CMOS Differential-Mode Exponential Voltage-To-Current Converter

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Abstract. A CMOS differential-mode exponential voltage-to-current converter is presented which is based on the approximated Taylor's series expansion. The proposed circuit has been fabricated in a 0.5 μm N-well CMOS process. Experimental results show that the output dynamic range of the proposed differential-mode exponential voltage-to-current converter can be 15dB while the nonlinear error is less than 1.35%. The experimental results are given to demonstrate the proposed circuits.

Key Words: CMOS, differential, exponential circuit

1. Introduction

The exponential function generator is one of the important building blocks in a variable gain amplifier (VGA) [1–3]. To realize the exponential function, the approximated Taylor's series expansion [4–6] and a pseudo-exponential function [7, 8] can be utilized. A differential-mode exponential voltage-to-current converter can realize the following equation,

$$I_{\text{out}} = \exp(V_1 - V_2) \quad (1)$$

where $(V_1 - V_2)$ is the voltage difference between two input signals and I_{out} is the output current. Among [4–8], although some of them can operate with a voltage-mode input signal, however, it is limited to be a single-ended one. Since $\exp(x - y) \neq \exp(x) - \exp(y)$, a differential-mode exponential voltage-to-current converter can not be realized by directly using two single-ended exponential voltage-to-current converters as the input stage. For example, there are two logarithmic current-to-voltage converters with the input currents, I_1 and I_2 , and the corresponding output voltages, V_{out1} and V_{out2} , where $V_{\text{out1}} = k \ln I_1$ and $V_{\text{out2}} = k \ln I_2$, respectively, and k is a constant. The ratio of the currents I_1 and I_2

can be obtained by connecting the differential-mode exponential voltage-to-current converter to the outputs of the logarithmic current-to-voltage-converters, i.e.,

$$\begin{aligned} I_{\text{out}} &= \exp(V_{\text{out1}} - V_{\text{out2}}) = \exp(k \ln I_1 - k \ln I_2) \\ &= \left(\frac{I_1}{I_2} \right)^k \end{aligned} \quad (1)$$

Therefore, the development of the differential-mode exponential voltage-to-current converter is of certain importance. In this paper, a differential-mode exponential voltage-to-current converter is presented. It is based on the approximated Taylor's series expansion. The proposed circuit has been fabricated in a 0.5 μm N-well CMOS process and the experimental results are given to demonstrate the proposed circuit.

2. Circuit Description

According to the Taylor's series expansion, a general exponential function can be expressed as

$$\exp(ax) \approx \frac{a^2 x^2}{2} + ax + 1; \quad \text{if } |ax| \ll 1 \quad (2)$$

The proposed differential-mode exponential voltage-to-current converter is shown in Fig. 1. Assume

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Fig. 1. Proposed differential-mode exponential voltage-to-current converter.

Fig. 2. The die photo of the proposed differential-mode exponential voltage-to-current converter.

that all the transistors in Fig. 1 are in saturation region without body effect. M8, M9 and M10, M11 are two level shifters. Supposed that M8–M11 are perfectly matched with a constant voltage V_c , one can obtain

$$V_{GS8} = V_{GS9} = V_{GS10} = \sqrt{\frac{2I}{k_n}} + V_{Tn} \triangleq V_B \quad (3)$$

where $k_n = \mu_n C_{ox} \frac{W}{L}$ is the transconductance parameter, V_{Tn} is the threshold voltage of M8–M11

and

$$I_9 = I_{11} = \frac{1}{2} k_n (V_c - V_{SS} - V_{Tn})^2 \triangleq I, \quad (4)$$

respectively. Again, assume that M1 and M2 are perfectly matched. According to the square-law characteristics of the MOSFET, one can obtain

$$I_1 = \frac{1}{2} k_p (V_1 - (V_2 - V_B) - |V_{Tp}|)^2 \quad (5)$$

and

$$I_2 = \frac{1}{2} k_p (V_B - |V_{Tp}|)^2 \quad (6)$$

where $k_p = \mu_p C_{ox} \frac{W}{L}$ is the transconductance parameter and V_{Tp} is the threshold voltage of M1 and M2, respectively. Since M3, M4 and M5–M7 realize two current mirrors, one can obtain

$$I_1 = I_6 = I_7 \quad (7)$$

Since $I_6 = I_1$, there is no current to flow into the node “a”. From equations (5)–(7), the output current of Fig. 1 can be given as

$$I_{out} = I_7 + I_2 = I_1 + I_2 = k_p (V_B - |V_{Tp}|)^2 \times \left(\frac{(V_1 - V_2)^2}{2(V_B - |V_{Tp}|)^2} + \frac{(V_1 - V_2)}{(V_B - |V_{Tp}|)} + 1 \right) \quad (8)$$

According to equation (2) and $V_B > |V_{Tp}|$, equation (8) can be written as

$$I_{out} \approx I_b \cdot \exp(a(V_1 - V_2)) \quad (9)$$

where $I_b = k_p (V_B - |V_{Tp}|)^2$ and $a = \frac{1}{V_B - |V_{Tp}|}$. From equation (9), a differential-mode exponential voltage-to-current converter can be obtained. Since M1 is assumed to be in saturation region, the input signal should be limited by

$$V_1 - V_2 \geq |V_{Tp}| - V_B \quad (10)$$

and

$$V_2 \geq V_B + V_{SS} + V_{Tn} - |V_{Tp}| \quad (11)$$

Assume that the transconductance parameters of M1 and M2 are mismatched due to the process variations,

