital fault recorders such as PMUs.

Based on the topology of the power system, this paper first proposes a minimal placement strategy of PMUs, and then develops an innovative fault-location algorithm for the transmission network. Using installed PMUs, the new fault-location scheme can enhance the protection function of the network. In the proposed scheme, every line section in transmission networks only needs one-side PMU installation. The proposed

Fig. 1. IEEE-14 test buses.

fault-location scheme can accurately and quickly determine a fault point no matter where it occurs on the power grid. Meanwhile, the prefault transmission network models are not necessary in this approach. The proposed algorithm is more appropriate to practical applications.

II. MINIMAL PMU PLACEMENT STRATEGY

Based on the purpose of this work, this section will present a simple but minimal PMU placement strategy such that the fault-location observability can be achieved. The PMUs placement proceeds with the following steps.

- Step 1) PMUs are placed on the buses with the largest number of connected branches and with the second largest number of connected branches.
- Step 2) PMUs are placed on the buses by the so-called “one-bus spaced deployment strategy,” which means that every two PMUs are spaced by one bus as possible as that the minimal PMUs are achieved and the proposed fault-location scheme can work.
- Step 3) Prune PMUs deployment after the above two steps and eliminate the redundant PMUs until it violates the one-bus-spaced deployment strategy.

Notably, the selection of PMU's placement location is not unique in Step 2). The criterion for the proposed fault-location scheme can work properly in that every line section in the protected network needs at least one-side PMU installation.

We illustrate the placement strategy with Fig. 1, which is the IEEE 14-bus system. The sample network shown in Fig. 1 can

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Fig. 2. Five subnetworks with one-bus-spaced deployment strategy.

be composed of five subnetworks, type-1 to type-5 as shown in Fig. 2. Each of the five subnetworks is placed with PMUs, which are called “one-bus-spaced deployment strategy.” Indeed, most power transmission networks can be composed of these five subnetwork configurations. The type-1 subnetwork is the basic target of our fault-location algorithm developed in Section III. The fault-location algorithm for the type-1 subnetwork will be called the “one-bus-spaced fault-location scheme.” The scheme can be directly applied to the fault-location calculation of type-2 to type-5 subnetworks. The reason is explained as follows.

In the type-2 subnetwork, since both ends of line 1-2 have been placed with PMUs, its fault-location task can be achieved using our previous two-terminal approach [9]. The configuration of line 1-3-2 of the type-2 subnetwork is the same with type-1. Also, the type-3 subnetwork can be viewed as a combination of two type-1 subnetworks connected in a parallel manner (i.e., line 1-2-4 and line 1-3-4). Therefore, a one-bus-spaced fault-location scheme can serve as the fault locator of these two cases.

As for type 4, except that there is one more line (i.e., line 1-4) than the type-3 subnetwork, the other portion of the type-4 subnetwork is the same with type-3. The faults in the type-4 subnetwork can also be located by the mentioned two-terminal approach [9] and one-bus-spaced fault-location scheme easily.

For type 5, finally, the lines 1-2-3, 1-2-4, or 3-2-4 are the same configuration of the type-1 subnetwork. Thus, if PMUs are placed on the buses 1, 3, and 4, and the one-bus-spaced fault-location scheme is applied twice, the fault-location observability of the type-5 subnetwork can be achieved. According to the proposed PMUs deployment strategy, any complicated transmission network can be divided into type-1 to type-5 subnetworks. Then, we apply the one-bus spaced fault-location scheme (proposed in Section III) and the two-terminal fault location algorithm developed in the previous paper [9] to the respective subnetworks. So, the fault-location observability can be achieved. As an example, we apply three steps to Fig. 1.

Fig. 3. Type-1 subnetwork with a fault.

First, according to Step 1), PMUs are placed on the buses 4, 5, 9, 2, and 6. Second, we place PMUs on buses 8, 10, and 13 according to one-bus-spaced deployment strategy. Finally, prune PMUs’ deployment in the entire network and eliminate the redundant PMUs. As a result, only eight PMUs are needed to be placed on buses 2, 4, 5, 6, 8, 9, 10, and 13 to achieve fault-location observability. Notably, the placement of PMUs based on the proposed strategy is not unique. But the number of PMUs is minimal based on a one-bus-spaced deployment strategy.

In [14], the largest number of PMUs for power state observability is derived below

$$v_{UB} = \left\lceil \frac{(N + \frac{m}{2})}{3} \right\rceil \quad (1)$$

where N is the number of buses and m is the number of unknown power injection. $\lceil x \rceil$ is an integer equal to or large than real number x .

