Analysis of propagation characteristics for an octagonal photonic crystal fiber (O-PCF)

Jung-Sheng Chiang^a, Tzong-Lin Wu^{b,*}

^a Department of Electrical Engineering, National Sun Yat-Sen University, 70 Lien-Hai Road, Kaohsiung 80424, Taiwan, ROC

^b Department of Electrical Engineering and Graduate Institute of Communication Engineering, National Taiwan University, No. 1, Sec. 4, Roosevelt Rd, Taipei 106, Taiwan

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Abstract

An octagonal photonic crystal fiber (O-PCF) structure with eight air-holes on the first ring is proposed based on a unit isosceles triangle. The propagation characteristics and cut-off behaviors of the O-PCF and the standard hexagonal PCF (H-PCF) are numerically investigated by combining the vector boundary method and the effective area method. The phase boundaries for cut-off, single-mode, and multi-mode operations between the O-PCF and H-PCF are calculated and compared. It is found that under the same pitch Λ and air filling fraction (AFF) of the air-holes the O-PCF has significantly wider wavelength range operating in single-mode region, more circular-like field distribution, and less confinement loss than the H-PCF.

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1. Introduction

Photonic crystal fibers (PCFs), or holey fibers, are constituted of a single material (generally silica) with multiple air-holes periodically arranged

around the core. By making a defect on the periodic lattice, light can be confined and guided along the fiber axis through the mechanism of either the total internal reflection (TIR) or the photonic bandgap (PBG). Typically, the air holes are arranged in the equal lateral triangular lattice with six holes in the first ring surrounding the cores, hereafter named as the hexagonal PCF (H-PCF). By suitably designing the air-hole pitch and diameter, the H-PCF possesses numerous unusual

* Corresponding author. Tel.: +886 2 23635251; fax: +886 2 23638247.

E-mail addresses: d9031822@student.nsysu.edu.tw (J.-S. Chiang), wtl@cc.ee.ntu.edu.tw (T.-L. Wu).

properties, such as wide single-mode wavelength range [1], anomalous group-velocity dispersion at visible and near-infrared wavelength [2], and extremely large effective-core-area at single-mode region [3].

Besides the hexagonal structure, other air-hole arrangements, such as honeycomb or kagomé, were proposed to design the PCF [4]. The square-lattice PCF was also studied for the specific dispersion characteristics [5]. Recently, the ring structure with the holes lie on circular ring was presented for enlarging the core size or achieving the cylindrically layered fibers [6,7]. These examples demonstrate that exactly periodic lattice arrangements for the air-holes are not absolutely necessary to achieve the guidance of the light based on the TIR mechanism. The design of TIR-type holy optical fibers is more flexible to have special propagation characteristics.

This paper proposes a new air-hole arrangement based on an isosceles triangle of $\pi/4$ vertex angle. By repeating the isosceles triangle around the core center, an octagonal PCF (O-PCF) with eight air-holes in the first ring can be developed. The propagation characteristics of the O-PCF are investigated based on the vector boundary element method (VBEM) [8,9]. Compared with the H-PCF, the O-PCF possesses the advantages of wider wavelength range operating in single-mode region, more circular-like field distribution, and significantly less confinement loss.

The paper is organized as follows. In Section 2, we introduce the octagonal symmetry design for PCF and geometrical parameter. The propagation characteristics calculated for O-PCF and compared with H-PCF are analyzed in Section 3. We conclude in Section 4 with a brief discussion of fabrication issue.

