Unit commitment by annealing-genetic algorithm

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Abstract

This paper presents the application of the annealing-genetics (AG) algorithm to solve the unit commitment (UC) problem. By exploiting the similarity between a minimization process and the cooling of a molten metal, the simulated annealing (SA) guides the search point toward a global minimum with probability 1 using a devised move generation strategy and cooling schedule. But the SA usually takes much computation time in order to arrive at a near-global minimum. The genetic algorithm (GA) is a general-purpose optimization technique based on the principle of natural selection and natural genetics. The GA is a fast algorithm but usually has the inferior solution quality compared to the SA. The AG algorithm incorporates the genetic algorithm into the simulated annealing to improve the performance of the simulated annealing in solving combinatorial optimization problems such as the unit commitment problem. Numerical results on two cases including a realistic Taiwan power (Taipower) system and comparisons with results obtained using the AG, the SA, the GA, the dynamic programming (DP) and the Lagrangian relaxation (LR), show that the features of easy implementation, fast convergence, and highly near-optimal solution to the UC problem can be achieved by the AG. © 2002 Elsevier Science Ltd. All rights reserved.

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1. Introduction

The task of the unit commitment (UC) problem involves scheduling the on/off status, as well as the real power outputs, of thermal units for use in meeting the forecasted demand over a future short-term (24–168 h) period. The resultant schedule should minimize the system production cost during the period while simultaneously satisfying the load demand, the spinning reserve, and physical and operational constraints of the individual units. Since an improved UC schedule may save the electric utilities millions of dollars per year in production costs, the UC problem is an important optimization task in the daily operational planning of modern power systems.

Mathematically, the UC problem may be formulated as a non-linear, large scale, mixed-integer combinatorial optimization problem. The exact solution to the problem can be obtained only by the complete enumeration, often at the cost of prohibitive computational time for realistic power systems [1]. Research endeavours, therefore, have been focused on efficient, near-optimal UC algorithms, which can be applied to large-scale power system and have reasonable storage and computational time requirements. A survey

of literature on the UC methods reveals that various numerical optimization techniques have been employed to approach the UC problem. Specifically, there are the priority list method [2,3], the integer programming [4,5], the dynamic programming [6–11], the Lagrangian relaxation method [12,13], the mixed integer programming [14], the branch-and-bound method [15,28], and so on. Among these methods, the priority list method is simple and fast, but the quality of final solution is quite far from the optimum. The dynamic programming method based on a priority list is flexible, but the computational time suffers from ‘curse of dimensionality’. The branch-and-bound method adopts a linear function to represent the fuel consumption and time-dependent start cost and obtains the required lower and upper bounds. The shortcoming of the method is that the execution time required increases rapidly for the large-scale UC problem. The integer and mixed-integer methods adopt linear programming techniques to solve and check for an integer solution. These methods have only been applied to the small UC problem and have required major assumptions that limit the solution space. The Lagrangian relaxation method provides a fast solution but it suffers from the numerical convergence and the solution quality.

Aside from the above methods, there is another class of numerical techniques applied to the UC problem. Specifically, there are the artificial neural network [16,17], the simulated annealing (SA) [18] and the genetic algorithm

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(GA) [19,20,23,24,29]. These methods can accommodate more complicated constraints and are claimed to have better solution quality. The SA is a powerful, general-purpose stochastic optimization technique, which can theoretically converge asymptotically to the global optimum solution with probability 1. One main drawback of the SA is, however, that it takes a great deal of CPU time to find the near-global minimum. The GA is a general-purpose stochastic and parallel search method based on the mechanics of natural selection and natural genetics. In contrast to the SA, the GA is a fast search method but has inferior solution quality. Recently, Wong and Wong [30] have developed the hybrid GAA2 algorithm for the UC problem. The GAA2 algorithm is based on the combination of the incremental GA approach and the SA technique with population size of 2 to minimize the memory requirement. Empirical results show that the GAA2 algorithm has excellent performance for the UC problem [25–26].

