Design of controllers for the damping of torsional oscillations using time-scale decomposition

Y-Y Hsu and L Wang

Department of Electrical Engineering, National Taiwan University, Taipei, Taiwan, Republic of China

A new technique is presented for the design of an excitation controller and a static VAR compensator which are used to improve the damping characteristics of torsional oscillations in a capacitor-compensated power system. To facilitate the controller design, the system matrix is reduced by means of multi-time-scale decomposition while preserving the mechanical shift modes, which are the ones to be damped by the proposed controller. The parameters of the controller are then determined on the basis of the reduced system by using modal control theory. It is found that model reduction can save the effort in controller design without any significant drift in the eigenvalues of the closed-loop system. To demonstrate the effectiveness of the proposed damping scheme under disturbance conditions, computer-simulated dynamic response tests based on a non-linear system model are also performed. It is concluded that all subsynchronous oscillation modes can be effectively damped by the proposed controller.

Keywords: power system stability, torsional oscillation, excitation controller, static VAR compensator, model reduction

I. Introduction

Various countermeasures to subsynchronous oscillations in a series capacitor-compensated power system have been proposed¹⁻¹⁰ since the two shaft failures at Mohave station¹¹. The damping scheme to be examined in this paper is a combination of an excitation controller^{2,3} and a static VAR compensator⁵⁻¹⁰.

The mathematical model that will be employed for the present study is the IEEE First Benchmark Model¹², which can be described by a system state equation with 34 state variables. In order to save the effort in controller design, several approaches have been developed to reduce the order of the system. Yan and Yu's reduced model³ was obtained by neglecting the turbine and governor equations as well as one small time constant of the exciter and by combining generator and exciter masses together. In addition, the high-pressure turbine mass and the medium-pressure mass were combined. The masses of the

two low-pressure turbines (LPA and LPB) were also lumped together. Thus the three oscillation modes (1, 2 and 5) which are less vulnerable to torsional oscillations were eliminated while the other poorly-damped modes (0, 3 and 4) were retained. The main drawback of this approach is the considerable eigenvalue drift during the reduction process. Furthermore, laborious work is needed to compute the equivalent spring coefficient for the lumped mass. Balda *et al.*⁴ presented an alternative method for model reduction and controller design based on plant input-output behaviour. However, the accuracy of the reduced model depends heavily on the choice of the reduction matrix **R** and the weighting matrix **Q**. Usually, it takes a lot of time before a proper choice for these matrices can be reached.

In this paper a novel approach based on multi-time-scale decomposition^{13,14} is employed to reduce the order of the system. Controller design based on the reduced model is then carried out using modal control theory. The accuracy of the reduced model is demonstrated by comparing the eigenvalues of the reduced system and the original system. Time-domain simulations using both linear and non-linear system models are also performed. It is concluded that a very accurate reduced model can be achieved by the proposed model reduction technique. Moreover, the combination of the excitation controller and the static VAR compensator, the parameters of which are determined using the reduced model, can offer satisfactory damping effects for all torsional oscillation modes.

II. Notation

General

A, B, C	system matrix
G(s)	transfer function of the system output-input ratio
H(s)	transfer function of the compensator
ω, θ	rotor speed and rotor angle
δ	generator torque angle
R, X	resistance and reactance
B_L, B_C	susceptances of the inductor and capacitor
E_{FD}	per unit output voltage of exciter

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V_R	voltage regulator output
V_x	infinite bus voltage
F	mechanical torque distribution fraction
H	moment of inertia
K	spring constant
K_P, K_I, K_D	PID controller gain
T_w	wash-out term time constant
T	torque
i_L	transmission line current
i_s	current of B_L
a	actuator signal
e_s	voltage of B_C
g	governor opening

Subscripts

d, q	d-axis and q-axis stator quantities
F	field circuit quantities
kd, fq, kq	damp windings quantities
H	high-pressure turbine quantities
I	medium-pressure turbine quantities
A, B	low-pressure turbine, A, B quantities
X	exciter's mechanical quantities
S	static VAR compensator quantities
E	excitation system quantities
L	transmission line quantities

Prefix

Δ	linearized quantities
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III. Description of the study system

The system under study is the IEEE First Benchmark Model², the one-line diagram which is shown in Figure 1. A static VAR compensator¹⁵ of thyristor-controlled reactor/fixed capacitor (TCR-FC) type, which is connected to the generator bus via a step-down transformer, is also incorporated into the model. System parameters and operating conditions taken from References 2, 3, 12 and 16 are given in Appendix 1.

