Steady-State Analysis and Simulation of a BJT Self-Oscillating ZVS-CV Ballast Driven by a Saturable Transformer

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Abstract—The steady-state oscillation of a zero-voltage-switching clamped-voltage self-oscillating ballast driven by a saturable transformer is analyzed and simulated. Its self-oscillating operation is divided into six stages according to the hysteresis B–H loop of saturable transformer cores. Stage-wise circuit analysis shows the saturable transformer limits the lamp current and dominates the switching frequency of the ballast. The saturating behavior of driving and switching devices clearly distinguishes this self-oscillating inverter from an external-drive high-quality-factor resonant inverter. Analytical results are verified by mathematical simulation and laboratory experiment.

Index Terms—Bipolar transistor oscillators, circuit analysis, circuit simulation, resonant power conversion, RLC circuits.

I. INTRODUCTION

A DISCHARGE fluorescent lamp normally exhibits negative resistance characteristics in its operation region. Therefore, it can be roughly operated under open-loop control. If it is directly connected to a voltage source high enough to produce ionization, the discharge will cause overcurrent. Hence, it requires a current-limiting device, the ballast. Filaments could be employed, but the ballast efficiency is seriously degraded. Traditionally, leakage transformers or choke coils are utilized as ballasts. However, their size and weight are considerable and efficiency still deserves improvement.

In recent years, electronic ballasts have attracted much research interest [1]–[15] due to the merits of high-frequency ballasting, such as high efficacy, low audible noise, small volume, and light weight. At present, push–pull [4], [8]–[11] or half-bridge [5]–[7], [12] series–parallel resonant inverters have been employed for electronic ballasts. Nevertheless, based on the considerations of cost and reliability, most off-line electronic ballasts adopt self-oscillating half-bridge inverters. A thorough analysis for this ballasting circuit is, indeed, timely and relevant.

In the literature, most of the topics of electronic ballasts are aimed at the circuit topology and the loaded-resonant characteristics of inverters [16]–[20]. The design of ballasts always relies upon the specific characteristics of discharge lamps, as well as the selected switching frequency. Presently, the fundamental approximation [5]–[7] has been adopted for the analysis and design of resonant ballasts. It should be noted that the approximation holds only under high quality factors, linear loads, and near-resonant frequency. For fluorescent lamps, the discharge is nonlinearity and varies with the lifetime and ambient temperature of lamps. Therefore, the value of loaded quality factor \((Q)\) is not constant and the output of a high-\(Q\) resonant ballast may be easily changed by operating condition.

In this paper, an off-line low-\(Q\) self-oscillating half-bridge ballast driven by a series saturable transformer is analyzed and simulated. Mathematical equations are derived under steady state, since the equivalent resistance of fluorescent lamps dramatically changes during the startup moment. The dc input voltage of load-resonant networks (75 or 150 V) is always higher than the steady-state operating voltage of lamps (55–110 V). Depending on the circuit design, its oscillation frequency is usually around tens of kilohertz and can be increased by shortening the storage time of bipolar junction transistors (BJT’s).

II. CIRCUIT DESCRIPTION

Fig. 1(a) shows the circuit scheme of a half-bridge inverter. Under inductive loads, it features the function of zero-voltage-switching and clamped-voltage (ZVS-CV). Namely, the reverse-recovery problem of freewheeling diodes can be avoided and the turn-off loss of transistors can be reduced by two transition capacitors. Basically, two transition capacitors are in parallel when both transistors are off. They can be merged into one capacitor and placed in parallel to either one transistors. Similarly, two dc capacitors are in parallel when one switch conducts; they also can be merged into one capacitor and connected to either rail of the dc bus. Fig. 1(b)
shows the circuit alternatives with dot lines [19]. To obtain the ZVS-CV operation, the series–parallel resonant network \((L_p, C_{ig}, R_{\text{lamp}}, C_s)\) should be inductive. In Fig. 1(b), the steady-state voltage of the lamp may be higher or lower than the fundamental voltage across the series–parallel resonant network. As the latter is higher than the former, then “parallel resonance” is the required resonant mode of the series–parallel network. The parallel-loaded ballasts considered in [5] and [6] belong to this category. For ZVS operations, the switching frequency of the half-bridge inverter should be higher than the resonant frequency that forms the boundary between capacitive and inductive loads. By fundamental approximation, this boundary frequency is \(f_r = f_p(1 - 1/Q_p^{1/2}), Q_p = R_{\text{lamp}}(C_{ig}/L)^{1/2}, f_p = 1/2\pi(LC_{ig})^{1/2}\).

Fig. 2(a) depicts the frequency responses of the series–parallel network when the series capacitor \((C_s)\) is large enough and can be ignored. The parallel-loaded quality factor \((Q_p)\) and the selected switching frequency codetermine the voltage gain of parallel resonance. For parallel-loaded resonant ballasts [5], [6], the value of \(Q_p\) is mostly around 1.3. However, it is not a constant, since the lamp resistance \((R_{\text{lamp}})\) may vary with the lifetime and temperature of lamps. As the value of \(Q_p\) is equal to or less than unity, the “parallel resonance” disappears and the series–parallel network becomes a purely inductive load. In other words, when the resonance effect of the ignition capacitor \((C_{ig})\) fades away, the steady-state lamp-tube voltage will be lower than the fundamental voltage across the series–parallel network.

The ballast considered in this article is to be operated from a dc source (150 or 300 V). The fundamental voltage across the series–parallel resonant network (about \(300/\pi\) or \(600/\pi\) V) is higher than the operating voltage of fluorescent lamps (about 55–110 V). In order to improve the current crest factor and reduce the size of the series capacitor, “series resonance” is the principal resonant mode of the series–parallel resonant network and the series-loaded quality factor \((Q_s)\), \(Q_s = (L/C_s)^{1/2}/R_{\text{lamp}}\), is normally less than unity for practical application. Fig. 2(b) depicts the frequency responses of the series–parallel network when the effect of the ignition capacitor on the resonant current is small enough and can be ignored. It shows that the low-\(Q\) series resonance behaves with wide-bandwidth characteristics and its voltage gain is insensitive to the frequency variation. Therefore, the low-cost self-oscillating inverter can suit this ballast application. Currently, based on the consideration of cost, most off-line ballasts adopt the voltage-driven half-bridge self-oscillating inverters.

