Submillimeter-Wave Phasor Beam-Pattern Measurement Based on Two-Stage Heterodyne Mixing With Unitary Harmonic Difference

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Abstract—A near-field phasor beam measurement system is developed for the characterization of heterodyne receiver optics at submillimeter-wave frequencies. The system synthesizes a pair of submillimeter-wave signals as the RF and local oscillator (LO) sources from common reference sources. The synthesized harmonic numbers of the RF and LO sources are arranged with difference by one, which makes this a new configuration with a unitary harmonic difference. The coherent RF and LO signal are down-converted by the receiver under test, then mixed with the microwave-frequency common reference signal to generate the second-order IF signal around 100 MHz for amplitude and phase comparison. The amplitude and phase fluctuation of the measurement system at 683 GHz is within ± 0.2 dB and $\pm 4^\circ$ in a 1-h period, respectively. The system dynamic range at 683 and 250 GHz can be as high as 43 and 47 dB, respectively. The system is then used to measure the receiver beam patterns at 683 and 250 GHz with different RF transmitting probe antennas.

Index Terms—Frequency conversion, Gaussian beams, phase measurement, submillimeter-wave receivers.

I. INTRODUCTION

WITH THE increasing demands on atmospheric observing and radio astronomical instruments, the development of the submillimeter-wave heterodyne receivers with high sensitivity and accurate pointing is required [1]–[3]. The highly sensitive heterodyne receiver can be implemented by using a nearly quantum limited superconductor–insulator–superconductor (SIS) mixer or a hot-electron bolometer mixer as the front-end [4]–[6]. For accurate pointing of the receiver optics, a useful and efficient diagnostic instrument is required to measure the beam pattern to analyze the optical parameters of the receiver at the operating frequency.

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A typical phasor antenna pattern measurement system consists of a coherent transmitter–receiver module, or so-called phasor network analyzer, combined with a precision scanner to compare the amplitude and phase between transmitted and received signals at different relative positions. For frequencies below 500 GHz, a commercially available millimeter-wave phasor network analyzer was introduced recently [7]. For higher frequencies below 1 THz, a millimeter-wave vector network analyzer is also available with user-reconfigurable test sets.¹ Previous research on antenna or telescope-optics pattern measurement using commercially available phasor network analyzers has also been published [8]–[17], and the amplitude dynamic range can be as high as 60 dB at 860 GHz. However, these commercially available phasor network analyzers are based on the harmonic mixer as the down-converting device, which limits its application to the integrated measurement of the antenna system with a heterodyne receiver. To solve this problem, some research for frequency ranging from 200 to 1500 GHz has also been published [18]–[22]. These receiver beam pattern measurement configurations typically require complicated frequency synthesis and conversion. Phase stability is an important factor in the measurement system. For a system with phase sensitive to the environmental temperature, the scanner carried RF transmitter requires an axis along the beam propagation to scan the beam pattern at different propagation positions. However, once the measurement configuration exhibits stable phase, the measured amplitude and phase data secured by scanning at a fix plan perpendicular to the beam axis are sufficient to extract the beam parameters.

In this paper, the simple, but stable measurement configuration for submillimeter-wave phasor beam pattern and receiver optics alignment described in [23] for 690 GHz is adapted to multiple frequency measurement in Section II. The configuration only needs two frequency standards—one microwave swept frequency source and one fixed reference frequency source—to synthesize and convert the frequencies required for the measurement. When measuring the receiver systems equipped with multiple channels for different frequency bands, this configuration can be used for measuring beam pattern by replacing the transmitting antenna, rotating the polarization of the RF transmitting module, and switching the active receiver channel. A theoretical formulation of the phase stability of the measurement configuration is derived in Section III. Both the theoretical prediction and

¹Type MVNA-8-350 Millimeter-Wave Network Analyzer, AB Millimetre, Paris, France.

the measurement show that this method can reduce the system phase variation to as low as $\pm 4^\circ$ during a 1-h period.

