POLARIZATION DEPENDENCE ON ELECTROMAGNETIC SCATTERING OF
PERFECTLY CONDUCTING CONCAVE CORNERS BY MAGNETIC FIELD INTEGRAL EQUATIONS

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ABSTRACT — Due to the presence of multiple reflections, the induced current on perfectly conductive concave corners is difficult to analyze accurately, especially when different polarizations are taken into account. In this paper, the magnetic field integral equation formulation is employed for both the cases that the incident waves are horizontally and vertically polarized. The method of moments, together with the conjugate gradient solution technique, is then implemented on a personal computer to solve the induced current on the conductor surfaces and the far-field scattering coefficients. Numerical computations are executed for a conductor of "L" shape. The induced current near the concave corner are compared with that near the convex corner for both polarizations. Also presented are the polarization dependence on back-scattering coefficients of the "L" conductor for various frequencies and incident angles.

INTRODUCTION

The electromagnetic (EM) scattering of conducting objects is greatly influenced by the polarizations of incident waves [1]. The polarization diversity finds its applications in the radar sensitivity optimization, the noise reduction of rain, snow, or jamming objects, and the target classification [2]. Hence, it becomes increasingly important to study the polarization dependence on the EM scattering, especially in the realm of high-resolution radars.

A thorough study of the physical mechanism inherent in the wave-conductor interaction calls for the analysis of induced current on the conductor surfaces. From which, the scattered field can be deduced accordingly. In the use of polarization diversity, it is necessary to find the scattering matrix [1]. In other words, not only the magnitude but also the relative phase of the scattered field for two orthogonally polarized incident waves are required.

Various high-frequency methods, such as GO, PO, PTD, GTD, and UTD, have been well established in the literature [3] for the scattering analysis of electrically large convex conducting objects. Recently, these methods have been combined to discuss the EM scattering of concave structures such as the dihedral reflectors, where multiple scattering happens [4], [5]. However, these methods, relying more or less on the ray theory, still find difficulty in accurately unveiling the induced current near the concave corners where the wave mechanism is dominant.

In this respect, we apply the method of moments (MoM) [6] to solve integral equations for the induced current. This approach directly originates from the Maxwell equations and can be more accurate to deal with the concave corners. However, it is very numerically incentive and is too time-consuming to tackle three-dimensional electrically large structures. Hence, we consider here only the cases of arbitrarily shaped conducting cylinders.

Although the MoM solution for plane wave scattering by a two-dimensional cylinder was discussed long time ago, most approaches employed electric field integral equation (EFIE) for the vertical polarization

cases and magnetic field integral equation (MFIE) for the horizontal polarization cases [7], [8]. For vertical polarization cases, EFIE can model the infinitesimally thin corners. However, only the sum of the current on the two surfaces of one plate is available and then the induced current near the concave corner is mixed with that near the other convex side. Hence, the characteristics of the induced current near the concave corners are remained comparatively unexploited.

Here, we consider the conductors of "L" shape where the plates are of a small thickness. The MFIE, which can be applied for both polarizations as well, are thus employed to investigate the polarization dependence on the induced current and scattering characteristics of concave and convex corners.

MAGNETIC FIELD INTEGRAL EQUATIONS

Fig.1 shows a conducting cylinder, which is uniform in the z-direction, illuminated by a plane wave of unit strength. By the MFIE formulation [4], the induced current density on the conductor surface Γ satisfies

$$\frac{1}{2}\vec{J}(\vec{r}) - \hat{n} \times \int_{\Gamma} \nabla \times G(\vec{r}, \vec{r}') \vec{J}(\vec{r}') d\vec{r}' = \hat{n} \times \vec{H}^i(\vec{r}) \quad (1)$$

Here, $\int$ means the Cauchy principal integral, $\vec{H}^i$ is the given incident magnetic field, and the two-dimensional Green function G is related to the Hankel function by $G(\vec{r}, \vec{r}') = H_0^{(2)}(k|\vec{r} - \vec{r}'|)/4j$.

When the incident wave is horizontally polarized, or TE case, only the transverse current component J_s is induced. Eq. (1) becomes

$$\frac{1}{2}J_s(\vec{r}) - \int_{\Gamma} \frac{\partial G}{\partial n} J_s(\vec{r}') d\vec{r}' = -H_z^i(\vec{r}) \quad (2)$$

where $\partial G / \partial n$ is the derivative of G with respect to the outward normal direction at position $\vec{r}'$.