For the illustrated sample system, m is equal to 14. So, it is found that, at most, seven PMUs should be installed to observe the operation state of the transmission network under normal conditions. Although, 8 PMUs from the proposed strategy are somewhat more than 7 PMUs placement for power state n observability, the latter cannot locate the fault for transmission networks when a fault occurs.

III. ONE-BUS-SPACED FAULT-LOCATION SCHEME

Consider Fig. 3, which is the type-1 subnetwork with a fault. The PMUs are only installed in both bus i and bus k . The type of subsystem connected to bus j has no limitation. It can be generators, loads, or combined systems. Bus j is connected at a distance of L_{ij} away from bus i and at a distance of L_{jk} away from bus k , respectively. The distributed transmission line model is used to develop the fault-location algorithms.

Based on the measurement configuration, the fault-location algorithm for a transmission network can be developed. Fig. 4 illustrates the flowchart of the one-bus-spaced fault-location scheme for the sample network. The three-phase synchronized phasors of voltages and currents are measured simultaneously by PMU i and PMU k . The phasor quantities are then decoupled into sequence components using symmetrical transformation and the symmetrical components are used as the input data for further computations. The fault-location algorithm developed here is based on symmetrical components of voltages and currents rather than directly using phase values since the former allows the three-phase system to be treated like three single circuits. By virtue of these decoupled single-phase circuits,

Fig. 4. Flowchart of the one-bus spaced fault-location scheme for the type-I subnetwork.

the fault location of the original three-phase transmission line system can be also obtained and, thus, the computation burden is reduced significantly.

First, the fault detection indices from bus i or bus k can be used to trigger the proposed fault-location scheme from sleep state to activation state. Since the fault detector can not discriminate the true fault side between line i and line k , two identical subroutines (subroutine 1 and subroutine 2) are used to calculate the fault location in line i and k , respectively. Then, the proposed fault-side selector is used to identify the correct fault side and fault location. Finally, the correct fault side and fault location are confirmed and displayed to operators.

The assumption of pure fault resistance [1], [2] is used to develop the proposed fault-location algorithm in subroutine 1 and subroutine 2. We will show in our simulation results that the accuracy of the proposed fault-location algorithm will not be affected if the pure fault resistance assumption is violated. Since the fault-location algorithms in subroutine 1 and subroutine 2 are identical, only subroutine 1 is explained. The proposed fault-location algorithms and fault-side selector are developed as follows.

A. Fault-Location Algorithm

All variables in Fig. 3 can be thought of as symmetrical components vectors. As shown in Fig. 3, a fault labeled as F is assumed to occur on line k at a distance of D_k (p.u.) away from bus k . When only bus k is measured, the fault-location calculation of line k is similar to that of the single-terminal fault location problems [1], [2]. However, the problem is more complex in this case, since the configuration of the subsystem is completely unknown.

In order to solve the fault location in line k , the measurements in bus i are used to build the boundary conditions on bus j . First,

Fig. 5. Two-port hyperbolic π -model.

the transmission line between bus j and the fault location F is replaced by the two-port hyperbolic π -model depicted in Fig. 5. The variables in Fig. 5 are defined as follows:

$$\begin{aligned} Z_e &= Z_{ck} \sinh(r_k(1 - D_k)L_{jk}) \\ \frac{Y}{2} &= \frac{1}{Z_{ck}} \tanh(r_k(1 - D_k)L_{jk}) \end{aligned} \quad (2)$$

where $(1 - D_k)L_{jk}$ (kilometers) is the line length of the π -model transmission line. Z_{ck} and r_k are the characteristic impedance and propagation constant of the transmission line k , respectively. The two-port π -network has four terminal variables V_j , I_{jk} , V_F , and I_{kF} . If we got any two of them as known variables, the remaining two unknowns can be easily solved via basic circuit computations [15].

Although bus j is not measured, bus voltage V_j can be calculated via the measurements at bus i and is given in (3)

$$V_j = \frac{(V_i + I_{ij}Z_{ci})}{2 \exp(r_i L_{ij})} + \frac{(V_i - I_{ij}Z_{ci})}{2} \exp(r_i L_{ij}) \quad (3)$$

where Z_{ci} and r_i are the characteristic impedance and propagation constant of the transmission line i , respectively. Thus, the left terminal voltage V_j of the two-port π -network is known but the left terminal input current I_{jk} is still unknown since the injection current of the subsystem is unknown. In addition, since the fault location is unknown, both the right terminal voltage and currents are also unknown variables. However, the right terminal voltage V_F is only dependent on the unknown fault location D_k . If the fault location D_k is obtained, the left terminal current I_{jk} can also be obtained.