In this paper, we apply an Annealing-Genetics (AG) algorithm to solve the UC problem. The basic idea of the AG is that the GA is incorporated into the SA in such a way that it reduces the computational time of the SA while retaining the main characteristics of the SA, i.e. the ability to achieve the near-global minimum in a finite time. The AG has been successfully applied to many difficult combinatorial optimization problems [22]. A description of the AG algorithm is presented in Section 2. Then a detailed application of the AG algorithm to the UC is given in Section 3. Numerical tests on two cases using the AG, the SA, the GA, the dynamic programming and the Lagrangian Relaxation are presented in Section 4. Finally, a summary conclusion is given in Section 5.

2. Annealing-genetics algorithm

For the AG algorithm to be efficiency we require (1) the gap between the final solution and the optimal solution should be less than that of the GA and (2) the computational time must be less than that of the SA. These two goals are achieved by designing an annealing schedule that has incorporated the advantages of the GA. The important parameters affecting the efficiency of the SA are the initial temperature, the total length of the Markov chain at each temperature, the equilibrium condition, and the frozen condition. The AG uses the notions from the GA to initiate and control these parameters. A brief description of the AG algorithm is given in the remainder of this section. A detailed AG algorithm can be found in [22].

The AG algorithm starts with a randomly generated population P'_0 . Next, the genetic operators are applied to produce a new population P_0 by rejecting the higher cost offspring so that the average cost must be less than the random cost P'_0 . Then, the Markov chains are generated from P_0 by a small value of temperature until the next generation P_1 is created. That is, starting with a random point from P_0 , the next point

is generated from the current point by the move generation strategy. An explanation of this strategy applied to the UC problem is given in Section 3.1.5. The initial value of temperature is obtained by dividing the difference between the points of highest cost and the lowest cost of P_1 by half of the population size, i.e.

$$\text{Unital temp} = \frac{(\text{the highest cost} - \text{the lowest cost})}{(\text{the population size}/2)}$$

In this way, the acceptance probability of the largest detrimental move will approximately be $0.6 (\cong \exp(-\Delta C/T))$, where $\Delta C = \text{new cost} - \text{old cost}$.

Progressively, for $k = 1, 2, \dots$, starting with the point of the lowest cost of P_k , the next point is generated from the current point by the move generation strategy. If it is accepted by the Metropolis criterion [18], the next point not only becomes the current point but also a point of the quasi-generation P'_{k+1} ; otherwise, a new starting point from P_k is attempted. After the population P'_{k+1} has been generated at each temperature T_k , the genetic operators such as crossover, mutation, and reproduction are applied to produce a new population P_{k+1} of the next generation.

An explanation of these genetic operators applied to the UC problem is given in Section 3.1.3. Finally, when more individuals of the population in a certain generation have the same lowest cost, the frozen condition is signaled.

Consequently, the AG algorithm consists of a two-stage cycle. The first stage is the search by the simulated annealing and the second stage is the evolution by the genetic operators. Fig. 1 shows the basic configuration of the AG approach. In each search stage, an iterative generate-and-test procedure is executed to generate many search paths that create a quasi-population. In each evolution stage, a set of genetic operators of the production function is applied to the quasi-population to evaluate their fitness and to produce the population of the next generation. Usually, the production function makes the next generation more mature than the last generation and creates a set of schemata of the population. On the other hand, the schemata are also used to guide the searches of the search stage in the next cycle. They have the power to guide the directions of the search paths toward to high performance regions.

Our AG approach has the following features: (1) it can be viewed as a simulated annealing algorithm with the population-based state transition and with the genetic-operator-based quasi-equilibrium control, (2) it can be viewed as a genetic algorithm with the Boltzmann-type selection operator.

3. Application to unit commitment problem

In this section, we first formulate the UC problem, and then present a detailed AG algorithm for solving the UC problem.

