Column Parity Row Selection (CPRS) BIST Diagnosis Technique: Modeling and Analysis

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Abstract—Column Selection Row Parity (CPRS) diagnosis is an X-tolerant and low aliasing technique that is suitable for the BIST environment. A row selection LFSR randomly selects outputs of multiple scan chains so that unknowns can be tolerated. Column and row parities of selected outputs are observed to solve linear equations for the error positions. Experimental data show that CPRS achieves nearly perfect diagnosis, even in the presence of 1 percent unknowns. CPRS compresses the diagnosis data because only parities of circuit responses, instead of responses themselves, are observed. Two error distribution models (scattered and clustered) are developed and analyzed to show the effectiveness of CPRS. The analytical results are demonstrated to be accurate by more than 10,000 experiments.

Index Terms—Diagnostics, reliability, testing, fault tolerance.

1 INTRODUCTION

WITH the advance of system-on-chip technology, output response compression and built-in self test (BIST) are becoming popular test solutions for embedded cores [1], [2]. Diagnosis of BIST failures, however, is very difficult due to the constraint of compressed outputs with unknowns (X) involved. The first challenging task of BIST diagnosis is to precisely identify the failing patterns and the failing locations [3]. Failing patterns are the patterns for which errors occur. An error is a mismatch between the expected good outputs and the actually observed circuit outputs. Failing locations are two-dimensional indexes of failing scan chains and failing scan cells where errors are observed. Once the failing patterns and failing locations are available, traditional scan-based diagnosis techniques (such as [4]) can then be applied to identify the faults.

Past research in BIST diagnosis can be summarized as follows: Early techniques, such as reciprocal polynomial [5] and diagonal matrix [6], can diagnose only one failing location. Observing MISR quotients or unloading MISR remainders identifies both the failing patterns and the failing locations [7], [8]. They are, however, limited in diagnosing multiple errors because of aliasing and masking problems of signature analyzers [9]. Error correcting code techniques, such as Reed-Solomon codes [10] and BCH codes [11], are mathematically feasible but practically too complex to be implemented. Cycling registers [12], [13], serial LFSR [14], and programmable MISR [15] enhance the diagnosis resolution by diversified polynomials. The LFSR-based random selection techniques apply repeated BIST sessions with different LFSR seeds to diagnose multiple failing locations in a chain [16], [17]. The counter-based

selection performs binary searches for failing locations in multiple chains [3].

The abovementioned MISR-based or LFSR-based diagnosis techniques suffer from the unknowns, which is very common in BIST. Because of the MISR/LFSR feedback, one single unknown corrupts the entire signature. The first solution is to add bypass circuitry that dumps scan chain contents without compression [18], [19], [20]. Although bypass techniques are useful to diagnose failing locations, they have difficulty in searching for failing patterns. The diagnosis data volume can therefore be very large. The second solution is to continuously observe the output responses in a “streaming” way. The X-compact technique compresses outputs using an XOR network [21]. The parity technique observes the column parity and the row parity, which is cleared after unloading scan chains, so unknowns do not accumulate [22]. The convolutional compactor eliminates the feedback so unknowns disappear after a few scan cycles [23]. The above X-tolerant techniques are very effective in reducing the number of output pins, but their diagnosis resolutions may not be satisfactory. Specifically, if 100 scan chains are compressed into one single output, fewer than 15 percent of failing locations are correctly diagnosed in the presence of 1 percent unknowns [23].

Several improvements on the compactor-based diagnosis were recently proposed. A selective convolutional compactor masks the X-generating scan chains so that the diagnosis resolution is greatly enhanced [1]. Heuristic methods, such as [24], have been proposed to search for a particular solution, but their results can be wrong when multiple solutions exist. The compactor independent diagnosis is suitable for various kinds of compactors, such as the X-compactor or the convolutional compactor [25]. This technique cannot be applied to the conventional MISR because of its infinite memory.

The column parity row selection (CPRS) technique has been shown to be an effective X-tolerant BIST diagnosis technique [26]. CPRS combines the advantages of the random selection technique [16] and the parity technique [22]. CPRS

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Manuscript received 5 Jan. 2006; revised 27 July 2006; accepted 21 Aug. 2006; published online 22 Jan. 2007.

For information on obtaining reprints of this article, please send e-mail to: tc@computer.org, and reference IEEECS Log Number TC-0005-0106.