Therefore, there are two unknown variables I_{jk} and D_k left to be solved. Herein, the Gauss-Seidel numerical iteration method is used to solve the problem. According to the circuit theories and assumption constraint, the two groups of equations for two unknown variables are defined as

Group-1: Network equations

$$F_1(I_{jk}, D_k, \theta) = 0;$$

Group-2: Constraint equations

$$F_2(I_{jk}, D_k, \theta) = 0$$

where θ represents the known variables, such as V_i , V_k , V_j , etc., and I_{jk} and D_k are the unknown variables. **Group-1** equations are obtained from the two-port π -network enclosed by dashed

Fig. 6. Flowchart for the procedure of calculating D_k and I_{jk} .

line in Fig. 5 to solve the unknown variable I_{jk} . **Group-2** equations are obtained from the assumption of pure resistance fault impedance to solve the fault location D_k . Since the simultaneous equations of **Group-1** and **Group-2** are nonlinear, numerical method [16] is needed to find the solutions of I_{jk} and D_k .

The calculation procedure is depicted in Fig. 6 and summarized as follows.

- 1) Given an initial value of D_k .
- 2) Substitute D_k into **Group-1** equations to calculate I_{jk} .
- 3) Substitute I_{jk} into **Group-2** equations to calculate D_k .
- 4) Repeat steps 2) and 3) until the final results of D_k and I_{jk} are obtained.
- 5) The **Group-1** and **Group-2** equations for different fault types are summarized in **Appendix A**.

Apply the same procedures to subroutine 2 computation; fault location D_i (p.u.) away from bus j can also be obtained.

B. Fault-Side Selector

Simultaneously applying the proposed fault-location computation to subroutines 1 and 2, two fault locations D_k and D_i can be obtained, respectively. Since only one is the correct result, a fault-side selector is developed to select the correct solution.

The resulting solutions of D_k and D_i in subroutines 1 and 2 can be classified into the following four conditions:

- 1) $D_k \in (0, 1)$, $D_i \notin (0, 1)$;
- 2) $D_k \notin (0, 1)$, $D_i \in (0, 1)$;
- 3) $D_k \in (0, 1)$, $D_i \in (0, 1)$;
- 4) $D_k \notin (0, 1)$, $D_i \notin (0, 1)$.

In order to obtain the correct solution, this work uses a technique proposed by Yu *et al.* [17]. In our extensive simulations and a simple proof given in Appendix B, we find that the calculated fault resistances R_F of the correct and incorrect side are positive and negative, respectively. Moreover, since D_k and D_i are both per-unit representations of the fault location in line k and line i , respectively, the correct D_k or D_i must obey the assumptions: $D_k \in (0, 1)$ or $D_i \in (0, 1)$. Thus, the correct solution in condition 1, 2, and 3 can be easily selected as D_k and D_i ,

Fig. 7. Logic block of the proposed fault site selector.

respectively, by choosing corresponding positive R_F . If there is no corresponding positive R_F for condition 1 to condition 3, then the selector output is an external fault. As for condition 4, since the computed fault locations D_k and D_i do not fall into the interval $(0, 1)$, the fault is identified as an external event.

From the above discussions, the fault-side selector can be easily implemented by the logic circuit shown in Fig. 7. In this case, let the logic signal of the computed fault locations $(D_k, D_i) \in (0, 1)$ and positive R_F be state “1;” otherwise, the logic signals are set to be state “0.” For example, when $(D_k, D_i) \notin (0, 1)$, the two AND gates on the left-hand side of Fig. 7 will output logic “0” whatever R_F values are computed. Then, the scheme classifies the fault as an external fault.

Note that the proposed fault-location algorithm uses the voltage and current measurements from both ends of the protected line even though only one PMU is placed at either end of the line. For example, suppose that a fault occurred at line k is shown in Fig. 4. Although only one PMU k is placed at Bus k of Line k , the voltages and currents of the no-PMU Bus j can still be estimated by the data provided from PMU i at Bus i . This work can be achieved by applying two groups of equations: **Group-1** network equations and **Group-2** constraint equations. Therefore, the proposed method can be considered as a two-terminal-based fault-location algorithm. This means that the prefault load condition does not affect the fault-location accuracy of the proposed algorithm [18]. Therefore, it is not necessary to make any assumption, such as source impedance, source voltage, current distribution factor, etc., during the development of the proposed fault-location algorithm.