The objective of the UC problem is the minimization of

Fig. 1. The basic configuration of the AG algorithm.

the total production costs over the scheduling horizon. Therefore, the objective function is expressed as the sum of fuel and start-up costs of the generating units. Mathematically, the function is as follows:

$$C(X, P) = \sum_{i=1}^N \sum_{h=1}^H [X_{ih} F_i(P_{ih}) + ST_i X_{ih} (1 - X_{i(h-1)})] \quad (1)$$

where

$F_i(P_{ih})$: fuel cost function of the i -th unit with generation output, P_{ih} , at the h -th hour. Usually, it is a quadratic polynomial with coefficient a_i , b_i and c_i as follows:

$$F_i(P_{ih}) = a_i P_{ih}^2 + b_i P_{ih} + c_i$$

ST_i : the start-up cost of the i -th unit,

P_{ih} : the generation output of the i -th unit at the h -th hour,

P : the generation matrix with P_{ih} , $i = 1, 2, \dots, N$; $h = 1, 2, \dots, H$, as elements,

X_{ih} : the on/off status of the i -th unit at the h -th hour, and $X_{ih} = 0$ when off; $X_{ih} = 1$ when on,

X : the schedule matrix with X_{ih} , $i = 1, 2, \dots, N$; $h = 1, 2, \dots, H$, as elements,

N : the number of units,

H : the number of hours.

Now, we consider the constraint equations. Basically, the constraints result from the following consideration of the system operation:

1. balance of power supply and load demand;
2. spinning reserve;
3. generation limit;
4. minimum up-and down-time;
5. ramp rate limit.

Mathematically, these equality and inequality constraints can be expressed in the following vector forms:

$$L(X, P) = 0 \quad (2)$$

$$G(X, P) \leq 0 \quad (3)$$

Note that the inequality vector means that every component of the vector is an inequality equation.

In summary, combining Eqs. (1)–(3) yields a complete formulation of the UC problem as the following:

Minimize

$$C(X, P) \quad (4)$$

Subject to

$$L(X, P) = 0 \quad (5)$$

$$G(X, P) \leq 0 \quad (6)$$

Note, there are two matrices X , P to be solved in the above equations to yield the minimum production cost. In this paper, the AG algorithm searches X and P is computed by the Lambda-iteration method [1]. A step-by-step AG algorithm for the UC problem is outlined as follows.

3.1. The AG algorithm for UC

Step 1: Initialize the parameters such as the random number seed, the cooling rate (α), the Boltzman factor, the size of population, the mutation rate, the crossover rate, the penalty factor and the number of moves at each temperature;

Step 2: Randomly generate the initial population P'_0 ;

Step 3: Apply genetic operators to P'_0 to create P_0 ;

Step 4: Perform the Lambda_iteration of each decoded point in P_0 ;
Calculate the fitness value and the cost of each point in P_0 ;

Step 5: Solution_vector = current_point = the point of the lowest cost in P_0 ; $k = 0$;

Step 6: While (the system is not frozen) do
{no_of_point = 0;

Step 7: While (no_of_point $\leq$ population size) do
 {Generate next_point from current_point using a move generation strategy; check the feasibility of the next-point, and if any constraint is violated then goes to step 7; $\Delta C := (\text{cost of next_point}) - (\text{cost of current_point})$;

$$P_r := \min \left[1, \exp \left(\frac{-\Delta C}{T_k} \right) \right];$$

if ($P_r > \text{random}[0,1]$) then
 {put next_point into P'_{k+1} ;
 current_point := next_point;
 no_of_point := no_of_point + 1;}
 else
 {pick another point from P_k as
 current_point;}
 } end while

Step 8: Apply the genetic operators to P'_{k+1} to create P_{k+1} ;

Step 9: Perform the Lambda_iteration of each point in P_{k+1} ;
 Calculate the fitness and the cost of each point in P_{k+1} ;

Step 10: If (point with the lowest cost in $P'_{k+1} < \text{solution_vector}$) then
 {solution_vector := the point with the lowest cost in P_{k+1} ; current_point := the point with the lowest cost in P_{k+1} ;}
 }

Step 11: If (it is the initial stage) then
 {determining the initial temperature T_1 ;
 $T_1 := (\text{the highest cost} - \text{the lowest costs}) / (\text{the population size} / 2)$;
 }
 else { $T_{k+1} = T_k * \alpha$;}
 }

Step 12: $k := k + 1$;

Step 13: If (frozen condition is signaled) then
 {set system to be frozen;}
 }

Step 14: } end while

Step 15: Print out the solution-vector as the final solution;

Some explanations regarding the AG algorithm are given in the following.