The eigenvalues of the open-loop system without any controllers are listed in the second column of Table 1. It can be observed that there exists an unstable SSR mode (mode 3) and a mode with poor damping (mode 4) under the given operating condition. As the degree of series compensation (X_C/X_L) is varied, the real parts of the eigenvalues for the SSR modes will change in the manner shown in Figure 2.

It is concluded from Figure 2 that, of the six modes of concern, only modes 2 and 5 remain stable for any degree of series compensation. Therefore it is necessary to install some kinds of dynamic compensators to stabilize the four unstable modes (modes 0, 1, 3 and 4). To this end, a combination of an excitation controller and a static VAR compensator, the block diagram of which is shown in Figure 3, will be employed.

Before we tackle the problem of controller design, we first introduce the technique of time-scale decomposition in order to reduce the order of the system so that controller design can be carried out on the basis of a smaller reduced system.

IV. Model reduction using multi-time-scale decomposition

To reduce the order of the system using time-scale decomposition^{13,14}, the state equations of the linearized

system are first written as

$$\dot{\mathbf{X}}(t) = \mathbf{A}\mathbf{X}(t) + \mathbf{B}\mathbf{U}(t) \quad (1)$$

$$\mathbf{Y}(t) = \mathbf{C}\mathbf{X}(t) \quad (2)$$

where

$$\mathbf{X} = [\Delta\omega_H, \Delta\theta_H, \Delta\omega_I, \Delta\theta_I, \Delta\omega_A, \Delta\theta_A, \Delta\omega_B, \Delta\theta_B, \Delta\omega, \Delta\delta, \Delta\omega_X, \Delta\theta_X, \Delta a, \Delta g, \Delta T_H, \Delta T_I, \Delta T_A, \Delta e_{cd}, \Delta e_{cq}, \Delta e_{sd}, \Delta e_{sq}, \Delta E_{fd}, \Delta i_d, \Delta i_F, \Delta i_{kd}, \Delta i_q, \Delta i_{fq}, \Delta i_{kq}, \Delta i_{Ld}, \Delta i_{Lq}, \Delta i_{sd}, \Delta i_{sq}, \Delta B_L, \Delta V_R]^T$$

is the state vector,

$$\mathbf{Y} = \Delta\omega$$

is the output signal and

$$\mathbf{U} = [U_E, V_S]^T$$

is the input vector.

It can be observed from Table 1 that the oscillation frequencies of the torsional modes (modes 0–5) to be retained in the final representation are lower than those of some other modes ($e_{sd}, e_{sq}, \dots$) but are higher than those of the modes in the other group ($a, g, T_H, \dots$). Thus it is necessary to perform time-scale decomposition twice during the model reduction process. In the first stage the system states are divided into a group of slow states and another group of fast states, with the desired torsional modes included in the group of fast-varying states. The resultant state equations are

$$\dot{\mathbf{X}}_1(t) = \mathbf{A}_{11}\mathbf{X}_1(t) + \mathbf{A}_{22}\mathbf{X}_2(t) + \mathbf{B}_1\mathbf{U}(t) \quad (3)$$

$$\dot{\mathbf{X}}_2(t) = \mathbf{A}_{21}\mathbf{X}_1(t) + \mathbf{A}_{22}\mathbf{X}_2(t) + \mathbf{B}_2\mathbf{U}(t) \quad (4)$$

$$\mathbf{Y}(t) = \mathbf{C}_1\mathbf{X}_1(t) + \mathbf{C}_2\mathbf{X}_2(t) \quad (5)$$