Fig. 1. CPRS hardware architecture.

provides fine diagnosis resolutions for multiple errors in multiple scan chains. What is more, CPRS solves the linear equations to find the exact solution, rather than heuristically searching for one possible solution. CPRS is X-tolerant because the row parity register is cleared after every scan unload. Experimental results show that CPRS correctly diagnoses all failing locations, even in the presence of 1 percent unknowns. On top of the abovementioned advantages, the diagnosis resolution of CPRS can be improved by additional deterministic diagnosis. Most of the past BIST diagnosis techniques show their effectiveness by experiment rather than probability analysis. This paper develops two error distribution models for analysis: the clustered model and the scattered model. The former is good for single fault, while the latter is useful for multiple faults. The probability analysis provides very good insight into the CPRS diagnosis effectiveness and helps to predict the number of sessions needed. The analytical results are demonstrated to be accurate by more than 10,000 experiments.

The organization of this paper is as follows: Section 2 introduces the CPRS hardware and its diagnosis flow. Section 3 shows the calculation methods. Sections 4 and 5 show the probabilistic analysis and experimental results, respectively. Sections 6 and 7 are discussions and summary.

2 CPRS HARDWARE AND DIAGNOSIS FLOW

2.1 Hardware Architecture

The CPRS hardware architecture is shown in Fig. 1. During every scan cycle, the scan outputs from m scan chains are randomly selected by the row selection LFSR (RS-LFSR). The row selection hardware is made up of m AND gates. The selected scan outputs are XORed together to produce one bit of column parity (CP), which is observed every scan cycle. The selected scan output of every scan chain is XORed with its own row parity (RP), which is accumulated in the row parity register. The row parity is shifted out after scan chains are fully unloaded. (The row parity scan chain is not drawn in the figure for clarity.) The scan chains, the RS-LFSR, and the row parity register are all synchronized to the same clock.

2.2 Diagnosis Flow

The overall CPRS diagnosis flow is shown in Fig. 2. The CPRS diagnosis flow can be divided into two phases: the random diagnosis and the deterministic diagnosis. The

Fig. 2. Overall CPRS diagnosis flow.

circuit under test (CUT) is first diagnosed by the random diagnosis. If a unique diagnosis result is obtained, the flow is terminated; otherwise, the deterministic diagnosis is needed.

2.2.1 Random Diagnosis

Fig. 3 shows the steps of random diagnosis data collection. They are explained as follows:

1. Test the CUT in BIST mode with all scan chains selected. Observe CP and RP to identify failing patterns. The RS-LFSR outputs are ties to all ones, so all scan chains are selected.
2. Start BIST all over again and pause after a particular failing pattern is loaded.
3. An RS-LFSR seed is loaded and the row parity register is cleared.
4. Apply one system clock.
5. Unload the scan chains. The RS-LFSR and the row parity register are also clocked while unloading the scan chains. One bit of CP is observed every scan cycle.
6. After unloading all scan cells, the row parity is shifted out and observed.
7. If there are more sessions to go, the procedure goes back to step 2; otherwise, the random diagnosis data collection is finished.

Fig. 3. Random diagnosis data collection.

A *diagnosis session* consists of Steps 2 to 6. Every diagnosis session has a distinct RS-LFSR seed. The RS-LFSR seeds are randomly generated for now because no failing locations information is available at this time. More diagnosis sessions provide finer diagnosis results at the cost of longer diagnosis time.

2.2.2 Deterministic Diagnosis

The deterministic diagnosis data collection is very similar to what is shown in Fig. 3. One major difference is that the RS-LFSR seed in deterministic diagnosis is calculated based on the random diagnosis results. Another difference is that deterministic diagnosis does not need to identify failing patterns, which are already known from the random diagnosis.

2.3 Diagnosis Data Volume

Without knowing the failing patterns, the direct dumping technique has to unload all scan chain contents after applying every pattern. Please note that binary search for failing patterns is not suitable in the BIST environment because the unknown scan outputs can corrupt the MISR signature. Therefore, the diagnosis data volume is as large as $m \times l \times P_{total}$, where P_{total} is the number of patterns applied. By contrast, CPRS reduces the amount of diagnosis data because only the CP and RP are observed when identifying the failing patterns (Step 1 in Fig. 3). After that, multiple random diagnosis sessions are applied to every failing pattern. The amount of data required is therefore $(m + l)P_{total} + s(m + l)P_{failing}$, where s is the number of sessions and $P_{failing}$ is the number of failing patterns. Please note that, although random diagnosis requires multiple sessions, the total amount of data is still small because $P_{failing}$ is usually much smaller than P_{total} . The deterministic diagnosis is not counted here because the number of deterministic sessions is usually much smaller than that of random diagnosis.