Fig. 3 displays the off-line half-bridge self-oscillating ballast studied in this article. It can be divided into five parts according to specific functions. For 220-V ac line source, Box I may be a full-wave rectifier and the obtained dc-bus voltage around 300 V. However, for a 110-V ac line source, Box I may be a full-wave rectifier or a half-wave voltage doubler, depending on the required operating voltage of fluorescent lamps. If Box I includes a boost-type power-factor-correction circuit, the dc-bus voltage is usually about 300 V. Box II is a widely used starting circuit. It consists of a DIAC (diode ac switch, \(D_{ac}\)), a discharging diode \((D_{q})\), and an RC charging circuit \((C_{dt} and R_{\text{dt}})\). Box III is a ZVS-CV half-bridge inverter using BJTs as switching devices. Its driving circuit is shown in Box IV. Feedback signals are derived from the resonant path via a three-winding \((N_p : N_{q1} : N_{q2} = n : 1 : 1)\) saturable transformer \((T_{r})\) [21]–[24]. Finally, Box V is the series–parallel load-resonant network \((L_p, C_s, C_{ig}, and a discharge fluorescent lamp)\) mentioned herein before.

At the startup moment, the lamp tube is regarded as open circuitry. The series-resonant current through the ignition capacitor heats the two filaments and the voltage across the ignition capacitor ignites the lamp. In the process of startup, the filament resistance rises and the lamp-tube resistance drops simultaneously. Usually, the filament resistance under normal operation is about fourfold of under room temperature, and the required ignition voltage of fluorescent lamps decreases with the rise of filament temperature. It should be noted that improper emission temperature normally shortens the lifetime of filaments and, hence, the startup current of filaments should be limited to a certain range. At the steady state, the current through the ignition capacitor reduces the current harmonics through the lamp and keeps the temperature of the filaments. For the studied ballast, the effect of ignition capacitance on the self-oscillating frequency is small and can be neglected.

Basically, owing to the Lenz’s law, the switching operation of the transformer-driven self-oscillating inverter takes place naturally when the resonant current drops from its maximum point. Hence, the self-oscillating switching frequency is always higher than the natural frequency of the series–parallel load-resonant network and the ZVS-CV operation can be naturally obtained. No matter the series-connected driving transformer is saturable or nonsaturable, the self-oscillating inverter can oscillate if and only if the Barkhausen’s criterion for oscillation is satisfied in stages 1 and 4(b). Since the iron core of the driving transformer is saturated, the self-oscillating inverter loses its driving signals when the driving transformer is
saturated by resonant current. In this way, both the inductor current and the self-oscillating frequency are limited by the volt-time value of the saturable transformer. Another advantage of the use of the series driving transformer is the “current feedback.” The base–drive (feedback) current can naturally lead the collector–load (resonant) current and directly proportional to the collector–load (resonant) current. No matter how the collector–load current changes, the Barkhausen’s criterion can be easily satisfied in stages 1 and 4(b). Namely, the characteristics of current feedback facilitates the startup operation of the self-oscillating inverter.

As the driving signals are derived from a nonsaturable transformer inserted in the resonant path, the inverter loses the merits of current feedback and the current-limit function. Particularly during the startup moment, the base–emitter junction of transistors may be damaged by excessive driving voltage resulting from the nonsaturable driving transformer. Based on the above, the studied BJT half-bridge self-oscillating ballast usually employs a series-connected saturable transformer (current transformer) as its driving device.

III. SELF-OSCILLATING OPERATION

In view of the square hysteresis of saturable cores, the piecewise linear modeling can be adopted for the saturable driving transformer ($T_r$). Fig. 4 depicts the B–H loop of saturable cores. Along the variation of transformer operating points, a complete self-oscillating operation of the ballast is divided into six stages. Six stage boundaries ($t_0$–$t_6$) are specified in the breakpoints of the B–H loop. Under inductive loads, the six stages of self-oscillating operation are described as follows.

Stage 1: $t_0 < t < t_1$ ($T_r$ unsaturated, $Q_2$ conducting).

At $t_0$, the operating point of $T_r$ leaves its saturation region and turns $Q_2$ on. Until the inductive
resonant current reverses, the current of $D_2$ starts to transfer to $Q_2$.

Stage 2: $t_1 < t < t_2$ ($T_r$ saturated, $Q_2$ conducting).

At $t_1$, the operating point of $T_r$ enters its saturation region. No energy is transferred to $Q_2$ base from $T_r$, but the stored charges of $Q_2$ base keep $Q_2$ conducting. Until $t_2$, the stored excess charges disappear and the operating point of $Q_2$ enters its active region.

Stage 3: $t_2 < t < t_3$ ($T_r$ saturated, $Q_2$ turning off).

The operating point of $Q_2$ traverses through its active region. As $Q_2$ gradually turns off, $C_Z$ is discharged by resonant current and its voltage starts to drop. At $t_3$, the voltage of $C_Z$ drops to zero and then $D_2$ starts to conduct.

Stage 4: $t_3 < t < t_4$ ($T_r$ unsaturated, $Q_1$ conducting).

At $t_3$, $T_r$ leaves its saturation region and saturates $Q_1$. As the current of $D_1$ falls to zero, the resonant current naturally transfers to $Q_1$. At $t_4$, $T_r$ comes into its saturation region again.

Stage 5: $t_4 < t < t_5$ ($T_r$ saturated, $Q_1$ conducting).

After $t_4$, no energy is transferred to $Q_1$ base from $T_r$. The excess charges stored in $Q_1$ base keeps $Q_1$ conducting.

Stage 6: $t_5 < t < t_6$ ($T_r$ saturated, $Q_1$ turning off).