Once the induced current is solved from (2), the scattered far-field with wavenumber vector $\vec{k}^s$ can be deduced. Then, the scattering coefficient for the horizontally polarized incident wave can be obtained by

$$S_{HH} = -\frac{1}{2}\sqrt{k} e^{-j3\pi/4} \oint_{\Gamma} e^{j\vec{k}^s \cdot \vec{r}'} \cos(\hat{n}, \vec{k}^s) J_s(\vec{r}') d\vec{r}' \quad (3)$$

Note that the vertically polarized component of the scattered field is zero, i.e., $S_{VH} = 0$.

For the vertically polarized incident wave, or TM case, the induced surface current flows only in the z-direction. After some simplifications to (1), the current density J_z satisfies

$$\frac{1}{2}J_z(\vec{r}) + \int_{\Gamma} \frac{\partial G}{\partial n} J_z(\vec{r}') d\vec{r}' = H_z^i(\vec{r}) \quad (4)$$

where $\partial G / \partial n$ is the derivative of G with respect to the outward normal direction at position $\vec{r}'$. The scattering coefficients are now $S_{HV} = 0$ and

$$S_{VV} = \frac{1}{2}\sqrt{k} e^{-j3\pi/4} \oint_{\Gamma} e^{j\vec{k}^s \cdot \vec{r}'} J_z(\vec{r}') d\vec{r}' \quad (5)$$

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Fig.1 EM scattering of two-dimensional conducting objects.

SOLUTION BY METHOD OF MOMENTS

Eqs. (2) and (4) can be solved by applying MoM. The conductor surface Γ is partitioned into N line segments, i.e., Γ_n ($n = 1, 2, \dots, N$). When the length of each segment, l_n , is chosen smaller than one tenth of the wavelength of the incident wave, the current density inside the same segment can be assumed nearly constant. That is, the unknown $J = J_s$ in (2) or $J = J_z$ in (4) is modeled as

$$J(\vec{r}) = \sum_{n=1}^N J_n B_n(\vec{r}) \quad (6)$$

where J_n is the current density in the n th segment Γ_n , while the basis function $B_n(\vec{r})$ is unity when $\vec{r} \in \Gamma_n$ and is zero elsewhere. Since the current density is larger near the edge, it is suggested that a denser division should be employed there.

Substituting (6) into (2) or (4) and applying the Galerkin method, we obtain the matrix equation

$$\sum_{n=1}^N G_{mn} J_n = b_m, \quad m = 1, 2, \dots, N. \quad (7)$$

The matrix element G_{mn} and RHS vector element b_m are

$$G_{mn} = \begin{cases} -\frac{1}{2} \int_{\Gamma_n} B_n(\vec{r}) d\vec{r} = -\frac{1}{2} l_n & \text{if } m=n \\ \int_{\Gamma_n} \int_{\Gamma_m} B_m(\vec{r}) \frac{\partial G}{\partial n} B_n(\vec{r}') d\vec{r}' d\vec{r} & \text{if } m \neq n \end{cases} \quad (8)$$

$$b_m = \int_{\Gamma_m} B_m(\vec{r}) e^{-j\vec{k}^i \cdot \vec{r}} d\vec{r};$$

for horizontally polarized waves, and are

$$G_{mn} = \begin{cases} \frac{1}{2} \int_{\Gamma_n} B_n(\vec{r}) d\vec{r} = \frac{1}{2} l_n & \text{if } m=n \\ \int_{\Gamma_n} \int_{\Gamma_m} B_m(\vec{r}) \frac{\partial G}{\partial n} B_n(\vec{r}') d\vec{r}' d\vec{r} & \text{if } m \neq n \end{cases} \quad (9)$$

$$b_m = -\int_{\Gamma_m} B_m(\vec{r}) e^{-j\vec{k}^i \cdot \vec{r}} \cos(\hat{n}, \vec{k}^i) d\vec{r}$$

for vertically polarized waves. Here, $\vec{k}^i$ is the wavenumber vector of incident waves. A careful check reveals that the matrix defined in (9) can be obtained from that in (8) by taking transpose and negating the diagonal elements. Once we calculate the matrix for one polarization, the matrix for the other polarization is available immediately.

It should be noticed that the matrix inherent in MoM is fully populated, and sometimes may even be ill-conditioned. A common solution approach of Gaussian elimination or LU-decomposition not only occupies a vast computer core memory but sometimes also results in a large numerical error [9]. A remedy is to

employ the conjugate gradient method (CGM), which is an iterative but finite-step method [10]. It calls for a very small computer core memory so that even a personal computer can accommodate easily.

