Analysis of low frequency harmonics for continuous-conduction-mode boost power-factor correction

C.-S.Lin, T.-M.Chen and C.-L.Chen

Abstract: The input current harmonics of continuous-conduction-mode boost power-factor correction (CCMPFC) are presented. Input current distortion of single-phase CCMPFC is analysed and harmonic components are mathematically derived. Upper limits of the boost inductance for CCMPFC are also discussed. Design examples of CCMPFC are demonstrated. Experimental recordings verify the analytical derivations.

1 Introduction

Power-factor correction (PFC) has attracted significant research interest in recent years. To reduce input current harmonics in power supplies, PFC shapes the AC line current to be sinusoidal in phase with the line voltage. With this approach, electronic equipment produces less harmonics to pollute the power system and, hence, decreases the necessity of power generation. Passive PFCs are commonly used in low-cost applications, because they only require a couple of simple components. However, the corrected input power factors of passive PFCs are not higher than 0.95. For high-quality requirements, products need to produce less harmonics and their input power factors are usually higher than 0.97. In these cases, active PFCs are introduced by specialist power electronics with control circuitry [1]. These controllers sense the line current and calculate the pulse duty for the power switch to shape the line current so that it becomes almost purely sinusoidal. The power factors of these techniques are extremely high and often up to 0.99 [2]. The harmonics of the line current are much less than when using passive PFCs.

During the past decade, several active single-phase power-factor correction topologies have been presented in the literature [1, 2]. Among them, boost PFC is the most popular circuit because of low cost and high efficiency. The input inductor current of the PFCs can be controlled in continuous or discontinuous conduction mode [3]. Discontinuous conduction mode (DCM) is usually used only in small power supplies for good quality high-frequency current ripples [4]. Continuous conduction mode (CCM) is widely utilised in high quality, medium and high power applications [5–7]. A large enough input inductor may limit the current harmonics around the switching frequency yet, in the meanwhile, this large inductor may arouse significant low-frequency distortion [2, 8]. For this reason, the rela-

tionship between the input inductance and the low-frequency current distortion of the CCMPFC is quantitatively analysed. An upper bound of the boost inductance is imposed on the input inductance to comply with the current harmonics regulations.

In this paper, the input voltage and current of PFCs are normalised into a per-unit system. Under the standard per-unit system, the input line current of the PFC is analysed with its harmonic components in detail. Design examples of the CCMPFC demonstrate the upper limits of boost inductance. Finally, experimental results verify the theoretical derivations.

Fig. 1 Single-phase boost power-factor corrector

Fig. 2 Inductor-current waveform of the CCMPFC

2 Analysis of line current distortion

A single-phase boost PFC is depicted in Fig. 1. The inductor current waveform which operates in CCM is demonstrated in Fig. 2. The switch S in Fig. 1 is considered an ideal switch, and the bulk capacitor C is large enough to maintain the output voltage of the PFC as a constant. There are two major components of current distortions in this waveform compared with an ideal sinusoidal current waveform i_D . The first one is the low-frequency component. During the starting of each half-cycle ($0 < \omega t < \beta$),

© IEE, 2001

IEE Proceedings online no. 20010120

DOI: 10.1049/ip-cpa:20010120

Paper first received 20th January and in revised form 31st May 2000

The authors are with the Power Electronics Laboratory, Department of Electrical Engineering, National Taiwan University, Taipei, Taiwan 10617

the instantaneous value of the input voltage, v_s , is too small to force the inductor current to follow the sinusoidal reference. Until the time instant β , the inductor current gains enough variation rate, di/dt , to rejoin the demand current i_D . The second component is the high-frequency switching current ripples. In low-frequency analysis, it is assumed that the high-frequency ripples are easily absorbed by the line filters, therefore it can be neglected.

For convenience, the input voltage and line current can be normalised as a per-unit system as follows:

$$V_{base} = V_S, \quad I_{base} = \frac{V_S}{\omega L}, \quad Z_{base} = \omega L, \quad \omega t = \theta$$

Per-unit circuit parameters are located with a ‘~’ above the symbol.