3.1.1. Explanation 1

A point in the AG corresponds to an encoded schedule matrix in the UC problem. The matrix form of a point is shown in Fig. 2.

The i -th row of the point denoting the states of the i -th unit is a binary string divided into many binary substrings with 4 bits each such as $Y_{i1}, Y_{i2}, Y_{i3}, Y_{i4}$. The first substring, Y_{i1} , of each row represents one operating state of on or off (with a '1' or '0' in the leading bit) and the number of hours where on_state (or off_state) lasts for. The other substrings, Y_{i2}, Y_{i3}, Y_{i4} , of each row denote one operating state of on or

Fig. 2. Matrix form of a point.

off (with a '1' or '0' in the leading bit) and the number of hours of the on_state (or off_state) which exceeds the minimum up_time (or down_time). In this manner, the constraints of minimum up-time and down-time can be embedded in the binary substrings, Y_{i2}, Y_{i3}, Y_{i4} and will be satisfied implicitly during the genetic operation to greatly reduce the dimension of search space. If the overall number of hours represented by these substrings is less than the total H hours, then the last substring Y_{i4} is appended by the operating state, which is the same as the previous state until the H -th hour. For example, the i -th unit has a minimum up-time of 2 h and a minimum down-time of 3 h. The actual state of the i -th row of the schedule matrix and corresponding encoded i -th row of a schedule matrix are shown in Fig. 3.

3.1.2. Explanation 2

The fitness function translates the objective function with the constraints into a fitness value. The violations of constraints except the minimum up-time and down-time are penalized by a penalty factor. In the AG, the proposed

actual state of the i -th unit

Hour	1	2	3	4	5	6	7	8	9	10	11	12
Actual State	1	1	1	1	1	0	0	0	0	0	0	0
Hour	13	14	15	16	17	18	19	20	21	22	23	24
Actual State	0	0	0	0	0	0	0	1	1	1	1	1

encoded i -th row of a schedule matrix

1	1	0	1	0	1	0	1	0	0	1	1	1	0	1	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Fig. 3. Actual state and corresponding encoded i -th row of a schedule matrix.

fitness function is expressed as follows:

$$F(X, P) = \frac{1}{1 + K \left(\frac{F_{\max}}{F_r} - 1 \right)},$$

where

K : a scaling coefficient,

$$F_r := \frac{1}{C(X, P) + \sum_{h=1}^H \alpha_h \cdot L},$$

$C(X, P)$: the objective function of the UC (the production cost function)

α_h : = 1, when the violation of the constraints exists at the h -th hour,

: = 0, otherwise,

$F_{\max}$: the maximum of F_r of an individual point within the population,

L : the penalty factor.

3.1.3. Explanation 3

There are three genetic operators in the AG, namely reproduction, crossover, and mutation. The reproduction operator is a prime selection operator that copies an old point into a ‘mating pool’ according to the ‘roulette wheel parent selection’ technique [21,27]. In this technique, the points with larger fitness value have a higher probability to be selected.

The crossover operator recombines the extremely important features of two points to make the offspring points not only inherit some important characteristics from their parent points but also have the chance to get closer to the optimal solution. In the AG, we adopt a new crossover technique known as ‘uniform crossover’ [27], which exchanges bits between the parent points to create two new offspring points by a randomly generated mask. The scheme of ‘uniform crossover’ is shown in Fig. 4. In the random mask, the ‘1’ represents bit swapping and ‘0’ denotes bit unchanged.

The mutation operator allows us to create new point in the population and provides background variation depending on a mutation probability. The scheme of mutation operator is shown in Fig. 5.

3.1.4. Explanation 4

The ‘roulette wheel parent selection’ technique is used to select the ‘better’ parent points according to their fitness. It consists of the following steps [21,27]:

- Step 1: Sum the fitness of all points in the population; call it the FITSUM.
- Step 2: Generate a random number, P , between 0 and FITSUM.
- Step 3: Return the first chromosome whose fitness, added to the fitness of preceding points, is greater than or equal to P .