2. O-PCF and geometrical parameters

Fig. 1 shows the structure of the octagonal PCF with the hole radius a and pitch Λ between the adjacent rings. The O-PCF with octagonal symmetry is constructed by repeating the unit isosceles triangle around the core center. The triangle has two legs (equal sides) of length Λ and the vertex an-

Fig. 1. Cross section of the O-PCF. The cladding consists of air-holes of diameter $2a$ arranged in an isosceles triangular lattice with leg of length Λ and octagonal symmetry.

gle of 45° . The air-hole is located at each corner of the triangles. Fig. 2(a) and (b) shows the unit triangle of the H-PCF and O-PCF, respectively. The main difference between them is that the unit triangle for the H-PCF is an equal lateral triangle with the vertex angle 60° , but it is isosceles triangle for the O-PCF. Although the air-holes of the O-PCF are not arranged from the concept of the periodic lattice, such as triangle or square, in 2D space, the propagation modes can exist due to the TIR mechanism, i.e., the lower index of the air-filled

Fig. 2. Schematics of the unit triangle of PCFs: (a) H-PCF is constructed by equal lateral triangle with vertex angle of 60° and (b) O-PCF is by isosceles triangle with vertex angle of 45° .

cladding and higher index of the silica fibres core. To compare the propagation characteristics of the O-PCF and H-PCF under the same air-hole area, we define the air-filling fraction (AFF) as

$$\text{AFF} = \frac{A_{\text{hole}}}{A_{\text{cell}}}, \quad (1)$$

where A_{cell} is the area of the unit triangle and A_{hole} is the area of the air-hole inside the unit triangle. It can be derived that $\text{AFF} = (2\pi/\sqrt{3})(a_h/\Lambda)^2$ for the H-PCF and $\text{AFF} = (2\pi/\sqrt{2})(a_o/\Lambda)^2$ for the O-PCF, where a_h and a_o is the air-hole radius of the H-PCF and O-PCF, respectively. Under the same AFF condition and air-hole pitch Λ , it can be shown that $a_o = 0.9036 \times a_h$, i.e., the air-hole radius of the O-PCF is about 10% smaller than that of the H-PCF.

3. Propagation characteristics

The propagation characteristics of the PCFs are simulated by the vector boundary element method (VBEM) [8,9], which is based on a full-wave formulation of the surface integral equations. Under the same criterion of the AFF and Λ , the propagation characteristics of the O-PCF and H-PCF, including the guided modes and field confinement, are investigated and compared.

3.1. Guided modes

Fig. 3(a)–(f) shows the vectorial mode patterns of the O-PCF for the $HE_{11}^x, HE_{11}^y, TE_{01}, TM_{01}, HE_{21}^x$, and HE_{21}^y with normalized wavelength $\lambda/\Lambda = 0.1$ and normalized air-hole-diameter $d/\Lambda = 0.6$. Similar to the step-index fibers, it is found that the fundamental modes, HE_{11}^x and HE_{11}^y , and the second-order modes, HE_{21}^x and HE_{21}^y , are both degenerate, but the two circularly symmetric modes, TE_{01} and TM_{01} , are non-degenerated. The effective mode indexes of the fundamental and second-order modes are calculated to confirm their degeneracy. The refractive index of silica is chosen to be 1.46. The effective index of the $HE_{11}^x, HE_{11}^y, HE_{21}^x$, and HE_{21}^y modes are 1.459186058, 1.459186057, 1.457943054, and 1.457943057, respectively. It is clearly seen that the degeneracy between the HE_{11}^x and HE_{11}^y modes and

HE_{21}^x and HE_{21}^y modes. It is seen in Fig. 3 that, similar to the H-PCF, the proposed O-PCF structure can support the propagation of the two degenerate fundamental modes and four nearly degenerate second-order modes.

Fig. 4 shows an example of the wavelength responses of the effective areas both for the second-order modes of H-PCF and O-PCF with $\text{AFF} = 0.327$ and $\Lambda = 1 \mu\text{m}$. There are three rings of air-holes for both cases. The effective area of the guided modes can be derived as:

$$A_{\text{eff}} = \frac{(\int |E|^2 ds)^2}{\int (|E|^2)^2 ds}, \quad (2)$$

where $|E|^2$ is the intensity distribution on the fiber cross-sections [10,11]. It is seen that at shorter wavelength the effective area is finite and constant, but at longer wavelength the mode fields spread to the cladding and the effective area increases significantly. The crossings of the dotted lines with the horizontal axis are defined as the cut-off wavelengths of the second-order modes. It is found that O-PCF has shorter cut-off wavelength (about $\lambda/\Lambda = 0.58$) than the H-PCF does (about $\lambda/\Lambda = 0.68$) under the same AFF and pitch Λ . Carrying out the same analysis for different AFF, a phase diagram between the single-mode and multi-mode regions can be constructed. Fig. 5 shows the phase diagram for the H-PCF and O-PCF. The data above the $\text{AFF} = 0.2$ are calculated by combining the VBEM and the effective area method, and the data below $\text{AFF} = 0.2$ is curve fitted using the calculated data above $\text{AFF} = 0.2$. To check the validity of our numerical approach, the phase boundary proposed by Kuhlmeier et al. [12] for the H-PCF is also presented by dots in Fig. 5. Good agreement between them is seen. As shown in Fig. 5, the H-PCF and O-PCF operate in the single mode region for the AFF less than 0.155 and 0.157, respectively. The corresponding normalized air-hole diameter (d/Λ) is about 0.413 and 0.416, respectively. It is also clearly seen that the phase boundary for the O-PCF is at the shorter normalized wavelength than the H-PCF.

It has been known the PCF behaves endless single-mode for the ideal cladding with infinite rings of air-holes, but practically the ring number

Fig. 3. Transverse magnetic-field vector distributions of: (a) the HE_{11}^x , (b) HE_{11}^y , (c) TE_{01} , (d) TM_{01} , (e) HE_{21}^x and (f) HE_{21}^y modes of multimode O-PCF for $\lambda/\Lambda = 0.1$.

is finite for the general PCF. Therefore, the fundamental modes will not be guided (or cut-off) in the practical PCF for the wavelength longer than some cut-off wavelength. Fig. 6 shows an example of the wavelength responses of the effective areas for the fundamental modes of the four-rings' H-PCF and

O-PCF with $AFF = 0.327$ and $\Lambda = 1 \mu\text{m}$. Similar to the definition of the cut-off wavelength for the second-order mode, it is found that O-PCF has longer cut-off wavelength (about $\lambda/\Lambda = 2.26$) than the H-PCF does (about $\lambda/\Lambda = 2.2$) under the same AFF and pitch Λ . To check the validity of the

Fig. 4. Diagram of the wavelength responses of effective area both for the second-order modes of O-PCF (solid curve) and H-PCF (dashed curve). The crossings of the dotted line with the horizontal axis indicate the cut-off for the second-order modes of two PCFs.

Fig. 6. Diagram of the wavelength responses of effective area both for the fundamental modes of O-PCF (solid curve) and H-PCF (dashed curve). The crossings of the dotted line with the horizontal axis indicate the cut-off for the fundamental modes of two PCFs.

Fig. 5. Single-mode/multi-mode phase diagram. The solid curve and the dashed curve show the phase boundary of O-PCF and H-PCF, respectively. The dots show the boundary of Kuhlmeier et al. [12].

effective area approach for finding the cut-off wavelength of the fundamental modes, another approach by comparing the cladding effective index and the effective mode index is employed to solve the cut-off behavior of the H-PCF. The cladding effective index is the fundamental space-filling mode (FSM) index of infinite rings of air-holes with the core being absent, and is solved by the two-dimensional finite-difference time-domain method with periodic boundary conditions [13].

Fig. 7. Effective index as a function of λ/Λ for H-PCF at $d/\Lambda = 0.6$. The solid curve and the dashed curve show the effective index of fundamental mode and FSM, respectively. The crossing of fundamental mode and FSM curve shows the cut-off for the fundamental mode of H-PCF.

Fig. 7 shows the wavelength responses of the cladding effective index and the fundamental mode index of the H-PCF with $AFF = 0.327$ and $\Lambda = 1 \mu\text{m}$. It is seen that the effective index of the fundamental mode is smaller than the cladding effective index at the normalized wavelengths higher than $\lambda/\Lambda = 2.15$. It implied this four-ring's H-PCF is cut-off at the wavelength $\lambda/\Lambda = 2.15$, which is reasonably consistent with the result solved by the effective area approach.