In Section IV, a multichannel SIS-based heterodyne receiver for the Submillimeter Array (SMA) telescope is measured as the receiver under test. Operating frequencies around 250, 349, 409, and 683 GHz are chosen to measure the receiver front-end quasi-optics beam pattern. The measurement results show a dynamic range around 47 dB for the 230-GHz band receiver and 43 dB for the 690-GHz band. A formulation for extracting the optical parameters of the truncated Gaussian Beam is also derived. The extracted parameters are compatible with the theoretical prediction.

II. MEASUREMENT ARRANGEMENT

In order to configure the submillimeter-wave coherent transmitter–receiver module in a simpler approach with better performance, the frequency synthesis and conversion approaches of the measurement arrangements in [7]–[22] are investigated to find a method for improvement.

The millimeter-wave phasor network analyzer in [7] requires two microwave synthesizers to generate RF and local oscillator (LO) signal. An additional 20-MHz signal source is used as phase and amplitude comparison reference. The receiver inside the test sets uses harmonic mixers as down-converters, but it is still possible to adapt it for heterodyne receiver measurement. However, even if one can apply this configuration to the receiver beam pattern measurement, the disadvantage on the coherence between RF and LO signals due to the locking frequency at only 10 MHz limits its operation at higher submillimeter-wave frequencies. Typical operation of this kind of millimeter-wave phasor network analyzer requires 128 times averaging to reduce the random fluctuation, which leads to longer measurement time.

The other popular phasor network analyzers described in [8]–[17] provide flexible and different frequency synthesis and conversion approaches. When operating at a lower frequency, it utilizes a frequency multiplier to generate RF source and down conversion of the received RF signal to IF by harmonic mixer, which is similar to [7], for higher submillimeter-wave frequency it relies on one swept microwave synthesizer and two distinct low-frequency signal sources for phase and amplitude comparison [11]. Similar to the case in [7], this network analyzer is quite suitable for measurement related to an antenna without an integrated receiver, but for the measurement related to antenna–receiver integration, suitable adoption of the frequency synthesis approach is required.

The measurement configuration of [21] is another approach aiming to solve the antenna–receiver integrated measurement. The basic idea is to generate a pair of coherent RF and LO sources by using a harmonic-mixer-based phase-locked loop (PLL), followed by a cascading frequency multiplier to terahertz output frequency. A microwave synthesizer is used to provide the pumping of the harmonic mixers in a PLL pair, and the 10-MHz reference signal is multiplied to generate the reference signal for the LO PLL, the RF PLL, second down-conversion pumping, and phasor comparison. The arrangement of different reference signals for frequency synthesis is mainly due to the limited harmonic number suitable for frequency multiplication

to terahertz frequency. The main disadvantage of this configuration is the opposite sidebands of the RF and LO signals in the PLL, and the resulting IF leads to additive phase drifting, which will be discussed later. The measurement configurations in [18]–[20] are basically the hybrid types of [7] and [21].

Based on the above discussion, the principle to simplify the frequency synthesis and down-conversion to fulfill the antenna–receiver integration measurement can be summarized as follows.

- 1) Try to synthesize both RF and LO signal frequencies by the same signal source. For W - or F -band signal sources, which need a harmonic mixer in the PLL, a microwave synthesizer to pump the harmonic mixer with frequency f_{syn} and an 80–110-MHz reference signal with frequency f_{REF} distributed by power dividers are required.
- 2) Arrange the RF and LO PLL on the same sideband to avoid phase drifting enlargement.
- 3) For the receiver under test, the frequency of the IF signal may be too high to compare the amplitude and phase directly, second down conversion is required. To ensure coherence, the second-stage mixing pumping frequency and the second IF frequency should be f_{syn} and f_{REF} , respectively.

Based on the above principle, a new measurement arrangement is proposed, as shown in Fig. 1. In the configuration, the SIS mixer based heterodyne receiver, including the front-end optics and the LO, is the device-under-test. W - or F -band Gunn oscillators are used as the fundamental frequency sources to generate RF signal and LO signal for the receiver. The frequency relationship of this measurement arrangement is described as follows.