Fig. 3. Experimental results of the proposed differential-mode exponential voltage-to-current converter. (a) semi-logarithmic scale and (b) dB scale.

i.e., $k_{p1} = k_p$ and $k_{p2} = k_p + \Delta k_p$. According to equations (5) and (6), equations (8) can be rewritten as

$$I_{out} = k_p(V_B - |V_{Tp}|)^2 \left(\frac{(V_1 - V_2)^2}{2(V_B - |V_{Tp}|)^2} + \frac{(V_1 - V_2)}{(V_B - |V_{Tp}|)} + 1 \right) + \frac{\Delta k_p}{2}(V_B - |V_{Tp}|)^2 \quad (12)$$

where an additional constant current term is generated and it will result in the offset current with re-

spect to pseudo-exponential function. Suppose that the threshold voltages of M1 and M2 are not equal, i.e., $|V_{Tp1}| = |V_{Tp}|$ and $|V_{Tp2}| = |V_{Tp}| + \Delta V_{Tp}$. From equations (5), (6) and (8) can be rewritten as

$$I_{out} = k_p(V_B - |V_{Tp}|)^2 \left(\frac{(V_1 - V_2)^2}{2(V_B - |V_{Tp}|)^2} + \frac{(V_1 - V_2)}{(V_B - |V_{Tp}|)} + 1 \right) + \frac{\Delta k_p}{2}(\Delta |V_{Tp}|^2 - 2\Delta |V_{Tp}| \cdot (V_B - |V_{Tp}|)) \quad (13)$$

From equation (13), an additional constant current term is also generated. To avoid the body effect, all the PMOS transistors can be embodied in their individual wells and the sources of the NMOS transistors should be connected to the negative supply voltage. However, in the proposed circuit, M8 and M10 may be affected by the body effect. Assume that the threshold voltages of M8 and M10 are not equal due to the body effect, i.e., $V_{Tn8} = V_{Tn}$ and $V_{Tn10} = V_{Tn} + \Delta V_{Tn}$. From equation (3), it will result in $V_{GS8} \neq V_{GS10}$, i.e., $V_{GS8} = V_B$ and $V_{GS10} = V_B + \Delta V_B$. According to equations (5), (6), and (8) can be rewritten as

$$I_{out} = k_p(V_B - |V_{Tp}|)^2 \left(\frac{(V_1 - V_2)^2}{2(V_B - |V_{Tp}|)^2} + \frac{(V_1 - V_2)}{(V_B - |V_{Tp}|)} + 1 \right) + \frac{k_p}{2} (2\Delta V_B(V_B - |V_{Tp}|) + \Delta V_B^2) \quad (14)$$

From equation (14), an unexpected offset current is generated. In order to reduce the offset currents in equations (12)–(14), careful layout is necessary. If the offset currents are significant, long channel devices for M1 and M2 can be used to reduce the currents.

Finally, the Monte Carlo analysis shows that, as a 10% variation of the transconductance parameter and the threshold voltage of M1 and M2, after 30 times iterations, the variations of the output voltage are 0.207% and -0.243%, respectively. Also, for simulating the body effect, assume that there is 10% variation of the threshold voltage of M8 and M10, the Monte Carlo analysis shows that, the corresponding variations of the output voltage is 0.246%.

3. Experimental Results

The proposed differential-mode exponential voltage-to-current converter has been fabricated in a 0.5 μm N-well CMOS process. The die photograph is shown in Fig. 2. The aspect ratios of the transistors of the proposed circuit in Fig. 1 are all $(\frac{W}{L}) = (\frac{1 \mu\text{m}}{1 \mu\text{m}})$. To test the proposed differential-mode exponential voltage-to-current converter, the experiment were performed with the supply voltages $V_{DD} = |V_{SS}| = 2.5 \text{ V}$ and the constant voltage $V_c = -0.75 \text{ V}$. As the input voltage V_1 varies from -0.5 V to 0.85 V, the input voltage V_2 was set to -0.1 V, 0 V and 0.1 V, respectively. The measured results are shown in Fig. 3(a) and (b). According to Fig.

Fig. 4. Measured errors between the measured results and the theoretical values of the proposed differential-mode exponential voltage-to-current converter.

Fig. 5. Measured frequency response of the proposed differential-mode exponential voltage-to-current converter.

3(b), all the measured output currents exhibit a linear range of about 15 dB. Figure 4 shows the errors between the measured results and the theoretical values calculated by equation (9). For a 15 dB linear output range, the maximum measured errors for the different input considerations were 1.35% (for $V_2 = 0.1 \text{ V}$), 1.15% (for $V_2 = 0 \text{ V}$) and 1.16% (for $V_2 = -0.1 \text{ V}$), respectively. However, most the measured errors are less than 1%. Also the measured maximum power consumption is less than 0.96 mW (at $V_1 = 0.25 \text{ V}$, $V_2 = -0.1 \text{ V}$). The frequency response is shown in Fig. 5 which was measured under the conditions of the dc input voltage $V_1 = 0.3 \text{ V}$, ac input voltage $V_1 = 0.03 \text{ V}$ and the dc input voltage $V_2 = 0 \text{ V}$, respectively. For a 10 K Ω resistor load, the measured result shows that the corresponding -3 dB bandwidth can be 111.2 MHz.

4. Conclusion

In this paper, a CMOS differential-mode exponential voltage-to-current converter has been developed. Experimental results have been given to confirm the validity of the theoretical analysis. The proposed circuit is expected to be useful in the design of VGA and other analog signal processing applications.

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