Transistor $Q_1$ enters its active region and gradually turns off. The voltage of $C_Z$ increases. At $t_6$, the voltage of $C_Z$ rises to dc-bus voltage and $D_2$ starts to conduct.

Fig. 5 describes the corresponding component waveforms of the six stages. Because the resonant current commutates and flows through different switching devices in stages 1 and 4, both stages can be, respectively, redivided into two substages and the commutating points are designated as their boundaries ($t_{1a}$ and $t_{4a}$). To satisfy the Barkhausen’s criteria for oscillation, transistor $Q_2$ ($Q_1$) should be saturated in stage 1a (4a). From Fig. 5, it can be seen that the turn-off loss of transistors. The transformer voltage $V_T$, naturally leads the resonant inductor current $i_L$. The current of diode $D_2$($D_1$) is naturally commutated by the antiparallel transistor $Q_2$($Q_1$). Each transistor is turned on at zero voltage and each diode turns off at zero current. There is no reverse-recovery problem in the diodes ($D_2$, $D_1$) and the switch voltage $V_{C_Z}$ is clamped at the dc input voltage. That is, this transformer-driven self-oscillating inverter obtains the ZVS-CV operation naturally.

**IV. ANALYSIS AND SIMULATION**

Due to the nonlinear behavior of the power transistors, the discharge lamp, and the saturable transformer, it is difficult to exactly analyze the steady-state oscillation of the ballast. In order to reduce the analytical complexity, some simplifications are adopted in this analysis. First, based on the fact that the effect of ignition capacitance on the resonant current is small and the current harmonics arisen from the nonlinearity of discharge lamps are bypassed by the ignition capacitor and utilized for heating the two filaments, therefore, the lamp tube and the ignition capacitor can be globally viewed as an equivalent resistive component. Secondly, in view of the inductance of the driving transformer is normally less than one-tenth of the inductance of the series inductor, the series-connected driving transformer can be viewed as a current transformer driven by sinusoidal resonant current. Thirdly, the loss of iron cores and the junction capacitance of bipolar junction transistors are neglected because their effects on the self-oscillating inverter are very small. Fourthly, the employed large-signal parameters, such as the value of inductance, resistance, capacitance, minority carrier lifetime, and transit time, are all assumed nearly constant.

**A. Series–Parallel Resonant Network**

Fig. 6 illustrates the equivalent circuit model of the self-oscillating ballast in a half cycle ($t_0$–$t_3$). Based on the first simplifications, the series–parallel load-resonant network is roughly viewed as a second-order $RLC$ series-resonant circuit. And, the load effects of reflected resistance $R_L$ and magnetizing inductance $L_m$ of the saturable transformer are considered in stage 1a (4a) and 1b (4b), respectively.

For the off-line ballast, the series-loaded quality factor ($Q_s$) is normally less than unity but greater than 0.5, $Q_s = \frac{\omega_s}{2\alpha}$, $\alpha = R/2L$, $\omega_s = 1/(LC)^{1/2}$. Hence, the damping
factor $\alpha$ significantly affects the circuit characteristics. Theoretically, the resonance may be underdamped ($Q_s > 0.5$), overdamped ($Q_s < 0.5$), or critically damped ($Q_s = 0.5$). Except for the critical case, the inductor current of each stage with initial current $i_{L,0}$ and capacitor voltage $u_{C,0}$ can be written as

$$i_L(t) = \frac{V_i - u_{C,0} + i_{L,0}s_1L}{(s_1 - s_2)L} \exp(s_1t) - \frac{V_i - u_{C,0} + i_{L,0}s_2L}{(s_1 - s_2)L} \exp(s_2t)$$

$$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}. \quad (1)$$

Practically, the ballast is mostly designed as underdamped to improve the lamp current crest factor. From (1), the inductor current of each stage can be written as

$$i_L = \left(\frac{V_i - u_{C,0} - i_{L,0}\alpha L}{\omega_d L}\sin \omega_dt + i_{L,0} \cos \omega_dt\right) \cdot \exp(-\alpha t) \quad (2)$$

where $\omega_d$ stands for the damped natural radian frequency,

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}.$$

In each stage, the voltage of the series capacitor derived from the integration of (2) is

$$u_C(t) = \left(\frac{i_{L,0} + (u_{C,0} - V_i)\alpha C}{\omega_d C}\sin \omega_dt + (u_{C,0} - V_i) \cos \omega_dt\right) \exp(-\alpha t) + V_i. \quad (3)$$

The simplified RLC parameter values of each stage are listed in Table I. The table shows that the dc input voltage source ($V_i$) is effectively connected to the ballast circuit only in stages 1 and 2. Hence, if the onset time of each stage is zero, the energy sent into the ballast during a complete switching cycle, $E_{SW}$, can be expressed as

$$E_{SW} = \int_{t_0}^{t_0 + T_{SW}} i_L(t) \, dt \quad (4a)$$

where the resonant current $i_{L,1a}(t)$ charges the dc input voltage source. The practical power consumed in the lamp can be expressed by the energy variation of the series capacitor.

$$P_{lamp} = \frac{1}{2} \frac{V_i}{C_s} \left[ (\max V_{C,1a}(t))^2 - (\min V_{C,1a}(t))^2 \right] / T_{SW}. \quad (4b)$$

In stages 3 and 6, the transition capacitor $C_z$ takes part in the series-resonant operation since it is discharged in stage 3 and charged in stage 6 by inductor current. Fig. 5 shows that the inductor current changes slightly to make the switch voltage of $C_z$ rises to dc input voltage linearly. That is, the transition capacitor should be small enough and, hence, the equivalent series capacitance of stages 3 and 6 is dominated by the value of $C_z$. Notice that improper transition capacitance may destroy the zero-voltage turn-off operation of transistors. Usually, ignoring the current of the input voltage source, the transition capacitance can be approximately expressed as

$$C_z = \frac{t_3 - t_2}{t_2} i_L, 2(t_2 - t_1). \quad (5)$$

### B. Saturable Driving Circuit

The volt-time value of the saturable driving transformer can be roughly written as

$$VT = N_p B_{sat} A_{eff} = L_m [i_{Lm}(t) - i_{Lm}(t_{1a})] \quad (6a)$$

$$i_{Lm}(t') = \int_{t_{1a}}^{t'} i_L(t') \exp \left(\frac{-R_d t}{L_m} t\right) \, dt, \quad \epsilon = t - t_{1a} \quad (6b)$$

where $B_{sat}$ and $A_{eff}$ stand for the saturation flux density and the effective area of saturable iron cores, respectively.
Based on (2) and (6), the base current of transistor \(Q_2\) can be described as

\[
i_B(t') = n[i_L(t') - i_m(t')], \quad t' = t - t_{1a}.
\]  