Let $f_{\text{osc,RF}}$ and $f_{\text{osc,LO}}$ be the frequencies of the phase-locked oscillators for RF and LO sources, and let f_{syn} and f_{REF} be the frequencies of the synthesizers for the PLL harmonic-mixing pumping LO source and reference source. One can then express the frequencies as

$$f_{\text{osc,RF}} = n_{p,\text{RF}} f_{\text{syn}} + f_{\text{REF}} \quad (1a)$$

$$f_{\text{osc,LO}} = n_{p,\text{LO}} f_{\text{syn}} + f_{\text{REF}} \quad (1b)$$

$$f_{\text{RF}} = m_{\text{RF}} f_{\text{osc,RF}} \quad (2a)$$

$$f_{\text{LO}} = m_{\text{LO}} f_{\text{osc,LO}} \quad (2b)$$

In (1), $n_{p,\text{RF}}$ and $n_{p,\text{LO}}$ are the harmonic numbers of the harmonic mixers in the RF and LO PLLs, respectively. In (2), m_{RF} and m_{LO} are the multiplying factors of the frequency multipliers for the RF and LO sources, respectively. Please note that, in (1), the PLL is arranged at the same sideband, and the upper sideband is expressed in the formulation. This arrangement is to minimize the harmonic difference in the IF output of the receiver under test.

The RF and LO frequency difference should be within the heterodyne receiver IF frequency passband. From (1) and (2), one can get

$$\begin{aligned} f_{\text{IF}} &= |f_{\text{RF}} - f_{\text{LO}}| \\ &= |(m_{\text{RF}} n_{p,\text{RF}} - m_{\text{LO}} n_{p,\text{LO}}) f_{\text{syn}} + (m_{\text{RF}} - m_{\text{LO}}) f_{\text{REF}}|. \end{aligned} \quad (3)$$

Since $n_{p,\text{RF}}$, $n_{p,\text{LO}}$, m_{RF} , and m_{LO} are all integers, and the IF signal of the receiver is only down-converted once, the best

Fig. 1. System block diagram of the submillimeter-wave heterodyne receiver field pattern measurement arrangement.

way is to choose the IF frequency to be equal to $f_{\text{syn}} + f_{\text{REF}}$. The reference frequency based on the above-described principle is much lower than the IF frequency. Therefore,

$$f_{\text{IF},L} \leq f_{\text{syn}} \leq f_{\text{IF},H} \quad (4)$$

and

$$|m_{\text{RF}} n_{p,\text{RF}} - m_{\text{LO}} n_{p,\text{LO}}| = 1 \quad (5)$$

$$|m_{\text{RF}} - m_{\text{LO}}| = 1. \quad (6)$$

$f_{\text{IF},L}$ and $f_{\text{IF},H}$ in (4) are the lower and higher edges of the IF passband. Since (5) and (6) represent the harmonic difference of the synthesizer and the phase-locked reference signal sources in the heterodyne receiver output, and these differences are both equal to 1, one can call this approach the “unitary harmonic difference” method.

In operation, one of the four frequency bands in the receiver system to be measured can be selected by the microwave switches and the rotating optical mirror. An open-ended waveguide antenna with suitable aperture to keep single mode propagation is installed at the output of the RF transmitter when measuring the different frequency bands.

Concerning the mapping approaches, we choose conventional motorized XY scanning. Considering the 90° rotation on the polarization of the different frequency bands, an additional

manual rotation stage is used for installing the RF transmitter. For the application to other situations, for example, measuring the large-aperture reflector antenna, the proposed frequency synthesis approach can be used along with an innovative hologram-based CATR for large-aperture antenna measurement [13]–[16].

III. PHASE STABILITY CONSIDERATION

The phase fluctuation of the measurement system can be analyzed as the following. Consider the signal frequencies existing in the beam pattern system as f_{RF} , f_{LO} , f_{IF} , f_{syn} , and f_{REF} with the corresponded phase errors $\Delta\theta_{\text{RF}}$, $\Delta\theta_{\text{LO}}$, $\Delta\theta_{\text{IF}}$, $\Delta\theta_{2\text{ndIF}}$, $\Delta\theta_{\text{syn}}$, and $\Delta\theta_{\text{REF}}$. In addition, the phase variations induced by the cable length variation due to ambient temperature change are denoted as $\Delta\theta_{\text{syn,RF}}$, $\Delta\theta_{\text{syn,LO}}$, $\Delta\theta_{\text{syn,IF}}$, $\Delta\theta_{\text{REF,RF}}$, $\Delta\theta_{\text{REF,LO}}$, and $\Delta\theta_{\text{REF,IF}}$. The phase errors introduced by the phase-locked circuits of the RF and LO modules are denoted as $\Delta\theta_{\text{PLL,RF}}$ and $\Delta\theta_{\text{PLL,LO}}$.