Figure 1. The IEEE First Benchmark Model with a static VAR compensator

Table 1. System eigenvalues (rad/s⁻¹) under the operating conditions $V_t=1.05$ p.u., $P_G=0.9$ p.u. and PF=0.9 lagging

States		System conditions			
		Open-loop system	Open-loop system after time-scale decomposition	System with the PID controller designed using full model	System with the PID controller designed using reduced model
X_3 states	e_{sd}, e_{sq}	$-3.533 \pm j724.704$	$-3.53 \pm j724.706$	$-1.301 \pm j736.132$	$-1.512 \pm j736.974$
	i_{sd}, i_{sq}	$-0.229 \pm j376.99$	$-0.229 \pm j376.951$	$-0.242 \pm j376.988$	$-0.243 \pm j376.987$
	i_{Ld}, i_{Lq}	$-3.462 \pm j1485.46$	$-3.462 \pm j1485.46$	$-3.983 \pm j1479.839$	$-3.945 \pm j1479.424$
	i_d, i_q	$-6.964 \pm j590.597$	$-6.969 \pm j590.597$	$-6.253 \pm j590.24$	$-6.197 \pm j590.237$
	E_{fd}	-499.933	-499.933	-500.724	-500.720
X_2 states	Mode 5: λ_H	$-0.182 \pm j298.178$	$-0.182 \pm j298.177$	$-0.182 \pm j298.176$	$-0.182 \pm j298.176$
	Mode 4: λ_1	$-0.026 \pm j202.695$	$-0.026 \pm j202.694$	$-0. \pm j203.2^*$	$-0.59 \pm j203.117$
	Mode 3: λ_A	$0.765 \pm j160.691$	$0.765 \pm j160.696$	$-2.0 \pm j161.9^*$	$-2.054 \pm j160.998$
	Mode 2: λ_X	$-0.642 \pm j172.065$	$-0.642 \pm j127.065$	$-0.825 \pm j127.064$	$-0.816 \pm j127.049$
	Mode 1: λ_B	$-0.11 \pm j99.43$	$-0.11 \pm j99.43$	$-2.0 \pm j99.9^*$	$-1.986 \pm j99.727$
	Mode 0: λ_G	$-0.155 \pm j9.721$	$-0.156 \pm j9.721$	$-3.0 \pm j10.0^*$	$-3.203 \pm j9.69$
	V_R	-101.578	-101.578	-121.288	-121.818
	e_{cd}, e_{cq}	$-6.984 \pm j161.833$	$-6.982 \pm j376.951$	$-4.217 \pm j161.241$	$-4.246 \pm j162.161$
	i_F	-10.293	-10.292	$-6.22 - j0.192$	$-5.721 - j0.352$
	i_{kd}	-31.595	-31.595	-44.686	$-54.893 \pm j7.082$
	k_{kq}	-24.783	-24.783	-24.788	-24.787
	B_L	-6.0	-5.979	$-6.22 \pm j0.192$	$-5.721 \pm j0.352$
	PID controller			-100.0	-100.0
				-64.209	$-54.893 - j7.082$
X_1 states	i_{fq}	-1.705	-1.705	-2.063	-4.782
	T_H, T_I, T_A	-0.142	-0.142	-0.142	-0.142
	a, g	$-3.016, -4.477$	$-3.016, -4.459$	$-2.916, -4.743$	$-2.979 \pm j0.331$
		$-4.684 \pm j0.46$	$-4.704 \pm j0.449$	$-4.452 \pm j1.408$	$-3.639 \pm j2.422$

* Denotes exactly-assigned eigenvalues

Figure 2. The real parts of the eigenvalues associated with torsional modes for the open-loop system as a function of X_C/X_L

Here $X_1=[\Delta a, \Delta g, \Delta T_H, \Delta T_I, \Delta T_A, \Delta i_{fq}]^T$ is the vector comprising all slowly-varying state variables to be reduced and X_2 is the vector composed of the other state variables. Using the time-scale decomposition technique described in References 13 and 14, the original system can

be decoupled into two subsystems, the fast one being the desired reduced model:

● slow subsystem

$$\dot{X}_1(t) = \bar{A}_{11}X_1(t) + \bar{B}_1U(t) \tag{6}$$

● fast subsystem

$$\dot{X}_2(t) = \bar{A}_{22}X_2(t) + \bar{B}_2U(t) \tag{7}$$

$$Y(t) = \bar{C}_2X_2(t) \tag{8}$$

The matrices $\bar{A}_{11}$, $\bar{B}_1$, $\bar{A}_{22}$, $\bar{B}_2$ and $\bar{C}_2$ can be computed iteratively from the equations given in Appendix 2.

To further reduce the order of the model, the system described by equations (7) and (8) is again decomposed into two subsystems, with one fast system (X_3) comprising the fast-varying state variables to be eliminated and another slow system (X_4) containing the torsional modes to be retained. The resultant equations are as follows:

● fast subsystem

$$\dot{X}_3(t) = \bar{A}_{33}X_3(t) + \bar{B}_3U(t) \tag{9}$$

● slow subsystem

$$\dot{X}_4(t) = \bar{A}_{44}X_4(t) + \bar{B}_4U(t) \tag{10}$$

$$Y(t) = \bar{C}_4X_4(t) \tag{11}$$

Here $X_3 = [\Delta e_{sd}, \Delta e_{sq}, \Delta E_{fd}, \Delta i_d, \Delta i_q, \Delta i_{Ld}, \Delta i_{Lq}, \Delta i_{sd}, \Delta i_{sq}]^T$ is the vector comprising the state variables to be eliminated and X_4 is the vector consisting of all the states that are to be retained. Once again, the matrices $\bar{A}_{33}$, $\bar{B}_3$, $\bar{A}_{44}$, $\bar{B}_4$, and $\bar{C}_4$ can be obtained by the method described in Appendix 2.

Thus the reduced model (equations (10) and (11)) can be reached by two successive time-scale decompositions on the original system (equations (1) and (2)). It should be noted that the fast-slow ratios cannot be fixed in this study since the eigenvalues are spread over a wide range of frequencies. The dynamic response of the decomposed system will differ very much from that of the original system if a fixed fast-slow ratio is employed.

Figure 3. Block diagram of the system with an excitation controller and a static VAR compensator

Before we proceed with the design of controllers using this reduced model, we first have a look at the effect of model reduction on the torsional mode eigenvalues. A comparison of the eigenvalues before and after time-scale decomposition, which are listed in the second and third columns of Table 1 respectively, indicates that the time-scale decomposition does not cause any significant eigenvalue drift.

To give an idea of how the system eigenvalues vary with the iteration process of time-scale decomposition, the eigenvalues for the torsional modes in each iteration during the process of reducing X_3 are listed in Table 2. It is observed that the torsional modes which are of primary concern to us have been retained during the reduction process. In addition, the decomposition process can be completed in five iterations.

V. Controller design using the reduced model

To determine the parameters of the excitation controller and the static VAR compensator using modal control theory^{17,18}, the state equations in the frequency domain are first obtained by taking the Laplace transformation of equations (10) and (11):

$$sX_4(s) = \bar{A}_{44}X_4(s) + \bar{B}_4U(s) \tag{12}$$

$$Y(s) = \bar{C}_4X_4(s) \tag{13}$$

Rearranging equation (12) gives

$$X_4(s) = (sI - \bar{A}_{44})^{-1}\bar{B}_4U(s) \tag{14}$$

Substituting equation (14) into equation (13) results in

$$Y(s) = G(s)U(s) \tag{15}$$

where

$$G(s) = \bar{C}_4(sI - \bar{A}_{44})^{-1}\bar{B}_4 \tag{16}$$

Since the control vector $U(s)$ can be written as

$$U(s) = \begin{bmatrix} U_E(s) \\ V_S(s) \end{bmatrix} = \begin{bmatrix} H_E(s) \\ H_S(s) \end{bmatrix} Y(s) \tag{17}$$