NUMERICAL RESULTS

Together with MoM and CGM, a Fortran program has been implemented in the IBM/PC-AT to solve the MFIE's. The program calculates the induced surface current and then the far-field scattering coefficients. It is applicable to any two-dimensional conducting structures including multiple objects. The incident waves can be of any polarization. Due to the efficiency of CGM in the computer core memory, the number of unknowns in the matrix equation (7) can be as large as several hundreds even by PC. Roughly speaking, our program in IBM/PC-AT can solve any two-dimensional structure of size smaller than ten wavelengths. By rule of thumb, the CPU time for a problem modeled with 100 unknowns is about 10³ minutes.

To discuss the scattering mechanism near a concave corner, we consider an "L"-shape conductor where the narrow and wide sides are of sizes a and $5a$, respectively. Fig.2 draws the magnitude of the induced surface current density when the incident wave is indexed to the concave corner A, i.e., the incident angle is 45°. The frequency is such that $ka = 1$. The solid and dashed curves denote the cases of vertical and horizontal polarizations, respectively.

In case of vertical polarization, an interesting contrast is found that the current density tends to infinity at the convex corner, e.g., the points B, C, and D; while is zero at the concave corner, i.e., the point A. In case of horizontal polarization, the current density is finite and continuous no matter the corner is convex or concave. In the meantime, no matter the wave is of which polarization, the induced current is totally deviated from the PO prediction even in the illuminated region, e.g., region A-B. This example reveals the deficiency of PO in handling concave structures where multiple scattering dominates.

When the incident angle varies, the back scattering coefficients change also. Fig.3(a) draws the back scattering pattern for this "L"-shape conductor with $ka = 1$. The solid and dashed curves denote the cases of vertical and horizontal polarizations, respectively. When the wave illuminates the wide side ($\theta \approx 180^\circ$), the scattering coefficients S_{HH} and S_{VV} are almost the same. In other words, the scattering of smooth planar surface is almost independent of the polarization. In this case, GO or PO approximation is enough.

As the incident angle deviates from 180°, the edge diffraction plays its role. The scattering characteristics show some dependence on the polarization. Satisfactory results are possible if we could include the GTD corrections. However, as the incident wave illuminates the concave region ($45^\circ \leq \theta \leq 90^\circ$), the multiple scattering plays the leading role. The strength of the scattered field may be large and the dependence on polarization is significant. The approximation by PO and/or GTD seems not promising, especially in the range of intermediate frequencies. The numerical approach is then of great value.

Fig.3(b) shows the back scattering pattern for the case of a higher frequency $ka = 7$, i.e., the wide side is about 5.5 wavelengths. In the figure, the scattered wave is more significant as the incident angle is near 45° or 180°. The former is due to the multiple scattering of the concave corner, while the latter is due to the scattering of the smooth plane. However, both behaves very different. As frequency increases, the peak due to smooth plane scattering goes sharper

Fig.2 Induced surface current density on "L"-shape conductor. Solid and dashed curves denote the cases of vertical and horizontal polarizations, respectively. Incident wave is of frequency $ka = 1$ and points to concave corner A.

Fig.3 Back scattering pattern of "L"-shape conductor. Incident wave is of frequency (a) $ka = 1$ and (b) $ka = 7$.

and sharper; while the peak due to multiple scattering sustains for a certain range of incident angles. In addition, the scattering coefficients S_{HH} and S_{VV} have the same phase for scattering of the smooth plane; while emerge with about 180° phase difference for multiple scattering of the concave corner considered here.

CONCLUSIONS

In this paper, a rigorous approach based on MoM and CGM has been implemented in IBM/PC-AT to solve the EM scattering of arbitrary two-dimensional conducting objects with the incident wave of any polarization. Due to the MFIE formulation, the matrix elements calculated for one polarization can be used directly for the other polarization. The solution of matrix equation by CGM calls for small computer core memory and the result is reliable even when the matrix is ill-conditioned. Hence, it is applicable in the range of small as well as intermediate frequencies.

This approach is applied to investigate the scattering characteristics of convex and concave corners under the incident waves of different polarizations. For vertically polarized waves, the induced current density near the convex and concave corners is infinite and zero, respectively. For horizontally polarized waves, the induced current density is finite and continuous no matter the corner is convex or concave. The mechanism related to the convex corner is the edge diffraction, whose effect on scattering field can be predicted by GTD. On the other hand, the mechanism related to the concave corner is the multiple scattering, whose effect can not be approximated satisfactorily by GO, PO, or GTD, especially when the polarization dependence is to be taken into well account. In this respect, the numerical approach presented here deserves further study.

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