According to (1), since both the RF and LO sources are phase locked to the upper sideband of the harmonic mixer, the phase fluctuations of RF and LO sources can be expressed as

$$\begin{aligned} \Delta\theta_{\text{RF}} &= m_{\text{RF}} [n_{p,\text{RF}} (\Delta\theta_{\text{syn,RF}} + \Delta\theta_{\text{syn}}) \\ &\quad + (\Delta\theta_{\text{REF,RF}} + \Delta\theta_{\text{REF}}) + \Delta\theta_{\text{PLL,RF}}] \\ \Delta\theta_{\text{LO}} &= m_{\text{LO}} [n_{p,\text{LO}} (\Delta\theta_{\text{syn,LO}} + \Delta\theta_{\text{syn}}) \\ &\quad + (\Delta\theta_{\text{REF,LO}} + \Delta\theta_{\text{REF}}) + \Delta\theta_{\text{PLL,LO}}]. \end{aligned} \quad (7)$$

From (5) and (6), the phase fluctuation of the IF signal is

$$\begin{aligned}\Delta\theta_{\text{IF}} &= |\Delta\theta_{\text{RF}} - \Delta\theta_{\text{LO}}| \\ &= |(m_{\text{RF}}n_{p,\text{RF}}\Delta\theta_{\text{syn,RF}} - m_{\text{LO}}n_{p,\text{LO}}\Delta\theta_{\text{syn,LO}}) \\ &\quad + \Delta\theta_{\text{syn}} + (\Delta\theta_{\text{PLL,RF}}^2 + \Delta\theta_{\text{PLL,LO}}^2)^{1/2} \\ &\quad + (\Delta\theta_{\text{REF,RF}} - \Delta\theta_{\text{REF,LO}})|. \quad (8)\end{aligned}$$

Similarly, from (5) and (6), the phase fluctuation for the second IF mixer output signal can be expressed as

$$\begin{aligned}\Delta\theta_{2\text{ndIF}} &= |\Delta\theta_{\text{IF}} - (\Delta\theta_{\text{syn,IF}} + \Delta\theta_{\text{syn}})| \\ &= |(m_{\text{RF}}n_{p,\text{RF}}\Delta\theta_{\text{syn,RF}} - m_{\text{LO}}n_{p,\text{LO}}\Delta\theta_{\text{syn,LO}}) \\ &\quad + \Delta\theta_{\text{syn,IF}} + (\Delta\theta_{\text{PLL,RF}}^2 + \Delta\theta_{\text{PLL,LO}}^2)^{1/2} \\ &\quad + (\Delta\theta_{\text{REF,RF}} - \Delta\theta_{\text{REF,LO}})|. \quad (9)\end{aligned}$$

In the measurement arrangement, the networks distributed to RF and LO modules for f_{syn} share a common 96-in-long coaxial cable connected to a power divider and followed by a pair of cables with the same length of 24 in. The link for f_{syn} to the second IF mixer is directly connected after the first-stage power divider, which means $\Delta\theta_{\text{syn,IF}}$ is ignored. Since all the cables are operated under the same environment with the same ambient temperature variation, one can assume $\Delta\theta_{\text{syn,RF}} = \Delta\theta_{\text{syn,LO}}$. In addition, the cable lengths of RF and LO reference signals are the same, or $\Delta\theta_{\text{REF,RF}} = \Delta\theta_{\text{REF,LO}}$. The second IF phase variation is then

$$\Delta\theta_{2\text{ndIF}} = \left| \Delta\theta_{\text{syn,RF}} + (\Delta\theta_{\text{PLL,RF}}^2 + \Delta\theta_{\text{PLL,LO}}^2)^{1/2} \right|. \quad (10)$$

From the phase-noise measurement on the phase-locked Gunn oscillator, the phase noise of the phase-locked circuit is approximately 2.0°. The phase stability is approximately 3.0° at 26.5 GHz for a 36-in-long cable,² which corresponds to 2.15° at 5.74 GHz for a 120-in-long cable. Therefore, from (10), the second IF output phase fluctuation is approximately 4.97°.