(7)

Owing to the ZVS operation, the quasi-static approximation for charge-control analysis [25]–[28] is adopted. Theoretically, the base–drive current \(i_B\) can be divided into two parts, \(i_B = i_{BS} + i_{BX}\). The first part \((i_{BS})\) supplies the basic base charge \((Q_{BS})\) which makes the operating point of transistor \(Q_2\) locate at the edge of its saturation region. The second part \((i_{BX})\) provides the excess base charge \((Q_{BX})\) which makes the operating point of transistor \(Q_2\) enter its hard-saturation region. Based on the quasi-static approximation, the base–drive current can be rewritten as

\[
i_B(t') = \frac{Q_{BS}(t' - t_{1a})}{\tau_B} + \frac{dQ_{BS}(t')}{dt'} + \frac{Q_{BX}(t')}{\tau_S} + \frac{dQ_{BX}(t')}{dt'}, \quad t' = t - t_{0}.
\]  

(8)

where \(Q_{BS}(t') = i_B(t')\tau_T\), \(\tau_T\) is the mean transit time of excess minority carriers, \(\tau_B\) is the mean lifetime of \(Q_{BS}\), \(\tau_S\) is the lifetime of \(Q_{BX}\), and the ratio of \(\tau_B\) to \(\tau_T\) is the common emitter dc current gain, \(h_{FE} = \tau_B/\tau_T\).

In stage 1b or 2, if the maximum resonant current occurs at time \(t_p\), then the current flowing through transistor \(Q_2\) can be roughly expressed by the current slope \(k_p\) derived from (2)

\[
i_{L, u}(t') = \frac{V_i}{L_{r/c}} \exp(-\alpha_{1b} t_p) \left[ \frac{\ln(s_{2, u}/s_{1, 1b})}{(s_{1, 1b} - s_{2, 1b})} \right].
\]  

(9a)

Based on (9a), the first part of the base–drive current can be rewritten as

\[
i_{BS}(t') = \frac{k}{h_{FE}} t' + k\tau_T = i_B(t') - i_{BS}(t'), \quad t' = t - t_{1a}.
\]  

(9b)

From (8) and (9b), the excess charge \(Q_{BX}\) accumulated in stage 1b can be derived as

\[
Q_{BX, u}(t') = \left( \tau_S I_{BF} + \frac{\tau_S^2 k}{h_{FE}} - \tau_S k t_T \right) \left[ 1 - \exp \left( -\frac{t'}{\tau_S} \right) \right] - \frac{\tau_S k}{h_{FE}} t', \quad t' = t - t_{1a}.
\]  

(10)

where \(I_{BF}\) is the mean value of the forward base–drive current \(i_B(t')\). If the mean value of the primary voltage of the driving transformer equals \(V_{NP}\) volts, the mean base–drive current \(I_{BF}\) can be roughly written as \(I_{BF} = V_{NP}/(nR_{D2})\).

When the driving transformer saturates, the forward base current \(I_{BF}\) quickly disappears and a reverse base current \(I_{BR}\) is naturally formed by the saturation voltage \(E_{BES}\) of base-to-emitter junction, \(I_{BR} = E_{BES}/R_{d2}\). \(E_{BES} \approx 0.75\) V. Similarly to the derivation of (10), the variation of the excess charge during stage 2 can be derived as

\[
Q_{BX, 2}(t') = Q_{BX, 1}(t) \exp \left( -\frac{t'}{\tau_S} \right) - \left( \frac{\tau_S^2 k}{h_{FE}} + \tau_S k t_T \right) \left[ 1 - \exp \left( -\frac{t'}{\tau_S} \right) \right] - \frac{\tau_S k}{h_{FE}} t', \quad t' = t - t_{1a}.
\]  

(11)

When the excess charge \(Q_{BX, 2}\) decays to zero, then transistor \(Q_2\) enters its active region and turns off. The interval \(t_{1a} - t_{2a}\) is commonly referred to as inductive storage time, since the collector–load current is inductive. The above analysis shows that the interval \(t_{1a} - t_{2a}\) significantly varies with the base–drive current, collector–load current, minority carrier transit time, and lifetime. For practical applications, the measurement of minority-carrier transit time and lifetime is troublesome, because exact values should be measured under practical working environments [29]–[31].

To simply simulate the storage time from the circuit model, the base of transistors is modeled as a lumped \(RC\) charging circuit with specific driving conditions. Since transistor \(Q_2(Q_1)\) is saturated in stage 1 (4), the base storage capacitance is the sum of base-to-collector and base-to-emitter capacitance, \(C_B = C_{be} + C_{bc}\). By an impedance analyzer (e.g., HP4194A), this base storage capacitance can be directly measured. The base-to-emitter resistance \(r_{be}\) can be obtained by measuring the cut-in voltage \(E_{be}\) and the corresponding current of the base-to-emitter junction.