The circuit parameters used in this paper are:

- a_n = n th sine coefficient of Fourier series
- b_n = n th cosine coefficient of Fourier series
- D = switching duty cycle
- f_S = switching frequency
- $\tilde{I}_D$ = RMS of per-unit demand line current
- $\tilde{i}_{n(RMS)}$ = RMS of n th order harmonic line current
- $\tilde{i}_S$ = per-unit instantaneous line current
- $\tilde{i}_{S(RMS)}$ = RMS of per-unit instantaneous line current
- PF = power factor
- P_s = average input power
- $\tilde{P}_s$ = average per-unit input power
- V_O = output voltage of PFC
- V_S = RMS input voltage
- $\tilde{v}_S$ = per-unit instantaneous input voltage
- $V_{S(min)}$ = minimum RMS input voltage
- $\tilde{v}_{S(RMS)}$ = RMS of per-unit instantaneous line voltage
- β = rejoined angle of line current and demand current
- ΔI = maximum peak-to-peak current ripple
- θ = instantaneous angle of input voltage or line current

In Fig. 2, the per-unit ideal sinusoidal demand current is

$$\tilde{i}_D = \sqrt{2}\tilde{I}_D \sin \theta \quad 0 \leq \theta \leq \pi \quad (1)$$

The per-unit input voltage and current waveforms can be expressed as follows:

$$\tilde{v}_S = \sqrt{2} \sin \theta \quad 0 \leq \theta \leq \pi \quad (2)$$

$$\tilde{i}_S = \sqrt{2}(1 - \cos \theta) \quad 0 \leq \theta \leq \beta \quad (3)$$

$$\tilde{i}_S = \sqrt{2}\tilde{I}_D \sin \theta \quad \beta \leq \theta \leq \pi \quad (4)$$

At angle $\theta = \beta$, $\tilde{i}_S$ in eqns. 3 and 4 rejoin, $\tilde{I}_D$ is given by

$$\tilde{I}_D = \tan \left(\frac{\beta}{2} \right) \quad (5)$$

This per-unit input current can be expressed by its harmonics with Fourier series representation:

$$\tilde{i}_S = \sqrt{2} \sum_{n=1}^{\infty} [a_n \sin(n\theta) + b_n \cos(n\theta)] \quad (6)$$

The fundamental components of the input current are [2]:

$$a_1 = \frac{\tan \frac{\beta}{2}}{\pi} (\pi + \sin \beta - \beta) \quad (7)$$

$$b_1 = \frac{1}{\pi} (\sin \beta - \beta) \quad (8)$$

The harmonic components can also be expressed in terms of β :

For even n , $a_{n(even)} = 0$ and $b_{n(even)} = 0$

For odd n , the harmonic components are

$$\begin{aligned} a_{n(odd)} = & -\frac{2 \tan \frac{\beta}{2}}{\pi} \left\{ \frac{\sin[(n-1)\beta]}{2(n-1)} - \frac{\sin[(n+1)\beta]}{2(n+1)} \right\} \\ & + \frac{2}{\pi} \left\{ \frac{\cos[(n-1)\beta] - 1}{2(n-1)} + \frac{\cos[(n+1)\beta] - 1}{2(n+1)} \right\} \\ & - \frac{2}{n\pi} \{ [\cos(n\beta)] - 1 \} \end{aligned} \quad (9)$$

$$\begin{aligned} b_{n(odd)} = & \frac{2 \tan \frac{\beta}{2}}{\pi} \\ & \times \left\{ \frac{\cos[(n-1)\beta] - 1}{2(n-1)} - \frac{\cos[(n+1)\beta] - 1}{2(n+1)} \right\} \\ & - \frac{2}{\pi} \left\{ \frac{\sin[(n-1)\beta]}{2(n-1)} + \frac{\sin[(n+1)\beta]}{2(n+1)} \right\} \\ & + \frac{2}{n\pi} \sin(n\beta) \end{aligned} \quad (10)$$

In eqns. 9 and 10, n must be an odd number and is equal to or larger than 3. The per-unit RMS of the n th order harmonic current is given by

$$\tilde{i}_{n(RMS)} = \sqrt{a_n^2 + b_n^2} \quad (11)$$

The average per-unit input power is given by

$$\tilde{P}_s = \frac{1}{\pi} \int_0^{\pi} \tilde{v}_S \tilde{i}_S d\theta = \frac{\tan \frac{\beta}{2}}{\pi} (\pi + \sin \beta - \beta) \quad (12)$$