For the amplitude fluctuation, it is basically due to the bias voltage variation controlled by the PLL and the size of the Gunn oscillator cavity resonator changing with temperature. It is believed the amplitude fluctuation can be minimized by proper temperature control, and it should be small if the Gunn oscillator is turned on for a long time to be thermally balanced to the ambient temperature.

For comparative purposes, the measurement arrangement is analyzed such as that in [21]. The arrangement requires RF and LO sources phase locked at two opposite sidebands, and the reference signals for RF, LO, and IF are 120, 200, and 240 MHz, respectively. The microwave synthesizer signal f_{syn} in the RF and LO signals is cancelled by the submillimeter-wave mixer and the IF frequency f_{IF} is

$$f_{\text{IF}} = m_{\text{RF}}f_{\text{REF',RF}} + m_{\text{LO}}f_{\text{REF',LO}}. \quad (11)$$

It is clear that the phase fluctuation at IF in [21] is additive. In addition, the 3-GHz LO signal for the second IF mixer, the reference signals of RF and LO, and the reference signal for the

TABLE I
HARMONIC NUMBERS AND SIGNAL FREQUENCIES FOR UNITARY
HARMONIC DIFFERENCE BEAM PATTERN MEASUREMENT

Band	176-256 GHz	250-354 GHz	320-420 GHz	600-696 GHz
m_{LO}	2	3	4	6
m_{RF}	3	4	5	7
$f_{\text{Gunn,LO}}$ (GHz)	88-128	83-118	82-105	100-120
$f_{\text{Gunn,RF}}$ (GHz)	58.6-85.3	62.5-88.5	66-86	86-103
n_{LO}	22	21	19	20
n_{RF}	15	16	15	17
f_{syn} (GHz)	4.0-6.0	4.0-6.0	4.0-6.0	4.0-6.0
f_{REF} (MHz)	109	109	109	109
RF Output Antenna	WR-3	WR2.2	WR 1.8	WR 1.2
<i>Example of Frequency Sets</i>				
f_{LO} (GHz)	244.418	343.677	414.636	689.454
f_{RF} (GHz)	250.077	349.236	409.295	683.823
$f_{\text{Gunn,LO}}$ (GHz)	122.209	114.559	103.659	114.909
$f_{\text{Gunn,RF}}$ (GHz)	83.359	87.309	81.859	97.689
f_{syn} (GHz)	5.55	5.45	5.45	5.74

vector voltmeter comparison are through frequency multiplying of a 10-MHz signal. If this 10-MHz signal drifts by temperature even with a very small value, then the second IF output phase fluctuation will be amplified. Accurate environmental temperature control is, therefore, required.

When scanning the field of the beam pattern, there are necessarily moving or flexing cables. The phase variation of these flexing cables may introduce phase error between the LO and RF source in the measured data. Considering the extreme case of the cable carrying f_{syn} , the phase change is $\pm 1.0^\circ$ for one bend around 2.25-in (57 mm) radius, which corresponds to $\pm 45.0^\circ$ for measurement at 250 GHz or $\pm 120.0^\circ$ at 685 GHz. However, during the measurement, the flexing cable is bent with a radius much larger than 57 mm, and the scanning distance is not larger than 150 mm, thus the corresponding phase change can be negligible. For the Gaussian beam measurement far from its beam waist, the phase variation at the scanning plane will be quite large, which will also make this cable bend effect much smaller. For measuring the antenna patterns with expected flat phase profile or with large scanning area such as a 1–2-m size, the pilot signal phase correction approach introduced in [24] will be very helpful to enhance the measurement accuracy.