Assume that $P_{failing}$ is much smaller than P_{total} and l is much larger than m . CPRS data volume is about $1/m$ of the direct dumping data volume:

$$\frac{DATA_{CPRS}}{DATA_{dump}} = \frac{(m + l) \times P_{total} + s(m + l) \times P_{failing}}{m \times l \times P_{total}} \quad (1)$$

$$\approx \frac{m + l}{m \times l} \approx \frac{1}{m} \quad (P_{failing} \ll P_{total}).$$

This is a very significant compression ratio since m is very large in the BIST environment. More generally,

$$\frac{DATA_{CPRS}}{DATA_{dump}} = \frac{(m + l) \times P_{total} + s(m + l) \times P_{failing}}{m \times l \times P_{total}} \quad (2)$$

$$\approx \frac{1 + sf}{m} \quad (P_{failing} = fP_{total}),$$

where parameter f is the ratio between $P_{failing}$ and P_{total} ($0 < f \leq 1$). CPRS data volume is about $(1 + sf)/m$ of that of direct dumping. Still, the CPRS data volume is smaller than direct dumping as long as sf is smaller than m . In the worst case, even if the fault under diagnosis is catastrophic and almost every pattern fails, one possible alternative solution is to unload RP after every scan clock. This is equivalent to a direct dumping of the scan chain contents.

Error Matrix					Selection Matrix					Selected Error Matrix				
E _{1,1}	E _{1,2}	E _{1,3}	E _{1,4}	E _{1,5}	0	0	1	0	0	0	0	E _{1,3}	0	0
E _{2,1}	E _{2,2}	E _{2,3}	E _{2,4}	E _{2,5}	0	1	1	1	1	0	E _{2,2}	E _{2,3}	E _{2,4}	E _{2,5}
E _{3,1}	E _{3,2}	E _{3,3}	E _{3,4}	E _{3,5}	1	0	1	1	1	E _{3,1}	0	E _{3,3}	E _{3,4}	E _{3,5}
E _{4,1}	E _{4,2}	E _{4,3}	E _{4,4}	E _{4,5}	0	1	1	1	0	0	E _{4,2}	E _{4,3}	E _{4,4}	0

Fig. 4. Error matrix, selection matrix, and selected error matrix.

In this situation, the CPRS diagnosis data volume is the same as the direct dumping technique.

3 CALCULATION METHODS

3.1 Solving Error Variables

The scan output data of a single failing pattern are represented by a two-dimensional *error matrix*. In the error matrix, every row corresponds to a scan chain and every element corresponds to a scan cell. If the CUT has m scan chains of length l , then the error matrix is of size $m \times l$. Fig. 4 shows an example error matrix in which m equals four and l equals five. A failing scan cell in the j th scan cell of the i th scan chain is represented by a one in the error variable $E_{i,j}$. A passing scan cell is represented by a zero in the corresponding $E_{i,j}$.

The *selection matrix* (S) is of the same size as the error matrix. A 1 in the selection matrix means the corresponding scan cell is selected by the row selection hardware; a 0 in the selection matrix means the corresponding scan cell is masked. The *selected error matrix* is obtained by ANDing the error matrix with the selection matrix. The AND operation is performed in a bit-by-bit way so the selected error matrix is also of the size $m \times l$. Fig. 4 shows an example of the selection matrix and the selected error matrix.

The *error column parity* (CP^E) is a row vector of size l , which represents the difference between the expected good column parity and the observed column parity. A one in the j th column of CP^E means that the column parity of the j th scan chain is different from its expected good value. Similarly, the *error row parity* (RP^E) is a column vector of size m , which represents the difference of the expected good row parity and the observed row parity. Let x denote the weight of RP^E and y denote the weight of CP^E . If two errors ($E_{1,3}$ and $E_{3,4}$) occur in Fig. 4, CP^E and RP^E are $[0 \ 0 \ 1 \ 1 \ 0]$ and $[1 \ 0 \ 1 \ 0]^T$, respectively. The weights x and y are both 2.