Fig. 4. Uniform crossover operator.

3.1.5. Explanation 5

The move generation strategy in step 7 of the AG consists of two steps as follows:

- Step 1: Randomly pick an element of encoded schedule matrix (a point).
- Step 2: Change the state of the element. Eliminate all resulting violations on constraints while making few changes on the commitment of unit in other hours.

4. Numerical simulations

In this section, two cases are studied to illustrate the effectiveness of the proposed AG algorithm in terms of its production cost and computational time. The first case compares the AG with deterministic algorithms, i.e. the dynamic programming (DP) and the Lagrangian relaxation (LR). The second case compares the AG with stochastic algorithms, i.e. the simulated annealing (SA) and the genetic algorithm (GA). After testing and evaluating different parameter combinations, the control parameter settings are the scaling coefficient $K = 1000$, the penalty factor $L = 99999$ and the cooling rate $\alpha = 0.93$. All programs are coded in Turbo C and implemented on a compatible personal computer (PC 486DX2-66). In order to avoid misleading results due to the stochastic nature of the AG, 20 runs were averaged for each case, with each run starting with different random populations.

Fig. 5. Mutation operator.

Table 1
Comparison performance among AG, DP and LR [29]

	Cost (\$) by DP	Cost (\$) by LR	Cost (\$) by AG
10 unit	565,825	565,825	564,005
20 unit	—	1,130,660	1,124,651
40 unit	—	2,258,503	2,249,072

4.1. Case 1: 10-, 20- and 40-unit systems by DP, LR and AG

In this case, the commitment schedule determined by the proposed AG technique is compared to that determined by the Lagrangian relaxation (LR) and the dynamic programming (DP) [29]. The system unit data and the load demands are shown in Appendix A. After testing and evaluating different parameter combinations, the control parameter settings of AG are the population size 70, the initial

temperature 900, the probability of crossover 0.8 and the probability of mutation 0.1. The performance of the AG compared with that of LR and DP is given in Table 1. From this result, it is shown that the solution by AG is better than that of LR and DP in terms of production cost.

4.2. Case 2: Taipower 40-unit system by SA, GA and AG

In this case, a practical power system, Taiwan Power system (Taipower system), was tested by using the SA, GA and AG. The Taipower system consists of 40 major thermal units. The parameters of operation of units are provided in Appendix B. The utility factors of the thermal system are varied from 72 to 94% for different seasons of the year to investigate the influences of the load curves on

Fig. 6. (a) The search process of AG, SA and GA for 94% utility factor; (b) The search process of AG, SA and GA for 87% utility factor; (c) The search process of AG, SA and GA for 80% utility factor; (d) The search process of AG, SA and GA for 72% utility factor.

Fig. 6. (continued)

the performance of the AG. The utility factor is defined as

$$\text{Utility factor} = \frac{\text{Max load}}{\text{Max available system capacity}}$$

The performance of the AG compared with that of the SA and the GA is given in Fig. 6 and Table 2. From these results, it is shown that the solution by the AG is better than that of the SA in terms of CPU time and better than that of the GA in terms of production cost.

The influences of the various control parameters on the performance of the AG were examined further. We changed one parameter at a time, keeping the other ones unchanged, and evaluated the performance of the AG for the load curve of 94% in Appendix B. The parameters concerned included the population sizes, and the probabilities of crossover and mutation. The performance for different population sizes is illustrated in Table 3. Effect

on the variation of the population size show that a large population requires more evaluations per generation, resulting in a relatively slow rate of convergence; however, a small population retains less variation of the individuals and results in a premature stopping of the AG. The performances for different probability of crossover and mutation are almost the same.