IV. MEASUREMENT RESULTS

In our experiment, the multiple channel receiver system of the SMA telescope is used as the receiver under test. Considering all the criteria given in (4)–(6) and the fact of the fixed harmonic number of the LO sources, the allowable harmonic numbers and its corresponding frequencies are listed in Table I.

A. Phase Stability Measurement

To verify the phase stability formulation described in (10), the 690-GHz heterodyne receiver is tested with the operating frequency listed in Table I. The amplitude and phase fluctuations over a reasonably long time are measured with the RF transmitter mechanical position fixed. The measurement results, as shown in Fig. 2, indicate the amplitude and phase fluctuation is within ± 0.2 dB and $\pm 4.0^\circ$ over a 1-h period, respectively. To be noted is that the testing environment is not air-conditioned. Table II shows the stability performance comparison of this measurement arrangement for other published results.

²W. L. Gore PHASEFLEX series microwave cable assembly

Fig. 2. Measured results of: (a) amplitude and (b) phase fluctuation of the field pattern measurement system at 690 GHz.

TABLE II
STABILITY PERFORMANCE COMPARISON OF VECTOR BEAM PATTERN MEASUREMENT SYSTEMS IN SUBMILLIMETER-WAVE FREQUENCIES

RF Frequency	Fluctuation		SNR	VNA / Receiver Type	Mapping Type	Ref.
	Amplitude	Phase				
345 GHz	-	-	50-60 dB	AB Millimetre	X-Y Scanning	[9]
840GHz	-	-	60 dB	AB Millimetre	X-Y-Z Scanning	[10]
652 GHz	-	-	80 dB	AB Millimetre	X-Y Scanning	[11]
625 GHz	-	-	60dB	AB Millimetre	X-Y Scanning	[13]
119GHz	0.8 dB	10°	32dB	HP-8510	Hologram CATR	[14]
654GHz	3 dB	40°	32-36 dB	AB Millimetre	Hologram CATR	[16]
322GHz	±0.6 dB	±125°	52 dB	AB Millimetre	Hologram CATR	[17]
200-700 GHz	0.1 dB	4 °	80 dB	AB Millimetre	X-Y Scanning	[18]
211 GHz	-	±4.0°	50 dB	SIS	X-Y Scanning	[19]
1028 GHz	±0.2 dB	±15.0°	40 dB	HEB	X-Y-Z Scanning	[22]
1464 GHz	±0.15 dB	±7.5°	25 dB	HEB	X-Y Scanning	[23]
683.62 GHz	±0.2 dB	±4.0°	43 dB	SIS	X-Y Scanning	This work

B. 2-D Beam Pattern Contour

After ensuring the stability of the system, the system is further scanned at the frequency listed in Table I. The optics of the lower frequency receiver bands (176–256 and 250–354 GHz) incorporates the wire grid polarizer, LO coupling mesh, and the turning mirrors. The optics of the higher frequency receiver band in 600–696 GHz is equipped with a Martin–Puplett diplexer (MPD) for LO–RF combining. Details of the receiver optics design are described in [25].

The RF source, which is composed of an 84–110-GHz Gunn oscillator, a WR-8 isolator, a crossguide coupler, and a diode frequency multiplier is mounted on a precision *X–Y* scanner. An open-ended waveguide with proper aperture size and integrated taper to WR-3 is used as the RF transmitting probe antenna. The frequency multiplier is originally equipped with a single-ended Schottky diode and the original multiplying factor is three. However, all the other harmonics can be generated by the diode [26]. As in [27] and [28], the radiation pattern of the probe antenna is regarded to be isotropic in the scanning area.

TABLE III
SCANNING PARAMETERS OF THE BEAM PATTERN MEASUREMENT

	240GHz 1-D Scan	690GHz 1-D Scan	240GHz 2-D Scan.	690GHz 2-D Scan.
Step (mm)	1.0	0.2	1.5	1.5
Scanning Range (mm)	140.0	80.0	140 × 140	90 × 90
Average Factor	4	4	4	4
Data Points	140	400	8931	4557

The total optical path from the RF transmitting probe antenna to the receiver horn, including the reflection within the MPD, is approximately 1600 mm for the 690-GHz band. The scanning step and points for the RF source are listed in Table III.