A linear approximation for per-unit input power and β is

$$\tilde{P}_s \approx 0.5\beta \quad (13)$$

The relation between the per-unit power and actual power is

$$P_s = V_{base} I_{base} \tilde{P}_s = \frac{V_S^2}{\omega L} \tilde{P}_s \quad (14)$$

The power factor can be calculated by

$$PF = \frac{\tilde{P}_s}{\tilde{v}_{S(RMS)} \tilde{i}_{S(RMS)}} \quad (15)$$

where

$$\begin{aligned} \tilde{i}_{S(RMS)} = & \sqrt{\frac{2}{\pi} \left[\frac{3\beta}{2} - 2 \sin \beta + \frac{\sin(2\beta)}{4} + \left(\frac{\pi - \beta}{2} + \frac{\sin(2\beta)}{4} \right) \tan^2 \frac{\beta}{2} \right]} \end{aligned} \quad (16)$$

$$\tilde{v}_{S(RMS)} = 1 \quad (17)$$

From the above analysis, the rejoined angle β is an important design parameter to determine limits of input inductance for given harmonic constraints. Fig. 3 shows the curves of the n th harmonics and angle β .

To determine the minimum value of the inductance, the relation between the inductor current ripple and inductance was presented in [5]:

$$L_{min} = \frac{\sqrt{2}V_{S(min)}D}{\Delta I f_S} \quad (18)$$

where

$$D = \frac{V_o - \sqrt{2}V_{S(min)}}{V_o} \quad (19)$$

Take a 3000W CCM-PFC, for example: assume the output voltage of the PFC is 400V; the minimum input voltage of the PFC is 85VRMS; the switching frequency is 50kHz and the maximum switching current ripple is 5A. According to eqns. 18 and 19, the minimum value of the input inductance is 0.43mH. If the 3rd harmonic current cannot exceed 0.3A, the maximum boost inductance is 0.96mH. The inductance is valid from 0.43mH to 0.96mH. Two practical design examples of the CCM-PFC are given in the next Section.

Fig.3 Curves of the n th harmonics and β

3 Design examples

A 1000W CCM-PFC allows AC line voltage from 90~260V. Its output is 400VDC. The limits of the input harmonic currents are:

- power factor > 0.99
- 3rd harmonic < 2% of RMS current
- 5th harmonic < 1% of RMS current
- 7th harmonic < 0.5% of RMS current
- 9th harmonic < 0.35% of RMS current

According to Fig. 3, the harmonics are not linearly distributed with the angle β , so it requires a detail search to find the minimum inductance. A 3-D plot between the input power, input voltage and required β is shown in Fig. 4. The maximum boost inductances for CCM-PFC from 90~260V and 100~1000W are listed in Table 1. The plot

Fig.4 Curves of the input voltage, input power and β

between the input power, input voltage and required inductances is shown in Fig. 5. From Table 1 and Fig. 5, the minimum input inductance L is 2mH at 90VAC and 1000W. The switching frequency is selected at 50kHz to reduce the switching loss and the size of the boost inductor. From eqns. 15 and 16, the power factor of the PFC can be calculated. If the maximum switching current ripple is 1A, from eqns. 18 and 19, the lower limit of the inductance is 1.7mH. The inductance is valid from 1.7mH to 2mH for low-frequency and high-frequency harmonic limits.

Fig.5 Curves of the input voltage, input power and L

To demonstrate the distortion of the CCM-PFC, another 500W CCM-PFC allowing AC line voltage from 90~260V is also established. Its output is 400VDC. The limits of the input harmonic currents are:

- power factor > 0.99
- 3rd harmonic < 5% of RMS current
- 5th harmonic < 3% of RMS current
- 7th harmonic < 2% of RMS current
- 9th harmonic < 1% of RMS current

The maximum boost inductance L of the 500W CCM-PFC is 7.5mH at 90VAC and 500W. The switching frequency is also 50kHz. The inductance is valid from 1.7mH to 7.5mH.