Mason's rule states that the characteristic equation for the closed-loop system with control signals from the excitation controller (i.e. $U_E(s)$) and the static VAR compensator (i.e. $V_S(s)$) can be expressed as

$$\Delta(s) = 1 - G(s)H(s) = 0 \tag{18}$$

Table 2. Shaft mode eigenvalues in the iteration process of time-scale decomposition

Iteration	Mode 0	Mode 1	Mode 2	Mode 3	Mode 4
$k=j=1$	$-0.159 \pm j9.713$	$-0.119 \pm j99.458$	$-0.637 \pm j127.072$	$0.423 \pm j160.448$	$0.024 \pm j202.703$
$k=j=2$	$-0.162 \pm j9.739$	$-0.106 \pm j99.428$	$-0.641 \pm j127.064$	$0.629 \pm j160.847$	$-0.023 \pm j202.678$
$k=j=3$	$-0.153 \pm j9.716$	$-0.110 \pm j99.430$	$-0.642 \pm j127.065$	$0.756 \pm j160.718$	$-0.024 \pm j202.692$
$k=j=4$	$-0.156 \pm j9.724$	$-0.110 \pm j99.430$	$-0.642 \pm j127.065$	$0.758 \pm j160.709$	$-0.025 \pm j292.693$
$k=j=5$	$-0.156 \pm j9.721$	$-0.110 \pm j99.430$	$-0.642 \pm j127.065$	$0.765 \pm j160.696$	$-0.026 \pm j202.694$

Figure 4. The real parts of the torsional mode eigenvalues as a function of X_C/X_L

where

$$\mathbf{H}(s) = \begin{bmatrix} H_E(s) \\ H_S(s) \end{bmatrix} \quad (19)$$

It is observed from Figure 3 that the transfer functions of the excitation controller, $H_E(s)$, and the static VAR compensator, $H_S(s)$, are given by

$$H_E(s) = \frac{sT_{\omega E}}{1 + sT_{\omega E}} \left(K_{PE} + \frac{K_{IE}}{s} + K_{DE}s \right) \quad (20)$$

and

$$H_S(s) = \frac{sT_{\omega S}}{1 + sT_{\omega S}} \left(K_{PS} + \frac{K_{IS}}{s} + K_{DS}s \right) \quad (21)$$

respectively. Note that controllers of the proportional-integral-derivative (PID) type have been employed in the present study.

When a set of eight desired locations on the complex plane are specified for the eigenvalues associated with the four unstable torsional modes (modes 0, 1, 3 and 4) and are substituted into equation (18), we get a set of eight simultaneous algebraic equations with eight unknown constants $T_{\omega E}$, K_{PE} , K_{IE} , K_{DE} , $T_{\omega S}$, K_{PS} , K_{IS} and K_{DS} . The desired controller parameters can thus be computed by solving these non-linear equations. The results are as follows:

- prespecified eigenvalues

$$\begin{aligned} -3.0 \pm j10.0 & \quad (\text{mode 0}) \\ -2.0 \pm j99.9 & \quad (\text{mode 1}) \\ -2.0 \pm j161.9 & \quad (\text{mode 3}) \\ -0.5 \pm j203.2 & \quad (\text{mode 4}) \end{aligned}$$

- controller parameters determined by using the reduced model excitation controller

$$T_{\omega E} = 0.01 \text{ s}, \quad K_{PE} = 0.68, \quad K_{IE} = 12.24, \quad K_{DE} = 0.06$$

static VAR compensator

$$T_{\omega S} = 0.01 \text{ s}, \quad K_{PS} = -6.54, \quad K_{IS} = -18.5, \quad K_{DS} = -0.1$$

The eigenvalues of the closed-loop system with the designed controllers installed at the generator are listed in the fifth column of Table 1. It is noted that only insignificant drift in the torsional mode eigenvalues has been observed owing to the model reduction introduced in the controller design.