As shown in Fig. 6, the series driving transformer injects feedback currents into the base of \(Q_2\). The charges stored in the base storage capacitance \(C_B\) can be directly derived as

\[
Q_{B, u}(t') = \frac{B}{(A - \alpha_{ul})^2 + \omega_{ul}^2} \left[ \left( A - \alpha_{ul} \right) \sin \omega_{ul} t' - \omega_{ul} \cos \omega_{ul} t' \right] \left[ \exp(-\alpha_{ul} t') + \frac{R_{d1} L}{(A - \alpha_{ul}^2 + \omega_{ul}^2)} \right] + \frac{R_{d2}}{C_t} \left[ 1 - \exp(-A t') \right] + C_t E_{be} \exp(-A t') t' = t - t_{1a}.
\]  

(12)

\[
A = \frac{1}{r_{be} L}, \quad B = \frac{k_p}{\sqrt{2\pi M/T}} \exp(-\omega_{ul} t_{1a})
\]

where \(k_p\) is the mean current ratio of feedback current to resonant. Here, the \(k_p\) is estimated by the fundamental approximation and \(T\) is the time interval before the driving transformer saturates, \(T = V_T/V_{NP} = t_1 - t_{1a}\).
In stage 2, the driving transformer saturates and the reverse base current is \( I_{BR} = E_{BES}/R_{ds} \). Similarly to (12), the base charge of stage 2 can be directly derived as

\[
Q_{B,2}(t') = [Q_{B,1}(t_1 - t_{1a}) - C_{1e}E_{be} + I_{BR}R_{be}C_{b}] \\
\quad \cdot \left[ 1 - \exp \left( -\frac{t'}{R_{be}C_{b}} \right) \right] + C_{1e}E_{be}(t_1),
\]

where the value of transistor current gain \( h_{FE} \) should be measured under the edge of saturation region and the practical dc-bus voltage, since it normally changes with temperature, load current, and collector-to-emitter voltage.

At the end of stage 2, the operating point of transistor \( Q_2 \) enters its active region. That is,

\[
h_{FE}i_{b,2}(t_2 - t_1) = i_{L,2}(t_2 - t_1)
\]

where the value of transistor current gain \( h_{FE} \) should be measured under the edge of saturation region and the practical dc-bus voltage, since it normally changes with temperature, load current, and collector-to-emitter voltage.

In this way, similar analytical procedures can be performed in the next half cycle because the circuit operation of the inverter is symmetrical. According to the derived equations, a simulation program is developed in MATLAB. The initial conditions and the ending condition of each stage for this simulation are summarized in Table II. For simplification, the numerical simulation starts at stage 1b. Basically, the initial capacitor voltage of stage 1b should be equal to its final value in stage 1a. It should be given in advance to proceed with the circuit analysis. However, due to the nonlinearity, it is almost impossible to directly solve for \( v_{C,0} \) from the derived equations.

A simplified flow chart depicting the numerical simulation procedure is shown in Fig. 7. Initially, a guess of \( v_{C,0} \) is made.
and the derived equations are solved numerically. At the end of stage 6, $C_s$ voltage is compared with $V_{O,0}$. If the mismatch, $\varepsilon$, is beyond tolerance, $\varepsilon > 0.2$, the difference between these two values times a scaling factor $\delta$ is added to $V_{O,0}$ as the new guess. $\delta$ is 0.5 in this simulation. Until the mismatch is within the tolerance, the computed resonant current and $C_s$ voltage are true values. However, according to (4b), if the calculated lamp power cannot satisfy the desired lamp power, the circuit parameter, such as $L_r$, or $C_s$, should be changed. Based on this simulation program, proper circuit parameters can be obtained.

V. EXPERIMENTAL VERIFICATION

To verify the accuracy of the analysis, the numerical simulation is performed and results are compared with experiments. The fluorescent lamp is an 18-W PL-lamp, the lamp current of which is 0.32 A. The used saturable core is TDK H5B2 T5-10-2.5, $A_{ef} = 6.25 \text{ mm}^2$, and $B_{sat} = 4000 \text{ G}$ at 30°C. The employed switching transistors are MJE-13005. $E_{be} = 0.55 \text{ V}$, $I_{be} = 3.55 \text{ mA}$, $C_g = 10 \text{ nF}$. Under 300-V dc bus, the measured current gain $h_{FE}$ is 100. Generally speaking, to obtain the desired feedback base-drive current, the value of turns ratio $n$ may be greater than unity or less than unity. If the turns ratio is less than unity, the saturable core saturates quickly and then self-oscillating frequency increases. In the following circuit design, the turns ratio $n$ is greater than unity. By the simulation program, the values of circuit parameters can be regulated to obtain the desired lamp power.

In this experimental 300-V dc-bus ballast, the used circuit parameters are as follows: $L_r = 2.6 \text{ mH}$, $L_m = 150 \mu\text{H}$, $C_s = 0.1 \mu\text{F}$, $C_{ig} = 3.3 \text{ nF}$, $C_z = 2.2 \text{ nF}$, $N_p = 8$, $N_{s1} = N_{s2} = 4$, and $R_{d1} = R_{d2} = 47 \Omega$.

Fig. 8(a)–(f) verifies the effects of the ignition capacitor on circuit characteristics by changing the ignition capacitance from 3.3 to 20 nF. Fig. 8(a) and (b) shows that the effects of ignition capacitance on the switching frequency and resonant current are very small. Fig. 8(c) and (d) displays the nonlinearity of lamps. Fig. 8(e) and (f) illustrates that the
**TABLE II**