IV. PERFORMANCE EVALUATION

Fig. 8 depicts a simulated 345-kV transmission network system encountered in Taiwan. The EMTP/ATP simulator [19] is used to model the system and generate fault data to test the performance of the proposed algorithm. The sample system consists of eight transmission lines and eight buses. The transmission lines are modeled with distributed parameters. The related parameters of the simulated system are given in Table I.

According to the developed placement strategy of PMUs, three PMUs are placed on buses 2, 4, and 7 in the sample system. Three identified fault-location zones are Zone_1 (line 2 and line 3), Zone_2 (line 5 and line 6), and Zone_3 (line 7 and line 8), respectively. Lines 1 and 4 are out of the fault location. Thus, according to our previous discussion, the three

Fig. 8. Sample network for simulation tests. Depending on the need for tested cases, the equivalent voltage sources and loads behind buses 3, 6, and 8 were separately connected into the system.

TABLE I
PARAMETERS OF THE SIMULATION SYSTEM

System voltage 345 kV		System frequency 60 Hz	
Source			
$E_{SS} = 1.0 \angle 20^\circ$ p.u.	$E_{RR} = 1.0 \angle 0^\circ$ p.u.	$E_{MM} = 1.0 \angle 10^\circ$ p.u.	
$Z_{SSI} = 0.238 + j5.72 \Omega$	$Z_{RRI} = 0.26 + j6.55 \Omega$	$Z_{MMI} = 0.26 + j6.55 \Omega$	
$Z_{SS0} = 2.738 + j10 \Omega$	$Z_{RR0} = 1.96 + j8.31 \Omega$	$Z_{MM0} = 1.96 + j8.31 \Omega$	
Load behind Bus 2, 4, 6, 7, and 8 are 30 MW + 20 MVar, respectively.			
Transmission line parameter:			
Line_2 (100 km), Line_7 (100 km)			
$R_1 = 0.0275(\Omega / \text{km})$	$C_1 = 9.483(\text{nF} / \text{km})$	$L_1 = 1.345(\text{mH} / \text{km})$	
$R_0 = 0.275(\Omega / \text{km})$	$C_0 = 6.711(\text{nF} / \text{km})$	$L_0 = 3.725(\text{mH} / \text{km})$	
Line_3 (100 km), Line_8 (100 km)			
$R_1 = 0.0385(\Omega / \text{km})$	$C_1 = 38(\text{nF} / \text{km})$	$L_1 = 0.5676(\text{mH} / \text{km})$	
$R_0 = 0.4175(\Omega / \text{km})$	$C_0 = 22(\text{nF} / \text{km})$	$L_0 = 1.644(\text{mH} / \text{km})$	
Line_1 (50km) Line_4 (80km) Line_5(100km), Line_6(100km)			
$R_1 = 0.0321(\Omega / \text{km})$	$C_1 = 38(\text{nF} / \text{km})$	$L_1 = 0.473(\text{mH} / \text{km})$	
$R_0 = 0.3479(\Omega / \text{km})$	$C_0 = 22(\text{nF} / \text{km})$	$L_0 = 1.37(\text{mH} / \text{km})$	

installed PMUs can provide fault-location observability for the transmission network area enclosed by buses 2, 3, 4, 6, 7, and 8.

This work uses a digital mimic filter [20] to eliminate dc offset and the phasors are estimated using the discrete Fourier transform (DFT) filtering algorithm working at 64 samples per cycle. The total simulation time is 100 ms and the percentage error of the estimated fault location is defined as

$$\% \text{ error} = \frac{\text{Estimated Location} - \text{Actual Location}}{\text{Total Line Length}} \times 100\%. \quad (4)$$

The fault locations, fault resistances, and the phase angles of fault currents are computed from subroutine 1 and subroutine 2 simultaneously.

To verify the performance of the proposed scheme, extensive simulation studies for various systems and fault conditions have been performed. The definitions for fault resistance locations in the multiphase transmission line are the same as those defined in [21] and [22]. Thus, for the ABCG fault, there are three identical fault resistances R_f placed between the fault point of each phase and common point, respectively. Then, the common point is shorted to ground. For the ABS fault, there is one fault resistance R_f placed between the fault points of A-phase and B-phase.