If controller design is based on the original full system (equations (1) and (2)), the following controller parameters will be reached by using the same prespecified eigenvalues as those employed for the reduced model:

- controller parameters determined by using the full model excitation controller

$$T_{\omega E} = 0.01 \text{ s}, \quad K_{PE} = 1.43, \quad K_{IE} = 19.82, \quad K_{DE} = 0.06$$

static VAR compensator

$$T_{\omega S} = 0.01 \text{ s}, \quad K_{PS} = -7.87, \quad K_{IS} = -122.4, \quad K_{DS} = -0.093$$

When the controllers with these parameters are installed at the system, the eigenvalues listed in the fourth column of Table 1 will be obtained. The prespecified eigenvalues have been exactly assigned by the designed controllers.

Although controller design using the reduced model will introduce some minor drift in the system eigenvalues, the shift is insignificant. Moreover, the required CPU time for controller design can be reduced from 12.92 s on a VAX-11/780 computer to 4.76 s on the same machine if the reduced model is employed to replace the original full model. This is particularly desirable if controller design has to be done many times for different prespecified eigenvalues.

The real parts of the torsional mode eigenvalues for the system equipped with the excitation controller and the static VAR compensator as a function of the degree of series compensation are shown in Figure 4. It is concluded that the closed-loop system, though equipped with a controller designed on the basis of an operating condition of 50% series compensation, can remain stable as long as $X_C/X_L \leq 75\%$.

Figure 5. Time-domain simulation results using the linear model

a

b

c

d

e

Figure 6. Time-domain simulation results using the non-linear model (open-loop system)

VI. Linear simulation

It was demonstrated in the last section that as far as system eigenvalues are concerned, the controller designed on the basis of the reduced model will yield almost the same result as that designed using the original system. To look further into the effect of model reduction on system performance, time-domain simulations using both the 36th-order closed-loop linear system and the 21st-order reduced closed-loop system were carried out for a pulsed torque disturbance (ΔT_e) of 5% lasting for four cycles. The results are compared in Figure 5. It can be observed from the response curves in Figure 5 that the curve of the reduced model matches very closely that of the full model.

VII. Non-linear simulation

Another point of interest is the dynamic performance of the system subject to a major disturbance. In this case the system behaviour will be affected by all kinds of non-linearities such as exciter ceilings, control signal limiters, etc. To examine the performance of the controller, which was designed on the basis of a particular operating condition, under these drastically-changing operating conditions, dynamic responses to a 10%, four-cycle pulsed torque (ΔT_e) are plotted in Figures 6 and 7. A set of 36th-order non-linear differential equations describing the system behaviour were solved by the fourth-order Runge-Kutta method on a VAX-11/780 computer. Figure 6 depicts the response curves for the open-loop system. Unstable responses have been observed. This is as expected since there exists an unstable torsional mode, as is clear from the eigenvalues listed in the second column of Table 1. The response curves in Figure 7 indicate that all torsional modes have been effectively damped by the added controller. It can also be concluded from Figure 8 that only small control signals are required by the proposed controller.

VIII. Conclusions

A multi-time-scale decomposition technique has been employed to reduce the order of the system for subsynchronous resonance studies. By separating the fast and slow states, we obtain a reduced model which can yield essentially the same dynamic responses as those obtained by using the original full system. Controller

Figure 7. Time-domain simulation results using the non-linear model (closed-loop system)

Figure 8. Control signals

design can thus be carried out by using the reduced model to save the computational effort.

To stabilize all torsional modes, it is essential that the eigenvalues associated with the unstable modes be shifted leftwards to certain desirable locations on the complex plane by using the excitation controller and the static VAR compensator. It was found that, by properly tuning the controller parameters using model control theory, the prespecified eigenvalues can be exactly assigned by the proposed controller. To examine the effectiveness of the controller under disturbance conditions, time-domain simulations of the system subject to a major disturbance

were performed using a non-linear system model. It was found that all torsional modes can be stabilized by the developed controller.

Since the approach is generally applicable to any large-scale system, it can be easily extended to multi-machine power system problems.

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Appendix 1. System parameters and operating conditions

The following parameters are all on an 892.4 MVA base¹².