5. Conclusion

This paper investigates the unit commitment (UC) problem by the Annealing-genetic algorithm (AG) which incorporated the genetic algorithm (GA) into the simulated annealing (SA) algorithm to improve the CPU time of the simulated annealing (SA) algorithm and the solution quality of genetic algorithm (GA). The numerical tests and results

Table 2

Comparison performance among AG, SA and GA for different utility factor

Utility factor	Cost (\$) by SA	Cost (\$) by GA	Cost (\$) by AG	CPU time (s) by SA	CPU time (s) by GA	CPU time (s) by AG HSAGA
94% ^a	2,735,427	2,775,344	2,734,402	2402	2099	1801
87% ^b	2,377,755	2,408,643	2,376,098	3092	1961	2371
80% ^c	2,112,288	2,118,544	2,111,830	3954	1882	2566
72% ^d	1,857,257	1,872,381	1,856,584	3313	1952	3270

^a After testing and evaluating different parameter combinations, the control parameter settings are as follows: SA: the initial temperature = 900; GA: the population size = 50; the probability of crossover = 0.6; the probability of mutation = 0.0333; AG: the population size = 50; the probability of crossover = 0.7; the probability of mutation = 0.1.

^b After testing and evaluating different parameter combinations, the control parameter settings are as follows: SA: the initial temperature = 900; GA: the population size = 50; the probability of crossover = 0.8; the probability of mutation = 0.0333; AG: the population size = 50; the probability of crossover = 0.8; the probability of mutation = 0.3.

^c After testing and evaluating different parameter combinations, the control parameter settings are as follows: SA: the initial temperature = 900; GA: the population size = 50; the probability of crossover = 0.8; the probability of mutation = 0.0333; AG: the population size = 50; the probability of crossover = 0.8; the probability of mutation = 0.1.

^d After testing and evaluating different parameter combinations, the control parameter settings are as follows: SA: the initial temperature = 900; GA: the population size = 50; the probability of crossover = 0.8; the probability of mutation = 0.0333; AG: the population size = 60; the probability of crossover = 0.8; the probability of mutation = 0.3.

show that better solution of the unit commitment (UC) problem can be obtained by the AG algorithm compared with the DP, the LR, the GA and the SA. Moreover, the AG algorithm has the potential to be a practical near-global optimal solution to the UC problem for the large-scale electric power systems.

Table 3

The performance of AG for different population size

Population size ^a	Cost (\$)	CPU time (s)
40	2,734,948	1815
50	2,734,402	1799
60	2,734,643	2481

^a After testing and evaluating different parameter combinations, the control parameter settings are given in the following: the probability of crossover = 0.8; the probability of mutation = 0.1.

Appendix A. Unit data and load demands for case 1 [29]

Data for the 10-unit system

Unit	$P_{\max}$	$P_{\min}$	a	b	c	Up	Down	Hot start cost (\$)	Cold start cost (\$)	Cold start (h)	it (h)
1	455	150	1000	16.19	0.00048	8	8	4500	9000	5	8
2	455	150	970	17.26	0.00031	8	8	5000	10000	5	8
3	130	20	700	16.60	0.002	5	5	550	1100	4	-5
4	130	20	680	16.50	0.00211	5	5	560	1120	4	-5
5	162	25	450	19.70	0.00398	6	6	900	1800	4	-6
6	80	20	370	22.26	0.00712	3	3	170	340	2	-3
7	85	25	480	27.74	0.0079	3	3	260	520	2	-3
8	55	10	660	25.92	0.00413	1	1	30	60	0	-1
9	55	10	665	27.27	0.00222	1	1	30	60	0	-1
10	55	10	670	27.79	0.00173	1	1	30	60	0	-1

unit: the number of the unit,

$P_{\max}$, $P_{\min}$: the maximum and minimum power generation,

a , b , c : the coefficients of the fuel cost function,

down, up: the minimum down and up time,

$$\text{start up cost} = \begin{cases} \text{hot start cost, if down time} \leq \text{cold start h} \\ \text{cold start cost, otherwise} \end{cases}$$

it: the initial time of the unit, if it is positive (or negative), indicates the number of hours the unit has been already up (or down).

The 24 hour load demand

Hour	P_{load} (MW)	Hour	P_{load} (MW)
1	700	13	1400
2	750	14	1300
3	850	15	1200
4	950	16	1050
5	1000	7	1000
6	1100	18	1100
7	1150	19	1200
8	1200	20	1400
9	1300	21	1300
10	1400	22	1100
11	1450	23	900
12	1500	24	800

The problem data were scaled appropriately for the problems with more units. The reserve was assumed to be 10% of the demand.