TABLE II
TEST CASES OF THE PROPOSED FAULT-SIDE SELECTOR

Case	Case description D_k or D_i : p.u.; R_f : Ω	Subroutine 1		Subroutine 2		Selected output
		D_k (p.u.)	R_f (Ω)	D_i (p.u.)	R_f (Ω)	
1	Line 3, $D_k = 0.2$, $R_f = 0.1$, AG	0.199	0.101	20.913	908.88	D_k
2	Line 2, $D_i = 0.8$, $R_f = 10$, ABCG	-8.218	-28.75	0.801	10.020	D_i
3	Line 6, $D_k = 0.3$, $R_f = 10$, BCG	0.302	9.899	7.060	-7369	D_k
4	Line 5, $D_i = 0.6$, $R_f = 0.1$, AG	-13.21	105.27	0.601	0.113	D_i
5	Line 7, $D_i = 0.8$, $R_f = 10$, ABS	2.788	4884	0.798	10.117	D_i
6	Line 8, $D_k = 0.9$, $R_f = 10$, ABCG	0.897	10.928	0.874	-173.3	D_k
7	Line 1, $D_i = 1.1$, $R_f = 100$, ABCG	~0.74	~120	X	X	#

Symbol "X" represents an indefinite value and "#" represents an external fault. Letters A, B, and C represent phase-a, phase-b, and phase-c, respectively. Letters G and S represent ground fault and short fault, respectively.

For the BCG fault, the fault points of B-phase and C-phase are shorted to a common point. Then, the common point is connected to the ground by one fault resistance R_f . Some results are presented and discussed below.

A. Test for the Proposed Fault-Side Selector

Extensive cases simulated by the EMTP/ATP simulator are utilized to examine the validity of the proposed fault-side selector. Some cases are briefly described in the second column of Table II. In case 2, for example, a three-line-to-ground fault occurs at 80 km (i.e., $D_i = 0.8$ p.u.) in line 2 away from bus 3. The fault resistance of this simulation case is assumed to be 10 Ω . In the tested cases 1 and 2, a generator modeled as a voltage source E_{MM} is connected behind Bus 3. In cases 3 and 4, a load modeled as 30 MW + 21 MVar at nominal voltage is connected behind Bus 6. In cases 5 and 6, a generator and load are connected behind Bus 8. The load is modeled as 30 MW + 21 MVar at nominal voltage and the generator is modeled as a voltage source E_{MM} . Case 7 is a three-phase-grounded fault in line 1 with a distance of 10 km away from Bus 2, in which the grounded resistance is set as 100 ω . Cases 1–5 belong to condition 1 or 2 of the proposed fault-side selector mentioned in Section III. Case 6 is the simulated cases for conditions 3 of the fault-side selector. The simulation results are summarized in the remaining portion of Table II.

For case 6, Fig. 9(a) and (b) illustrate the calculation responses of fault-location indices (i.e., D_k and D_i), and fault resistance R_f calculated by subroutines 1 and 2, respectively. The inspection of these responses reveals that both of the calculated location indices D_k and D_i enter soon into the setting range of condition 3 (i.e., D_k and $D_i \in (0, 1)$). In this case, the proposed fault-side selector can automatically take the calculated R_f and the phase angles of fault currents into consideration to select the correct fault location. In this case, the correct fault location is easily selected as $D_k = 0.897$ (p.u.) which has an estimated positive fault resistance and the fault location error is 0.251%. Case 7 is an external fault case in line 1. The fault event is a three-phase ground fault in line 1 which is 10 km away from Bus 2. The fault resistance is 100 ω . Fig. 10(a) and (b) show the responses of fault-location indices (i.e., D_k and D_i , and fault resistance R_f calculated by subroutines 1 and 2, respectively). An examination of the shown responses indicates that case 7 belongs to condition 1. And, the fault-side selector will output an external fault,

Fig. 9. Responses of fault-location indices and fault resistance calculated by subroutine 1 and 2 for simulation case 6.

Fig. 10. Responses of fault-location indices and fault resistance calculated by subroutine 1 and 2 for simulation case 7.

because R_F is negative for D_k . From Table II, it is clearly seen that the proposed fault-side selector determines the correct fault location.

B. Statistical Evaluation

To verify the robustness of the proposed scheme, extensive fault simulations under various system operations and fault events were conducted. This test considers different fault types, resistances, locations, and inception angles as statistical tests. Any test case could be one of the combinations of the following fault conditions:

- fault locations: 0.1, 0.25, 0.5, 0.75, and 0.9 p.u. of the line length in every tested line of the entire sample system;

Fig. 11. Statistical evaluation of the fault-location estimation with respect to different faults away from Bus 7 (receiving end) occurred in line 6.