Rotor mass-spring parameters^{2,3}

Mass	Shaft	H (sec)	K (p.u. torque/rad)	F (%)
HP		0.092897		30.0
	HP-IP		19.303	
IP		0.155589		26.0
	IP-LPA		34.929	
LPA		0.858670		22.0
	LPA-LPB		52.038	
LPB		0.884215		22.0
	LPB-GEN		70.858	
GEN		0.868495		0.0
	GEN-EXC		2.822	
EXC		0.0342165		0.0

Speed governor system¹⁶

$$\begin{aligned}
 K_G &= 25.0 & T_{CH} &= 0.3 \text{ s} \\
 T_{SR} &= 0.2 \text{ s} & T_{RH} &= 7.0 \text{ s} \\
 T_{SM} &= 0.3 \text{ s} & T_{CO} &= 0.2 \text{ s}
 \end{aligned}$$

Exciter and voltage regulator^{2,3}

$$\begin{aligned}
 K_A &= 50.0 & T_A &= 0.01 \text{ s} & T_E &= 0.002 \text{ s} \\
 E_{FD\max} &= 7.3 & U_{E\max} &= 0.12 \\
 E_{FD\min} &= -7.3 & U_{E\min} &= -0.12
 \end{aligned}$$

The synchronous generator (p.u.)

$$\begin{aligned}
 R_A &= 0.0015 & R_F &= 0.001 & R_{kd} &= 0.0037 \\
 & & R_{fq} &= 0.0053 & R_{kq} &= 0.0182 \\
 X_d &= 1.79 & X_{ad} &= 1.66 & X_F &= 0.04 & X_{kd} &= 0.006 \\
 X_q &= 1.71 & X_{aq} &= 1.58 & X_{fq} &= 0.115 & X_{kq} &= 0.245
 \end{aligned}$$

Transformer and series-compensated transmission line

$$\begin{aligned}
 R_T &= 0.01 \text{ p.u.} & X_T &= 0.14 \text{ p.u.} & R_L &= 0.02 \text{ p.u.} \\
 X_{L1} &= 0.50 \text{ p.u.} & X_{L2} &= 0.06 \text{ p.u.} & X_C/X_L &= 50\%
 \end{aligned}$$

Static VAR compensator^{15,19}

$$\begin{aligned}
 K_S &= 1.0 & T_S &= 0.15 \text{ s} & X_{TS} &= 0.08 \text{ p.u.} \\
 B_C &= 0.6 \text{ p.u.} & B_L &= B_{L0} + \Delta B_L & B_{L0} &= -0.45 \text{ p.u.} \\
 B_{L\max} &= -0.42 \text{ p.u.} & & & V_{S\max} &= 0.12 \text{ p.u.} \\
 B_{L\min} &= -0.48 \text{ p.u.} & & & V_{S\min} &= -0.12 \text{ p.u.}
 \end{aligned}$$

Operating conditions

$$P_G = 0.9 \text{ p.u.} \quad \text{PF} = 0.9 \text{ lagging} \quad V_i = 1.05 \text{ p.u.}$$

Appendix 2. Iterative equations for time-scale decomposition

The time-scale decomposition method^{13,14} can be used to decouple the system

$$\dot{X}(t) = AX(t) + BU(t) \quad (22)$$

$$Y(t) = CX(t) \quad (23)$$

into two subsystems:

- slow subsystem

$$\dot{\mathbf{X}}_1(t) = \bar{\mathbf{A}}_{11}\mathbf{X}_1(t) + \bar{\mathbf{B}}_1\mathbf{U}(t) \quad (24)$$