Appendix B. Taipower 40 units' data and load demands for case 2 [27]

Data for the 40-unit system

Unit	P_{max}	P_{min}	a	b	c	stc	Up	Down	it
1	80	40	170.44	8.336	0.03073	247.3962	2	3	3
2	120	60	309.54	7.0706	0.02028	248.1078	2	3	3
3	190	80	369.03	8.1817	0.00942	245.8377	2	3	3
4	42	24	135.48	6.9467	0.08482	246.2440	2	3	3
5	42	26	135.19	6.5595	0.09693	231.7961	2	3	3
6	140	68	222.33	8.0543	0.01142	267.5881	2	3	3
7	300	110	287.71	8.0323	0.00357	267.0076	2	3	3
8	300	135	391.98	6.999	0.00492	326.6603	2	3	3
9	300	135	455.76	6.602	0.00573	267.6292	2	3	3
10	300	130	722.82	12.908	0.00605	376.2845	2	3	3
11	375	94	635.20	12.986	0.00515	383.9118	2	3	3
12	375	94	654.69	12.796	0.00569	384.0180	2	3	3
13	500	125	913.40	12.501	0.00421	384.0245	2	3	3
14	500	125	1760.4	8.8412	0.00752	380.3792	2	3	3
15	500	125	1728.3	9.1575	0.00708	386.9936	2	3	3
16	500	125	1728.3	9.1575	0.00708	380.0032	2	3	3
17	500	125	1728.3	9.1575	0.00708	247.3962	2	3	3
18	500	220	647.85	7.9691	0.00313	248.1078	2	3	3
19	500	220	649.69	7.9550	0.00313	245.8377	2	3	3
20	550	242	647.83	7.9691	0.00313	246.2440	2	3	3
21	550	242	647.81	7.9691	0.00313	231.7961	2	3	3
22	550	254	758.96	6.6313	0.00298	267.5881	2	3	3
23	550	254	758.96	6.6313	0.00298	267.0076	2	3	3
24	550	254	794.53	6.6611	0.00284	326.6603	2	3	3
25	550	254	794.53	6.6611	0.00284	267.6292	2	3	3
26	550	254	801.32	7.1032	0.00277	376.2845	2	3	3
27	550	254	801.32	7.1032	0.00277	383.9118	2	3	3
28	150	10	1055.1	3.3353	0.52124	384.0180	2	3	3
29	150	10	1055.1	3.3353	0.52124	384.0245	2	3	3
30	150	10	1055.1	3.3353	0.52124	380.3792	2	3	3
31	70	20	1207.8	13.052	0.25098	386.9936	2	3	3
32	70	20	810.79	21.887	0.16766	380.0032	2	3	3
33	70	20	1247.7	10.244	0.26350	267.6292	2	3	3
34	70	20	1219.2	8.3707	0.30575	376.2845	2	3	3
35	60	18	641.43	26.258	0.18362	383.9118	2	3	3
36	60	18	1112.8	9.6956	0.32563	384.0180	2	3	3
37	60	20	1044.4	7.1633	0.33722	384.0245	2	3	3
38	60	25	832.24	16.339	0.23915	380.3792	2	3	3
39	60	25	834.24	16.339	0.23915	386.9936	2	3	3
40	60	25	1035.2	16.339	0.23915	380.0032	2	3	3

unit: the number of the unit;

$P_{\text{max}}, P_{\text{min}}$: the maximum and minimum power generation;

a, b, c : the coefficients of the fuel cost function;

stc: start up cost;

up, down: the minimum up and down time;

it: the initial time of the unit, if it is positive (or negative),

indicates the number of hours the unit has been already up (or down).

Load demand of 94% utility factor

Hour	P_{load} (MW)	Hour	P_{load} (MW)
1	700	13	1400
2	750	14	1300
3	850	15	1200
4	950	16	1050
5	1000	17	1000
6	1100	18	1100
7	1150	19	1200
8	1200	20	1400
9	1300	21	1300
10	1400	22	1100
11	1450	23	900
12	1500	24	800

The load demand was scaled appropriately for the problems with utility factor.

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