<table>
<thead>
<tr>
<th>Stage</th>
<th>Initial ( i_{L,0} )</th>
<th>Initial ( V_{C,0} )</th>
<th>Ending condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>( i_{L,1a}(t=0) )</td>
<td>( V_{C,1a}(t=0) )</td>
<td>( i_{L,1a}(t=0) = 0 )</td>
</tr>
<tr>
<td>1b</td>
<td>0</td>
<td>( V_{C,1a}(t=1a) )</td>
<td>( V_{T} = L_{a}[i_{L}(t)-i_{L}(t_{a})] )</td>
</tr>
<tr>
<td>2</td>
<td>( i_{L,2b}(t=2b) )</td>
<td>( V_{C,2b}(t=2b) )</td>
<td>( h_{T}L_{a}(t_{a}) = i_{L,2b}(t_{a}) )</td>
</tr>
<tr>
<td>3</td>
<td>( i_{L,3c}(t=3c) )</td>
<td>( V_{C,3c}(t=3c) )</td>
<td>( \frac{V_{i}}{C_{a}} \int_{0}^{t_{a}} i_{L}(t) , dt = V_{i} = 0 )</td>
</tr>
<tr>
<td>4a</td>
<td>( i_{L,4a}(t=4a) )</td>
<td>( V_{C,4a}(t=4a) )</td>
<td>( i_{L,4a}(t=4a) = 0 )</td>
</tr>
<tr>
<td>4b</td>
<td>0</td>
<td>( V_{C,4a}(t=4a) )</td>
<td>( V_{T} = L_{a}[i_{L}(t)-i_{L}(t_{a})] )</td>
</tr>
<tr>
<td>5</td>
<td>( i_{L,5b}(t=5b) )</td>
<td>( V_{C,5b}(t=5b) )</td>
<td>( h_{T}L_{a}(t_{a}) = i_{L,5b}(t_{a}) )</td>
</tr>
<tr>
<td>6</td>
<td>( i_{L,5c}(t=5c) )</td>
<td>( V_{C,5c}(t=5c) )</td>
<td>( \frac{V_{i}}{C_{a}} \int_{0}^{t_{a}} i_{L}(t) , dt = V_{i} )</td>
</tr>
</tbody>
</table>

**VI. CONCLUSIONS**

A self-oscillating electronic ballast driven by a series saturable transformer has been investigated. This analysis is performed for large signals to completely describe the circuit operation and explains the difference from an external-drive high-\( Q \) resonant inverter. Each oscillation cycle is divided into six stages according to the operating points of the saturable core. Equations describing the electrical phenomena (current, voltage) and the associated initial conditions and ending requirements in each stage are presented. Simulation based on the equations benefits the understanding and design of the ballast circuit. Laboratory experiments of an 18-W PL-lamp ballast are performed to test the accuracy of the analysis. Experimental recordings are compared with numerical simulation of the derived equations. With percentage error within 5\%, the correctness and accuracy of the stagewise circuit resonance described by the derived equations are verified.

Fig. 9. Experimental results.

The principal effects of the ignition capacitor are to “reduce the current harmonics through the lamp” and “heat the filaments”. Therefore, the ballast circuit can be roughly viewed as a second-order series-resonant circuit.

Fig. 9 gives the recorded current and voltage waveforms of the resonant inductor and the series capacitor. They are not purely sinusoidal as investigated. In the experiment, the natural frequency, damped natural frequency, and switching frequency are 9.9, 8, and 26 kHz, respectively. The fundamental approximation based on the switching frequency may lose accuracy.

Fig. 10 shows the simulated waveforms and Table III lists several important values for comparison. Percentage errors are all within 5\%.

**TABLE III**

<table>
<thead>
<tr>
<th>DC bus ( V_i = 300V )</th>
<th>Experimental</th>
<th>Simulated</th>
<th>Percentage error*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak to peak of ( i_L )</td>
<td>1.12A</td>
<td>1.129A</td>
<td>- 0.80 %</td>
</tr>
<tr>
<td>Peak of ( V_C )</td>
<td>183 V</td>
<td>191.9V</td>
<td>- 4.86 %</td>
</tr>
<tr>
<td>Valley of ( V_C )</td>
<td>123 V</td>
<td>128.5V</td>
<td>- 4.47 %</td>
</tr>
<tr>
<td>Switching period</td>
<td>38.5 ( \mu )s</td>
<td>40.10( \mu )s</td>
<td>- 4.16 %</td>
</tr>
</tbody>
</table>

* : percentage error = \frac{(experimental) - (simulated)}{experimental} \times 100\%

Experimental recordings are compared with numerical simulation of the derived equations. With percentage error within 5\%, the correctness and accuracy of the stagewise circuit resonance described by the derived equations are verified.
APPENDIX