Fig. 3 shows the amplitude and phase contour plots of the field pattern of the 600–696-GHz SIS receiver. The MPD and cryogenic optical axis of the system are aligned mechanically prior to the installation. It shows that the system has a 43.5-dB dynamic range for the 690-GHz band, which is comparable to 43 dB given in [23]. The phase contour is matched with the amplitude contour.

Fig. 3. Measured results of: (a) amplitude and (b) phase contour of the 600–696-GHz SIS receiver beam pattern.

Fig. 4. Measured results of: (a) amplitude and (b) phase contour of the 176–256-GHz SIS receiver beam pattern.

Fig. 4 shows the amplitude and phase contour plots of the field pattern of the 176–256-GHz SIS receiver. With sufficient RF power level, the system dynamic range is approximately 47 dB at 250 GHz. It is worth noting that the contour plots at lower frequencies are not as smooth as the measured results of 690 GHz, which indicates: 1) stronger background scattering from the metallic mechanical structures of the optics; 2) limited isolation on position-dependent impedance mismatch along the microwave distribution network; and 3) a larger machining error on the receiver corrugated horn antenna and Teflon receiver lens due to larger device size.

C. Gaussian Optics Parameter Analysis

In order to ensure that the optics are satisfactory with respect to the design specification, the Gaussian beam parameters are extracted from the 690-GHz measured beam pattern. The detailed formulation for parameter extraction is described as follows.

For a fundamental Gaussian beam without offset and shifting, the amplitude distribution of the vector potential of a

paraxial propagated electromagnetic (EM) wave is expressed as [29]

$$\begin{aligned}
 u_{00}(x, y, z) &= \frac{j}{\lambda(z + jb)} \exp \left[-jk \frac{x^2 + y^2}{2(z + jb)} \right] \\
 &= \sqrt{\frac{2}{\pi}} \frac{1}{w(z)} \exp \left[-\frac{x^2 + y^2}{w^2(z)} \right] \exp(j\phi) \\
 &\quad \times \exp \left[-jk \frac{x^2 + y^2}{2R(z)} \right] \\
 &= A_{00}(x, y, z) \exp[j\psi(x, y, z)] \quad (12)
 \end{aligned}$$

where $w(z) = \sqrt{2b/k} \sqrt{1 + (z^2/b^2)}$, $R(z) = ((z^2 + b^2)/z)$, $\phi = \arctan(z/b)$, beam waist radius $w_0 = \sqrt{2b/k}$, $k = 2\pi/\lambda$, and b is the confocal parameter.

At the beam center of $x = y = 0$, $A_{00}(0, 0, z) = \sqrt{2/\pi}(1/w(z))$, the relative amplitude pattern is given as

$$A_{R,00}(x, y, z) = \frac{A_{00}(x, y, z)}{A_{00}(0, 0, z)} = \exp \left[-\frac{x^2 + y^2}{w^2(z)} \right]. \quad (13)$$

Fig. 5. Measured results and the Gaussian fit of: (a) amplitude and (b) phase profile of E - and H -plane beam pattern.

TABLE IV
COMPARISON ON THE EXTRACTED AND THEORETICAL GAUSSIAN PARAMETERS OF THE 600–696-GHz RECEIVER IN TEST DEWAR

	Amplitude Peak (x_c, y_c) (mm)	Phase Peak (x_c, y_c) (mm)	w_0 (mm)	z_w (mm)	δ_a (rad)	δ_p (rad)	w (mm)
Extracted E-Plan	(1.60, -1.20)	(-2.40, -0.60)	7.03	-789	1.484×10^{-4}	1.097×10^{-4}	17.22
Extracted H-Plan			6.37	-840	-5.2228×10^{-4}	2.288×10^{-5}	19.48
Theoretical Values (I)	(N/A)	(N/A)	7.55	-783	(N/A)	(N/A)	16.31
Theoretical Values (II)	(N/A)	(N/A)	6.00	-890	(N/A)	(N/A)	22.34

Note: Theoretical calculation (I) is based on thin lens assumption and the theoretical calculation (II) is based on thick lens assumption.