4 Experimental results

Two single-phase CCM-PFCs are implemented for the current harmonics and distortion tests. The boost inductance of 1000W CCM-PFC is selected as 2mH. Measured RMS input harmonics and power factors are listed in Table 2. The boost inductance of 500W CCM-PFC is selected at

Fig.6 Simulated input-current waveform at 90VAC/500W

Table 1: Calculated boost inductances L (mH) at rated currents (A)

	100W	200W	300W	400W	500W	600W	700W	800W	900W	1000W
90V	17.31 1.11A	9.51 2.22A	6.45 3.33A	4.92 4.44A	4.01 5.56A	3.34 6.67A	2.86 7.78A	2.51 8.89A	2.23 10.00A	2.01 11.11A
100V	20.95 1.00A	11.53 2.00A	7.96 3.00A	6.08 4.00A	4.86 5.00A	4.12 6.00A	3.53 7.00A	3.09 8.00A	2.75 9.00A	2.48 10.00A
110V	24.85 0.91A	13.95 1.82A	9.64 2.73A	7.35 3.64A	5.89 4.55A	4.99 5.45A	4.28 6.36A	3.74 7.27A	3.33 8.18A	2.99 9.09A
120V	28.96 0.83A	16.60 1.67A	11.27 2.50A	8.60 3.33A	7.00 4.17A	5.84 5.00A	5.09 5.83A	4.45 6.67A	3.96 7.50A	3.56 8.33A
130V	33.28 0.77A	19.12 1.54A	13.22 2.31A	10.10 3.08A	8.22 3.85A	6.85 4.62A	5.97 5.38A	5.23 6.15A	4.65 6.92A	4.18 7.69A
140V	37.78 0.71A	22.18 1.43A	15.33 2.14A	11.71 2.86A	9.53 3.57A	7.94 4.29A	6.81 5.00A	6.06 5.71A	5.39 6.43A	4.85 7.14A
150V	42.43 0.67A	24.99 1.33A	17.29 2.00A	13.44 2.67A	10.75 3.33A	9.12 4.00A	7.82 4.67A	6.84 5.33A	6.18 6.00A	5.57 6.67A
160V	47.20 0.62A	27.89 1.25A	19.67 1.88A	15.02 2.50A	12.23 3.12A	10.37 3.75A	8.89 4.38A	7.78 5.00A	7.03 5.62A	6.33 6.25A
170V	52.08 0.59A	31.49 1.18A	22.21 1.76A	16.96 2.35A	13.81 2.94A	11.71 3.53A	10.04 4.12A	8.79 4.71A	7.81 5.29A	7.15 5.88A
180V	55.67 0.56A	34.62 1.11A	24.89 1.67A	19.01 2.22A	15.48 2.78A	12.90 3.33A	11.25 3.89A	9.85 4.44A	8.76 5.00A	8.01 5.56A
190V	60.52 0.53A	38.58 1.05A	27.23 1.58A	21.18 2.11A	17.25 2.63A	14.38 3.16A	12.54 3.68A	10.97 4.21A	9.76 4.74A	8.78 5.26A
200V	65.39 0.50A	41.91 1.00A	30.17 1.50A	23.05 2.00A	18.78 2.50A	15.93 3.00A	13.89 3.50A	12.16 4.00A	10.81 4.50A	9.73 5.00A
210V	68.40 0.48A	46.20 0.95A	33.27 1.43A	25.42 1.90A	20.70 2.38A	17.56 2.86A	15.06 3.33A	13.40 3.81A	11.92 4.29A	10.73 4.76A
220V	73.05 0.45A	49.69 0.91A	35.83 1.36A	27.89 1.82A	22.72 2.27A	19.27 2.73A	16.52 3.18A	14.71 3.64A	13.08 4.09A	11.77 4.55A
230V	77.63 0.43A	54.31 0.87A	39.17 1.30A	30.48 1.74A	24.83 2.17A	21.06 2.61A	18.06 3.04A	16.08 3.48A	14.29 3.91A	12.86 4.35A
240V	79.71 0.42A	57.93 0.83A	41.84 1.25A	33.19 1.67A	27.04 2.08A	22.54 2.50A	19.66 2.92A	17.21 3.33A	15.56 3.75A	14.01 4.17A
250V	83.88 0.40A	62.86 0.80A	45.40 1.20A	35.36 1.60A	28.82 2.00A	24.45 2.40A	21.33 2.80A	18.67 3.20A	16.88 3.60A	15.20 4.00A
260V	85.08 0.38A	66.57 0.77A	49.10 1.15A	38.25 1.54A	31.17 1.92A	26.45 2.31A	23.07 2.69A	20.19 3.08A	18.26 3.46A	16.44 3.85A