\[ P_{\text{amp}} = 18; \ J_{\text{amp}} = 0.32; \ RL = P_{\text{amp}}/P_{\text{amp}}^2; \ L_r = 2.59e - 3; \ L_m = 0.15e - 3; \]
\[ C_s = 0.1e - 6; \ c_2 = 2.2e - 9; \ R_d = 47; \ N_p = 8; \ N_s = 4; \ A_c = 6.25; \ B_s = 0.4; \]
\[ E_{bc} = 0.65; \ \alpha = 3.55e - 3; \ \alpha = 10e - 9; \ \beta = E_{bc}/I_{bc}; \ A = 1/(\beta \times \alpha); \]
\[ R_p = 100; \ \beta = 0.73; \ V_{IBR} = E_{bc}/R_d; \ n = N_p/N_s; \ V_T = N_p \times A_c \times B_s \times 1e - 6; \]
\[ T = 0.1 \times V_T; \ w = 2 \times \pi \times (1/T); \ \kappa = w \times L_m/\sqrt{2(w \times L_m)^2 + (n^2 \times R_d)^2}; \]
\[ V_i = 300; \ V_{\alpha} = 125; \ h = 1e - 7; \]
\[ \text{loop} = 0; \]
while 1
%******** Stage 1b ********
clear \(i_{L1b}\); clear \(v_{C1U}\); \(L = L_r + L_m; \) \(R = RL; \) \(C = C_s; \) \(a = R/(2 \times L); \)
\[ u_d = \sqrt{1/(L \times C)} - a^2; \]
\[ i = 2; \ t_{1a} = h; \ i_{L1b}(1) = 0; \ v_{C1U}(1) = V_{\alpha}; \ i_{L1b}(1) = 0; \ v_{C1U}(0) = 0; \]
\[ B = \kappa \times n \times (V_i - V_{\alpha})/(u_d \times L)/\kappa; \]
while 1
\[ i_{L1b}(i) = (V_i - v_{C1U}(1))/(u_d \times L) \]
\[ + \sin(u_d \times t_{1a}) \times i_{L1b}(1) \times \cos(w_d \times t_{1a}) \times \exp(-a \times t_{1a}) \]
\[ v_{C1U}(i) = (\alpha \times C \times (v_{C1U}(1) - V_i)/(u_d \times C) \times \sin(u_d \times t_{1a}) + (v_{C1U}(1) - V_i) \]
\[ \times \cos(w_d \times t_{1a})) \times \exp(-a \times t_{1a}) + V_i \]
\[ v = B \times \exp(-a \times t_{1a}) \times ((A - a) \times \sin(w_d \times t_{1a}) - u_d \times \cos(w_d \times t_{1a}))/(A - a)^2 \]
\[ + u_d^2) + B \times u_d/(A - a)^2 + u_d^2; \]
\[ v_{C1U}(i) = v + E_{bc} \times (1 - \exp(-A \times t_{1a})) + v_c \times \exp(-A \times t_{1a}); \]
if \((i_{L1b}(i) \times L_m/\sqrt{2(w \times L_m)^2 + (n^2 \times R_d)^2}) > VT)\]
end \(i_{L1b}(i) < i_{L1b}(i - 1); \) break, end
\[ t_{1a} = t_{1a} + h; \ i = i + 1; \]
end
%******** Stage 2 ********
clear \(i_{L2}; \) clear \(v_{C2}; \) \(L = L_r; \) \(R = RL; \) \(C = C_s; \) \(a = R/(2 \times L); \)
\[ u_d = \sqrt{1/(L \times C)} - a^2; \]
\[ i = 2; \ t_1 = h; \ i_{L2}(1) = i_{L1b}(\text{length}(i_{L1b})); \ v_{C2}(1) = \text{length}(v_{C1U}); \]
while 1
\[ i_{L2}(i) = (V_i - v_{C2}(1)) \]
\[ = \sin(u_d \times t_{1a}) \times v_{C2}(1) \times \cos(w_d \times t_{1a}) \times \exp(-a \times t_{1a}) \]
\[ v_{C2}(i) = v_{C2}(i - 1) + i_{L2}(i) \times h/C_s; \ v_{C2} = v_{C2} - E_{bc} + L_{BR} + u_e \]
\[ (1 \times \exp(-A \times t_{1a})) \times i_{L2}(i) = (\exp(-A \times t_{1a}) \times i_{L2}(i) < 0) \times (i_{L2}(i) < 0); \) break, end
\[ t_1 = t_1 + h; \ i = i + 1; \]
end
%******** Stage 3 ********
clear \(i_{L3}; \) clear \(v_{C3}; \) \(L = L_r; \) \(R = RL; \) \(C = C_s; \) \(a = R/(2 \times L); \)
\[ u_d = \sqrt{1/(L \times C)} - a^2; \]
\[ i = 2; \ t_1 = h; \ v_{C3}(1) = -V_i; \ i_{L3}(1) = i_{L2}(\text{length}(i_{L2})); \]
\[ v_{C3}(1) = v_{C3}(\text{length}(v_{C2})); \]
while 1
\[ i_{L3}(i) = (V_i - v_{C3}(1)) \]
\[ - \sin(u_d \times t_{1a}) \times v_{C3}(1) \times \cos(w_d \times t_{1a}) \times \exp(-a \times t_{1a}) \]
\[ v_{C3}(i) = v_{C3}(i - 1) + i_{L3}(i) \times h/C_s; \ v_{C3}(i) = v_{C3}(i - 1) + i_{L3}(i) \times h/C_s; \]
\[ \text{if} \ ((v_{C3}(i) \times C_s < v_{C3}(i - 1)); \text{break, end} \]
\[ t_2 = t_2 + h; \ i = i + 1; \]
end
%******** Stage 4a ********
clear \(i_{L4a}; \) clear \(v_{C4a}; \) \(L = L_r; \) \(R = RL + n^2 \times R_d; \) \(C = C_s; \) \(a = R/(2 \times L); \)
\[ u_d = \sqrt{1/(L \times C)} - a^2; \]
\[ i = 2; \ t_2 = h; \ i_{L4a}(1) = i_{L3}(\text{length}(i_{L3})); \ v_{C4a}(1) = v_{C3}(\text{length}(v_{C3})); \]
while 1
\[ i_{L4a}(i) = ((i_{L4a}(1) - a \times L)/(u_d \times L) \times \sin(w_d \times t_{3a}) + i_{L4a}(1) \times \cos(w_d \times t_{3a}) \times \exp(-a \times t_{3a}); \]
\[ v_{C4a}(i) = v_{C4a}(i - 1) + i_{L4a}(i) \times h/C_s; \text{if} \ ((i_{L4a}(i) < 0); \text{break, end} \]
\[ t_3 = t_3 + h; \ i = i + 1; \]
end
% stage 4b
clear $i_{Lb}$; clear $v_{C5}$; $L = L_{L} + L_{m}$; $R = RL; C = C_{a}; a = R/(2*L);$