Fig. 12. Statistical evaluation of fault-location estimation with respect to different faults away from Bus 4 (receiving end) occurred in line 8.

- fault types: A-G, A-B-S, A-B-G, B-C-G, A-B-C-G, etc., where letters S and G represent short circuit and ground, respectively;
- fault resistance: 0.1, 0.5, 1, 5, 10, and 100 ω ;
- fault inception angles: 0°, 45°, 90°, 135°, and 180° in relation to the zero cross of a-phase voltage at the receiving end bus of the considered line.
- tested lines: line 1–line 8, in which line 1 and line 4 are not included in the protection zone.

The statistical evaluations of different faults in line 6 and 8 are selected to be shown in Figs. 11 and 12, respectively. In these simulated cases, all of the fault-location errors for every tested fault location have been expressed as the average errors of the mentioned inception angles. From the figures, it is clearly seen that the fault-location error is relatively insensitive to the fault types. The maximum error usually occurs in the remote ends [i.e., bus 6 and 8 (the uninstalled PMU bus)], and the errors for the entire tested cases were much smaller than 0.5%.

Fig. 13 illustrates the impacts of different fault resistances under different fault distances in line 2. An examination of Fig. 13 clearly reveals that the proposed algorithm is hardly affected by the magnitudes of fault resistance. Because the proposed scheme does not involve the fault resistance, it is practically independent of fault resistance.

A total of more than 1800 simulation cases have been conducted with respect to various system situations and fault events for the entire sample networks. The statistical evaluations of the fault-location errors and the failure cases for the proposed algorithm under different fault resistances are briefly summarized in Table III.

Fig. 13. Effects of faults resistances on the fault-location estimation away from Bus 3 (receiving end) occurred in line 2.

TABLE III
SUMMARY FOR THE STATISTICAL EVALUATIONS OF
THE PROPOSED FAULT-LOCATION ALGORITHM

Line	Total tested cases $R_F = 0.1, 1, 10 \Omega$	Failed cases	Error (%)	Total tested cases $R_F = 100 \Omega$	Failed cases
1*	40	0	--	10	0
2	250	0	0.159	50	2
3	250	0	0.187	50	6
4*	40	0	--	10	0
5	250	0	0.184	50	4
6	250	0	0.177	50	1
7	250	0	0.145	50	1
8	250	0	0.162	50	3

* Lines 1 and 4 are out of the protection zone.

The scheme correctly detects all of the fault events with the fault resistance of less than 100Ω and provides very good accuracy in fault location. When the fault resistance is even up to 100Ω , only a few fault events cannot be detected by the proposed approach. The failure rate of this statistical study is very low, about $17/1800 = 0.94\%$. In all cases, the maximum error is 0.637% and the average error is about 0.169% .

C. Effects of Nonpure Fault Resistance

As in [1] and [2], our proposed fault-location algorithms also assume the fault impedance is a pure resistance R_f (R_f is placed on the fault path) and uses the assumption to establish the constraint equations in Appendix A. However, if the fault impedance is not pure resistance, we must investigate and estimate the impact of fault impedance on the proposed fault-location algorithm. To answer this question, we have conducted many simulations. For example, assume that a different type fault occurs in the location of 0.25 p.u. of line 2 in the sample system shown in Fig. 8. The original fault resistance is set to be 1Ω . Then, let the magnitude of the imaginary part of a fault impedance be increased gradually. When X/R is tuned to be 10% , the error of the fault location is just about 0.01% as shown in Table IV. The result is almost the same as the condition of pure resistances. Even though the ratio of X/R is increased to 50% , the fault-location error is only about 2.83% . Except the above-mentioned example, extensive simulations with respect to various ground faults, including single-phase to ground faults, also show similar results. Thus, it can be concluded that the impact of nonpure resistance of the fault path on the proposed algorithm is very slight.

TABLE IV
SUMMARY FOR THE IMPACT OF FAULT IMPEDANCE VARIATIONS ON THE
PERFORMANCE OF THE PROPOSED FAULT-LOCATION ALGORITHM

Case*	$R_F (\Omega)$	Calculated results D (p.u.)	$R_F (\Omega)$	Location Error (%)
1	1	0.2499	1	0.01
2	$1 + j0.02$	0.2499	1.0002	0.01
3	$1 + j0.04$	0.2499	1.0008	0.01
4	$1 + j0.06$	0.2499	1.0018	0.01
5	$1 + j0.08$	0.2499	1.0032	0.01
6	$1 + j0.1$	0.2499	1.005	0.01
7	$1 + j0.3$	0.2412	1.047	0.88
8	$1 + j0.5$	0.2217	1.129	2.83

* The simulation case is a 3-phase ground fault occurred at 0.25 p.u. in line 2.