- fast subsystem

$$\dot{\mathbf{X}}_2(t) = \bar{\mathbf{A}}_{22}\mathbf{X}_2(t) + \bar{\mathbf{B}}_2\mathbf{U}(t) \quad (25)$$

$$\mathbf{Y}(t) = \bar{\mathbf{C}}_2\mathbf{X}_2(t) \quad (26)$$

The matrices $\bar{\mathbf{A}}_{11}$, $\bar{\mathbf{B}}_1$, $\bar{\mathbf{A}}_{22}$, $\bar{\mathbf{B}}_2$ and $\bar{\mathbf{C}}_2$ can be solved iteratively using the following equations for the j th iteration^{13,14}:

$$\begin{aligned} \bar{\mathbf{A}}_{11,kj} &= \bar{\mathbf{A}}_{11,k} - H_{kj}\bar{\mathbf{A}}_{21,k} \\ \bar{\mathbf{A}}_{12,kj} &= \bar{\mathbf{A}}_{12,k} - H_{kj}\bar{\mathbf{A}}_{22,k} + \bar{\mathbf{A}}_{11,kj}H_{kj} \\ \bar{\mathbf{A}}_{21,kj} &= \bar{\mathbf{A}}_{21,k} \\ \bar{\mathbf{A}}_{22,kj} &= \bar{\mathbf{A}}_{22,k} + \bar{\mathbf{A}}_{21,k}H_{kj} \\ \bar{\mathbf{B}}_{1,kj} &= \bar{\mathbf{B}}_{1,k} - H_{kj}\bar{\mathbf{B}}_{2,k} \\ \bar{\mathbf{B}}_{2,kj} &= \bar{\mathbf{B}}_{2,k} \\ \bar{\mathbf{C}}_{2,kj} &= \bar{\mathbf{C}}_{2,k} + \bar{\mathbf{C}}_{1,k}H_{kj} \end{aligned} \quad (27)$$

H_{kj} can be computed iteratively by solving the equations

$$\begin{aligned} H_{ki} &= \mathbf{A}_{12}\bar{\mathbf{A}}_{22,k}^{-1} + (\bar{\mathbf{A}}_{11,k} - H_{k,i-1}\bar{\mathbf{A}}_{21,k})H_{k,i-1}\bar{\mathbf{A}}_{22,k}^{-1} \\ H_{k1} &= \bar{\mathbf{A}}_{12,k}\bar{\mathbf{A}}_{22,k}^{-1} \end{aligned} \quad (28)$$

and the matrices in the k th iteration, $\bar{\mathbf{A}}_{11,k}$, $\bar{\mathbf{A}}_{12,k}$, $\bar{\mathbf{A}}_{21,k}$, $\bar{\mathbf{A}}_{22,k}$, $\bar{\mathbf{B}}_{1,k}$, $\bar{\mathbf{B}}_{2,k}$ and $\bar{\mathbf{C}}_{2,k}$, can be obtained from the equations

$$\begin{aligned} \bar{\mathbf{A}}_{11,k} &= \mathbf{A}_{11} - \mathbf{A}_{12}L_k \\ \bar{\mathbf{A}}_{12,k} &= \mathbf{A}_{12} \\ \bar{\mathbf{A}}_{21,k} &= \mathbf{A}_{21} - \mathbf{A}_{22}L_k + L_k\bar{\mathbf{A}}_{11,k} \\ \bar{\mathbf{A}}_{22,k} &= \mathbf{A}_{22} + L_k\mathbf{A}_{12} \\ \bar{\mathbf{B}}_{1,k} &= \mathbf{B}_1 \\ \bar{\mathbf{B}}_{2,k} &= \mathbf{B}_2 + L_k\mathbf{B}_1 \\ \bar{\mathbf{C}}_{2,k} &= \mathbf{C}_2 \end{aligned} \quad (29)$$

where $\mathbf{A}_{11}$, $\mathbf{A}_{12}$, $\mathbf{A}_{21}$ and $\mathbf{A}_{22}$ are submatrices of the matrix $\mathbf{A}$ and L_k is computed iteratively by solving the equation

$$\begin{aligned} L_i &= \mathbf{A}_{22}^{-1}\mathbf{A}_{21} + \mathbf{A}_{22}^{-1}L_{i-1}(\mathbf{A}_{11} - \mathbf{A}_{12}L_{i-1}) \\ L_1 &= \mathbf{A}_{22}^{-1}\mathbf{A}_{21} \end{aligned} \quad (30)$$