Carrying out the same analysis for different AFF using the effective area approach, a phase diagram between the single-mode and cut-off regions can be constructed. Combining the results in Fig. 5, Fig. 8 shows the cut-off, single-mode, and multi-mode phase diagram for the H-PCF and O-PCF. The cut-off and single-mode phase boundary proposed by Shen et al. [14] for the H-PCF is also presented by dots in Fig. 8 to demonstrate the accuracy of our approach. Compared with H-PCF, it is seen that under the same AFF the O-PCF behaves as cut-off at longer wavelength

and operates at multi-mode region at shorter wavelength than the H-PCF does. In other word, under the same AFF conditions, the O-PCF has significantly wider wavelength range operated in the single-mode regions than the H-PCF.

3.2. Field confinement

Fig. 9(a) and (b) shows the magnetic field intensity distributions of the fundamental modes for the H-PCF and O-PCF, respectively, with the same $AFF = 0.327$, $\Lambda = 1 \mu\text{m}$, and wavelength $\lambda = 0.7 \mu\text{m}$. It is clearly seen that the field pattern shape of the O-PCF looks more like circular than the H-PCF. Because there are eight air-holes in the first ring of the O-PCF and these holes are kept in octagonal rotational symmetry, the O-PCF has the fundamental field distributions more similar to the standard step-index fibers. Another point worth to note is that there is more energy distributed between the air-holes of the first ring for the H-PCF than for the O-PCF under the same air-filling ratio. It causes more confinement loss for the H-PCF. Fig. 10 shows the normalized effective area (A_{eff}/Λ^2) of the H-PCF and O-PCF versus normalized wavelength for three different $AFF = 0.145$, 0.227 and 0.327 . Three rings of air-hole structure is used in both types of the PCF and the air-hole pitch is equal to $1 \mu\text{m}$. It has been known that the confinement loss of the PCF is strongly related

Fig. 8. Cut-off/single-mode/multi-mode phase diagram. The solid curve and the dashed curve show the phase boundary of O-PCF and H-PCF, respectively. The dots show the boundary of Shen et al. [14].

Fig. 9. The field intensity distributions of the fundamental modes with $AFF = 0.327$, $\Lambda = 1 \mu\text{m}$, and wavelength $\lambda = 0.7 \mu\text{m}$ for (a) H-PCF and (b) O-PCF.

Fig. 10. The normalized effective area (A_{eff}/A^2) of the H-PCF and O-PCF versus normalized wavelength for three different $AFF = 0.145$, 0.227 , and 0.327 .

to the effective area of the fiber. A higher effective area fiber has more confinement loss [10,15,16]. As shown in Fig. 10, there are three important phenomena. First, it is seen that for both types of PCF longer wavelength results in higher effective area. Second, lower AFF causes higher effective area both for H-PCF and O-PCF because there is more energy possibly escaped from the fiber core through the path between the air-holes for smaller AFF fiber. Third, it is clearly seen that the O-PCF always has smaller effective area than the H-PCF does under the same normalized wavelength and AFF. Based on the above discussion, it implies that the O-PCF structure has less confinement loss than the typical H-PCF under the same comparison criterion and the field pattern of the O-PCF for the fundamental mode is more similar to the circular shape of the standard step-index fiber than that of the H-PCF.

4. Conclusion

An octagonal PCF (O-PCF) is proposed based on repeating an isosceles triangle with 45° vertex angle. The propagation characteristics of the O-PCF are investigated by using the full-wave boundary element method. Compared with the

typical hexagonal PCF (H-PCF), the O-PCF has three special properties. First, the O-PCF has significantly wider wavelength range for single-mode operation than the H-PCF does under the same air-hole pitch and filling fraction ratio. Second, the O-PCF can have more circular-like field distribution than the H-PCF. Third, the O-PCF has less confinement loss than the H-PCF under the same AFF and air-hole pitch. One point worth to consider is the fabrication issue. The stack-and-draw procedure provides highly versatile and complex lattices to be assembled from individual stackable units of suitable size and shape. It has been confirmed by the commercial PCF manufacturer that the O-PCF structure can be fabricated [17].

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