Fig. 14. Status of PMU deployment in the middle-to-north portion of the TPC transmission network system.

D. Realistic Example

In Taiwan, the phasor measurement system was employed in Taiwan Power Company (TPC) [23]. The extra-high-voltage [(EHV), 345-kV] transmission network of TPC includes 22 substations. Presently, eight PMUs were installed at eight EHV substations and mainly used an online phasor monitoring system and fault locators for the transmission lines. For example, Fig. 14 depicts the PMU deployment in the middle-to-north portion of the TPC transmission line system, in which four PMUs have been placed on the buses of Lung-Tan North, 2nd Nuclear, Chung-Liao, and Lung-Tan South, respectively. However, due to the limited installation of PMUs, only the transmission line between Chung-Liao and Lung-Tan South buses encircled by a dashed line can use our previous two-terminal fault-location technique. All of the other transmission lines cannot be applied by the existing fault-location technique. In order to enlarge the fault-location observability, the fault-location scheme proposed in this paper can be applied

TABLE V
SUMMARY FOR GROUP EQUATIONS OF SUBROUTINE 1

Group 1: Network Equation (n = 0, 1, 2)	Group 2: Constraint Equation
$V_{Fn} = \frac{(V_{kn} + I_{kjn}Z_{ckn})}{2} e^{(r_{kn}D_k L_{jk})} + \frac{(V_{kn} - I_{kjn}Z_{ckn})}{2} e^{(-r_{kn}D_k L_{jk})}$	a-g fault $\begin{aligned} &\text{Re}\{I_{F0} + I_{F1} + I_{F2}\} \\ &\times \text{Im}\{V_{F0} + V_{F1} + V_{F2}\} \\ &= \text{Re}\{V_{F0} + V_{F1} + V_{F2}\} \\ &\times \text{Im}\{I_{F0} + I_{F1} + I_{F2}\} \end{aligned}$
$I_{jkn} = \frac{(V_{jn} - V_{Fn})}{Z_e} + V_{jn} \times \frac{Y}{2}$	b-c-s fault $\begin{aligned} &\text{Re}\{I_{F1} - I_{F2}\} \\ &\times \text{Im}\{V_{F1} - V_{F2}\} \\ &= \text{Re}\{V_{F1} - V_{F2}\} \\ &\times \text{Im}\{I_{F1} - I_{F2}\} \end{aligned}$
$I_{Fn} = \frac{(V_{kn} + I_{kjn}Z_{ckn})}{2} e^{(r_{kn}D_k L_{jk})} + \frac{(V_{kn} - I_{kjn}Z_{ckn})}{2} e^{(-r_{kn}D_k L_{jk})/Z_{ckn}}$	b-c-g fault $V_{F1} = V_{F2}$
$+ \frac{(V_{jn} + I_{jkn}Z_{ckn})}{2 \exp(r_{kn}L_R)} e^{(r_{kn}D_k L_{jk})} + \frac{(V_{kn} - I_{kjn}Z_{ck1})}{2 \exp(r_{kn}L_k)} e^{(-r_{kn}D_k L_{jk})/Z_{ckn}}$	a-b-c-g fault $\begin{aligned} &\text{Re}\{I_{F1}\} \times \text{Im}\{V_{F1}\} \\ &= \text{Re}\{V_{F1}\} \times \text{Im}\{I_{F1}\} \end{aligned}$

Fig. 15. One-line diagram for the demonstration of selecting an algorithm.

to the present system. As a result, the fault-location observability can be enlarged to the area encircled by dotted line except the line between Shen–Mee and Lung–Tan North buses. Notably, there is no extra installation of PMUs. Moreover, using the proposed scheme, only 3 PMUs should be added on the buses **Hsieh-Ho**, **1st Nuclear**, and **Pan-Chiao**, and the fault-location observability can be achieved over the entire middle-to-north TPC transmission network.

V. CONCLUSION

This work has successfully developed a new fault-location scheme for transmission networks using PMUs. The proposed fault-location scheme is economic in the sense that minimal PMUs are used. This paper presents a concept of fault-location observability. The scheme combines the fault-location algorithm and the fault-side selector. Extensive simulation results verify the proposed scheme.