\[ u_{d} = \sqrt{r(1/(L_{C} - a^{2})}; \]
\[ i = 2; \]
\[ t_{Lb}(i) = i_{Lb}(\text{length}(i_{Lb})); \]
\[ v_{C5}(1) = v_{C5}(\text{length}(v_{C5})); \]
\[ i_{Lm}(1) = 0; \]
\[ v_{C20} = 0; \]
\[ BB = k_{p} * n * v_{C5}(1)/(u_{dl} * L); \]
\[ \text{while} > 1 \]
\[ i_{Lb}(i) = ((-v_{C5}(1) - i_{Lb}(1) * (u_{dl} * L) * \sin(w_{u} * t_{u}) + i_{Lb}(1) * \cos(w_{u} * t_{u}) - \exp(-a * t_{u})); \]
\[ v_{C5}(i) = v_{C5}(i - 1) + i_{Lb}(i) * h/C_{a}; \]
\[ v_{C5}(t_{u}) = (v_{C5}(i - 1) + i_{Lb}(i) * h/C_{a}; \]
\[ v_{C5} = v_{C5}(1) - (v_{C5} - E_{be} + I_{BR} * n_{re} + (1 - \exp(-a * t_{u})); i_{L5} = v_{C5} - E_{be} / n_{re}; \]
\[ \text{if} (i_{L5}(i) < v_{C5}(i - 1)), \text{break}, \text{end} \]
\[ t_{u} = t_{u} + h; \]
\[ i = i + 1; \]
end
% stage 5
clear $i_{L5}$; clear $v_{C5}$; $L = L_{L}; R = RL; C = C_{a}; a = R/(2*L);$
\[ u_{d} = \sqrt{r(1/(L_{C} - a^{2})}; \]
\[ i = 2; \]
\[ t_{5}(i) = i_{L5}(\text{length}(i_{L5})); \]
\[ v_{C6}(1) = v_{C6}(\text{length}(v_{C6})); \]
\[ v_{C6} = v_{C6}(1) \]
\[ \text{while} > 1 \]
\[ i_{L6}(i) = ((-v_{C6}(1) - i_{L6}(1) * (u_{dl} * L) * \sin(w_{u} * t_{u}) + i_{L6}(1) * \cos(w_{u} * t_{u}) - \exp(-a * t_{u})); \]
\[ v_{C6}(i) = v_{C6}(i - 1) + i_{L6}(i) * h/C_{a}; \]
\[ v_{C6} = v_{C6}(1) - (v_{C6} - E_{be} + I_{BR} * n_{re} + (1 - \exp(-a * t_{u})); i_{L6} = v_{C6} - E_{be} / n_{re}; \]
\[ \text{if} (i_{L6}(i) < v_{C6}(i - 1)), \text{break}, \text{end} \]
\[ t_{5} = t_{5} + h; \]
\[ i = i + 1; \]
end
% stage 6
clear $i_{L6}$; clear $v_{C6}$; $L = L_{L}; R = RL; C = C_{a}; a = R/(2*L);$
\[ u_{d} = \sqrt{r(1/(L_{C} - a^{2})}; \]
\[ i = 2; \]
\[ t_{6}(i) = i_{L6}(\text{length}(i_{L6})); \]
\[ v_{C6}(1) = v_{C6}(\text{length}(v_{C6})); \]
\[ v_{C6} = v_{C6}(1) \]
\[ \text{while} > 1 \]
\[ i_{L6}(i) = ((V_{i} - v_{C6}(1) - (-v_{C6}(1) - i_{L6}(1) * (u_{dl} * L) * \sin(w_{u} * t_{u}) + i_{L6}(1) * \cos(w_{u} * t_{u}) - \exp(-a * t_{u})); \]
\[ v_{C6}(i) = v_{C6}(i - 1) + i_{L6}(i) * h/C_{a}; \]
\[ v_{C6} = v_{C6}(1) - (v_{C6} - E_{be} + I_{BR} * n_{re} + (1 - \exp(-a * t_{u})); i_{L6} = v_{C6} - E_{be} / n_{re}; \]
\[ \text{if} (i_{L6}(i) < v_{C6}(i - 1)), \text{break}, \text{end} \]
\[ t_{6} = t_{6} + h; \]
\[ i = i + 1; \]
end
% stage 1a
clear $i_{L1a}$; clear $v_{C1a}$; $L = L_{L}; R = RL + n^{2} * R_{d}; C = C_{a}; a = R/(2*L);$
\[ u_{d} = \sqrt{r(1/(L_{C} - a^{2})}; \]
\[ i = 2; \]
\[ t_{a}(i) = i_{L1a}(\text{length}(i_{L1a})); \]
\[ v_{C1a}(1) = v_{C1a}(\text{length}(v_{C1a})); \]
\[ v_{C1a} = v_{C1a}(1) \]
\[ \text{while} > 1 \]
\[ i_{L1a}(i) = ((V_{i} - v_{C1a}(1) - (V_{i} - v_{C1a}(1) * (u_{dl} * L) * \sin(w_{u} * t_{u}) + i_{L1a}(1) * \cos(w_{u} * t_{0}) - \exp(-a * t_{0})); \]
\[ v_{C1a}(i) = v_{C1a}(i - 1) + i_{L1a}(i) * h/C_{a}; \]
\[ v_{C1a} = v_{C1a}(1) - (v_{C1a} - E_{be} + I_{BR} * n_{re} + (1 - \exp(-a * t_{0})); i_{L1a} = v_{C1a} - E_{be} / n_{re}; \]
\[ \text{if} (i_{L1a}(i) > 0), \text{break}, \text{end} \]
\[ t_{a} = t_{a} + h; \]
\[ i = i + 1; \]
end
% if $(aB_{C}(\text{length}(v_{C1a}))) - V_{a0} < 0.2$, break, else
\[ V_{a0} = V_{a0} + 0.5 * (aB_{C}(\text{length}(v_{C1a}))) - V_{a0}, V_{p} = \text{max}(v_{C1a}), \]
end
\text{loop} = \text{loop} + 1;
\[ T_{a} = \text{length}(i_{L1a}) + \text{length}(i_{L2}) + \text{length}(i_{L3}) + \text{length}(i_{L4a}) + \text{length}(i_{L5}) + \text{length}(i_{L6}) + \text{length}(i_{L7a}); \]
\[ T_{a} = \text{length}(i_{L1a}) + \text{length}(i_{L2}) + \text{length}(i_{L3}) + \text{length}(i_{L4a}) + \text{length}(i_{L5}) + \text{length}(i_{L6}) + \text{length}(i_{L7a}); \]
\[ t = 0; \]
\[ f_{1} = [v_{C2}, v_{C2}, v_{C3}, v_{C4}, v_{C5}, v_{C6}, v_{C1a}]; \]
\[ f_{2} = [v_{C2}, v_{C2}, v_{C3}, v_{C4}, v_{C5}, v_{C6}, v_{C1a}]; \]
\text{subplot}(211), plot(t(1 : Len), f_{1}(1 : Len)), grid,
\text{subplot}(212), plot(t(1 : Len), f_{2}(1 : Len)), grid,
REFERENCES


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Chern-Lin Chen (S’86–M’90), for a photograph and biography, see p. 118 of the February 1999 issue of this TRANSACTIONS.