Table 2: Measured harmonics and power factor for 1000W CCMFPC

	Limits	Measured	Limits	Measured
V_{in} , V	–	87.8	–	266.7
F_s , Hz	–	60	–	60
I_{in} , A	–	12.2	–	4.3
P_{in} , W	–	1071	–	1147
PF	0.99	0.999	0.99	0.99
THD (%)	–	3.6	–	7.4
Harmonic 2, A	–	0.024	–	0.009
Harmonic 3, A	0.244	0.240	0.086	0.069
Harmonic 4, A	–	0.017	–	0.006
Harmonic 5, A	0.122	0.049	0.043	0.039
Harmonic 6, A	–	0.004	–	0.001
Harmonic 7, A	0.061	0.030	0.022	0.016
Harmonic 8, A	–	0.001	–	0.001
Harmonic 9, A	0.043	0.030	0.015	0.010

7mH to illustrate the input current distortion. The computer simulated input current waveform of the CCMFPC at 90VAC before the tests, is shown in Fig. 6. The measured

input current waveform of the CCMFPC is shown in Fig. 7 to be the same as the simulated waveform. Measured RMS input harmonic currents and power factors are also listed in Table 3. The harmonics all comply with the limits.

Fig. 7 Measured input-current waveform at 90VAC/500W

Table 3: Measured harmonics and power factor for 500W CCMPFC

	Limits	Measured	Limits	Measured
V_{inv} V	–	90.7	–	265.6
F , Hz	–	60	–	60
I_{inv} A	–	5.95	–	2.2
P_{inv} W	–	534	–	584
PF	0.99	0.996	0.99	0.99
THD (%)	–	4.3	–	9.6
Harmonic 2, A	–	0.005	–	0.009
Harmonic 3, A	0.298	0.087	0.11	0.12
Harmonic 4, A	–	0.005	–	0.002
Harmonic 5, A	0.179	0.074	0.066	0.036
Harmonic 6, A	–	0.002	–	0.003
Harmonic 7, A	0.119	0.064	0.044	0.033
Harmonic 8, A	–	0.001	–	0.001
Harmonic 9, A	0.059	0.052	0.011	0.005

5 Conclusions

Single-phase CCMPFC for low-frequency input current distortion is presented. RMS input current harmonics are analysed in detail. Two CCMPFCs have been implemented

for current harmonics and distortion tests. Experimental recordings have verified the theoretical derivations. The upper limits of input inductance of boost PFC are calculated under the constraints of harmonic currents. This is useful in high-quality power-supply design.

6 References

- 1 BOURGEOIS, J.M.: 'Circuits for power factor correction with regards to mains filtering'. SGS-Thomson application note, AN510, pp.1–9
- 2 SALMON, J.C.: 'Techniques for minimizing the input current distortion of current-controlled single-phase boost rectifiers', *IEEE Trans. Power Electron.*, 1993, **8**, (4), pp. 509–520
- 3 SEBASTIAN, J., JAUREGUIZAR, M., and UCEDA, J.: 'An overview of power factor correction in single-phase off-line power supply system'. Proceedings of the IEEE Industrial Electronics conference, 1994, pp. 1688–1693
- 4 ANDREYCAK, B.: 'Controlled on-time, zero current switched power factor correction technique'. Unitrode Switching Regulated Power Supply Design Seminar Manual, 1993, pp.12–1–12–11
- 5 DIXON, L.: 'High power factor switching preregulator design optimization'. Unitrode Switching Regulated Power Supply Design Seminar Manual, 1996, I3–1–I3–12
- 6 TODD, R. C.: 'UC3854 controlled power factor correction circuit design'. Unitrode application note, U134, pp.3–269–3–289
- 7 COMANDATORE, G., and MORICONI, U.: 'Designing a high power factor switching preregulator with the L4981 continuous mode'. SGS-Thomson application note, AN628, pp.1–23
- 8 ZHOU, C., RIDLEY, R.B., and LEE, F.C.: 'Design and analysis of a hysteretic boost power factor correction circuit', *IEEE Power Electron. Spec. Conf. Rec.*, 1990, pp. 800–807