APPENDIX A

GROUP EQUATIONS OF SUBROUTINES 1 AND 2

The group equations of subroutine 1 for different types of faults are derived and summarized in Table V. All variables in Table V are the same as those defined in Fig. 3. The variables

with subscript 0, 1, and 2 denote the 0, 1, and 2 sequence variables, respectively. The group equations of subroutine 2 also can be derived using the same ideas in Table V.

APPENDIX B

SIMPLE PROOF OF THE FAULT-SIDE SELECTOR

Herein, the proposed selector is proved via the short distance transmission-lines model shown in Fig. 15. The subsystem connected behind bus j can be a generator or load. The fault occurs at transmission line k . The fault location D_k (p.u.) is the correct fault location and the fault location D'_i (p.u.) is assumed to be the incorrect fault location. Meanwhile, the fault location D_k and D'_i are on transmission lines k and i , respectively.

Using KVL, the relations of V_k and V_i are obtained as follows:

$$\begin{aligned} V_k &= V_j - (I_{ij} + I_j)(1 - D_k)L_{jk}Z_k + I_{kj}D_kL_{jk}Z_k \\ V_j &= V_i - I_{ij}L_{ij}Z_i \end{aligned} \quad (\text{B.1})$$

$$\begin{aligned} V_k &= V'_j + I_{kj}L_{jk}Z_k \\ V'_j &= V_i - I_{ij}(I - D'_i)L_{ij}Z_i + (I'_j + I_{kj})D'_iL_{ij}Z_i. \end{aligned} \quad (\text{B.2})$$

From (B.1) and (B.2), the following relation is obtained:

$$\begin{aligned} (I_{ij} + I_j + I_{kj})L_{jk}Z_k - (I_{ij} + I_j + I_{kj})D_kL_{jk}Z_k \\ + (I_{ij} + I'_j + I_{kj})D'_iL_{ij}Z_i = 0 \end{aligned} \quad (\text{B.3})$$

where $(I_{ij} + I_j + I_{kj}) = I_F$ is the fault current of the correct side, and $(I_{ij} + I'_j + I_{kj}) = I'_F$ is the fault current of the incorrect side.

Rewrite (B.3) as

$$(1 - D_k)I_F L_{jk}Z_k + D'_i I'_F L_{ij}Z_i = 0 \quad (\text{B.4})$$

where $(1 - D_k)L_{jk}$ and $D'_i L_{ij}$ are all positive and real constants, Z_k and Z_i are per-unit-length impedances of transmission lines k and i , and can be expressed as $Z_k = |Z_k|\angle\alpha^\circ$ and $Z_i = |Z_i|\angle\beta^\circ$, respectively.

Rewrite (B.4) as

$$\begin{aligned} \frac{I_F}{I'_F} &= -\frac{D'_i L_{ij} Z_i}{(1 - D_k) L_{jk} Z_k} \\ &= -\frac{D'_i L_{ij} |Z_i| \angle\beta^\circ}{(1 - D_k) L_{jk} |Z_k| \angle\alpha^\circ} \\ &= -K \angle(\beta^\circ - \alpha^\circ). \end{aligned} \quad (\text{B.5})$$

Since K is positive and real constant, the phase difference between I_F and I'_F must be $180^\circ + \beta^\circ - \alpha^\circ$. Normally, $|\beta^\circ - \alpha^\circ| < 90^\circ$. So, the phase difference between I_F and I'_F must be $90^\circ \sim 270^\circ$. Meanwhile, due to the pure resistance assumption of the fault impedance, if $R_F R'_F > 0$ or $R'_F > 0$, the phase difference between V_F and V'_F must be $90^\circ \sim 270^\circ$. From conventional circuit theory, a phase difference of $90^\circ \sim 270^\circ$ between V_F and V'_F is impossible, since the phase difference between the sending end and receiving end is much less than 90° . If $R_F R'_F < 0$ or $R'_F < 0$ (since R_F is positive), the phase difference between V_F and V'_F must be $-90^\circ \sim 90^\circ$. A phase difference of $-90^\circ \sim 90^\circ$ between V_F and V'_F is possible, since the phase difference between sending end and receiving end is much less than 90° . The possible phase difference between V_F and V'_F

is less than 90° . Thus, the estimated fault location with positive fault resistance is selected as the correct solution. When the fault occurs at line i , the same results can be obtained by the similar procedures demonstrated before.

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