The beam radius at the scanning plane can then be calculated from the measured relative amplitude pattern as

$$w_x(z) = \sqrt{\frac{x^2}{-\ln[A_{R,00}(x, 0, z)]}} \quad (14a)$$

$$w_y(z) = \sqrt{\frac{y^2}{-\ln[A_{R,00}(0, y, z)]}}. \quad (14b)$$

From (12), the relative phase pattern is

$$\Psi_R(x, y, z) = -\frac{k}{2R(z)}(x^2 + y^2) \quad (15)$$

hence, the phase front curvature $R(z)$ can be calculated from the measured relative phase pattern as

$$R(z) = -\frac{k(x^2 + y^2)}{2\Psi_R(x, y, z)}. \quad (16)$$

For practical situations, the beam axis shift and offset should be considered. In other words, the term $x^2 + y^2$ becomes $x'^2 + y'^2 + 2R(z)\delta_x x' + 2R(z)\delta_y y'^2$, where $x' = x - x_c$, $y' = y - y_c$, and x_c , y_c is the beam center position, and δ_x and δ_y are the beam offsets in x - and y -axis. For the receiver system characterization demonstrated here, the optical beam propagation paths along the E - and H -plan are different due to the roof mirror pair, which leads to the different truncation of the beam in an orthogonal plan. As shown in Fig. 4, the amplitude pattern is elliptical and the phase pattern is almost circular. The slight misalignment introduced by the multiple reflection within the MPD also leads to a slightly difference

positions for the amplitude and phase peaks. Thus, the truncated Gaussian beam through an MPD then becomes

$$A_{R,00}(x, y, z) = \exp\left[-\left(\frac{x_a^2}{w_x^2(z)} + \frac{y_a^2}{w_y^2(z)} + k\delta_{xa}x_a + k\delta_{ya}y_a\right)\right] \quad (17a)$$

$$\Psi_R(x, y, z) = -k\left(\frac{x_p^2 + y_p^2}{2R(z)} + \delta_{xp}x_p + \delta_{yp}y_p\right). \quad (17b)$$

In (17), $x_a = x - x_{ca}$, $y_a = y - y_{ca}$, $x_p = x - x_{cp}$, $y_p = y - y_{cp}$, and (x_{ca}, y_{ca}) is the position of the amplitude peak and (x_{cp}, y_{cp}) is the position of the phase peak. Therefore, solving $w(z)$ and $R(z)$ from the measured pattern, the shift peak positions x_a , y_a , x_p , and y_p are determined directly from the measured data, and the tilt angles δ_{xa} , δ_{ya} , δ_{xp} , and δ_{yp} are extracted by

$$\begin{aligned} \frac{u_{00}(x', 0, z)}{u_{00}(-x', 0, z)} &= \exp(-2k\delta_{xa}x_a) \exp(-j2k\delta_{xp}x_p) \\ \frac{u_{00}(0, y', z)}{u_{00}(0, -y', z)} &= \exp(-2k\delta_{ya}y_a) \exp(-j2k\delta_{yp}y_p). \end{aligned} \quad (18)$$

The measured Gaussian parameters can then be extracted from (12)–(18). Please note that the E - and H - planes are along the x - and y -axis, respectively. With the dimensions of the receiver lens–horn combination, one can calculate the theoretical Gaussian beam results of the fundamental mode. Fig. 5 shows the measured beam patterns from [23] of the E - and H -planes, with 20- μ m scanning resolution. The extracted and theoretical Gaussian beam parameters are listed in Table IV. From this

table, one can see the difference between the theoretical values and measured results of the beam waist radius and positions. It can be regarded as the result of the truncation introduced by the MPD.

V. CONCLUSION

In this paper, the optics of a multiple-channel SIS heterodyne receiver has been measured by a new submillimeter-wave phase stable phasor beam pattern measurement configuration. The phase fluctuation at 690 GHz is as small as $\pm 4^\circ$ within 1 h. The measurement results indicate this new setup is an efficient tool to characterize and align the optics system of the sub-terahertz SIS receiver. With suitable frequency selection and hardware configuration, this method may be applied to other frequencies as high as 1 THz.

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