The *reduced error matrix* (E^R) is obtained from the selected error matrix by deleting the rows for which the corresponding RP^E is equal to zero and deleting the columns for which the CP^E is equal to zero. The E^R is therefore of size x by y . The row equations are obtained by summing error variables of every row in the E^R . The column equations are obtained by summing all error variables of every column in the E^R .

$$\text{row equations}(i) = \sum_{j=1}^y E^R_{i,j} = RP^E_i, \quad (3)$$

$$\text{column equations}(j) = \sum_{i=1}^x E^R_{i,j} = CP^E_j. \quad (4)$$

Fig. 5. Derivation of row and column equations.

A total of x row equations and a total of y column equations can be derived from E^R . The row equations and column equations form a system of linear equations, $Ae = B$, where matrix A is the *coefficient matrix*. The column vector e represents all error variables to be solved. The column vector B is the corresponding RP^E and CP^E for each equation. In the presence of Xs, the row and column equations with unknown parity are deleted from the selected error matrix, so no unknowns are involved in the equations.

Fig. 5 shows an example of finding the row and column equations. The first, the second, and the fifth columns are deleted from the selected error matrix because CP^E is $[0\ 0\ 1\ 1\ 0]$. The second and fourth rows are also deleted because RP^E is $[1\ 0\ 1\ 0]^T$. The resulting E^R is 2×2 in size and there are three error variables to be solved: $E_{1,3}$, $E_{3,3}$, and $E_{3,4}$. Two row equations and two column equations are derived from E^R . The coefficient matrix has four rows and three columns. Solving the system of linear equations produces a unique solution— $E_{1,3} = 1$, $E_{3,3} = 0$, and $E_{3,4} = 1$ —which is a correct diagnosis.

If there is more than one diagnosis session, the above process is still applicable except for two changes. First, every session s has its own distinct selection matrix, CP_s^E and RP_s^E . Second, a row is deleted from the selected error matrix if no mismatch ever occurs in any one session—that is, a row is included in E^R only if its row parity fails at least one diagnosis session. The final value of x and y is calculated by taking the union of RP^E and CP^E from all sessions, respectively.

$$x = \text{weight} \left[\bigcup_{\text{all sessions}} RP_s^E \right], \quad (5)$$

$$y = \text{weight} \left[\bigcup_{\text{all sessions}} CP_s^E \right]. \quad (6)$$

After E^R is obtained, the row and column equations of different diagnosis sessions are put together into the final linear equation system. Fig. 6 summarizes the whole flow to solve error variables for multiple sessions.

If an error variable cannot be uniquely solved, then its diagnosis result is *ambiguous*. Fig. 7 shows an example of the ambiguous diagnosis. The number of error variables is seven, but only six equations are available. After the Gauss-Jordan elimination, the coefficient matrix is converted to a form of $(D|U)$. The upper part of D is an identity matrix and

Fig. 6. Solving error variables for multiple random sessions.

the lower part of D is all zeros. If the U matrix is free of ones, all error variables are uniquely solved; otherwise, there are error variables that remain ambiguous. In this particular example, only one error variable, $E_{1,3}$, is uniquely solved and the other error variables are ambiguous. To diagnose the ambiguous error variables, CPRS requires more diagnosis sessions.

3.2 Finding RS-LFSR Seed

The RS-LFSR seeds of deterministic diagnosis are calculated based on the random diagnosis results. The purpose of deterministic diagnosis is to add new row equations and column equations which are linearly independent of the existing equations. Finding a deterministic RS-LFSR seed needs the equivalent of finding a *selection matrix desired* (S^{MD}), which represents the specific scan cells to select. S^{MD} is of the same size ($m \times l$) as the selection matrix. A 1 in $S_{i,j}^{MD}$ represents that scan cell i, j is required to be selected. A 0 in $S_{i,j}^{MD}$ represents that the corresponding scan cell should be masked. S^{MD} is not unique and finding an S^{MD} is similar to a covering problem. A greedy algorithm (Fig. 8) is implemented to solve this problem. Note that the greedy algorithm may not always produce the optimal solution, but it is sufficient to demonstrate the feasibility of the deterministic diagnosis.

First, S^{MD} is initialized to all Xs, which stand for “not yet specified.” For all unsolved error variables $E_{i,j}$, fill in zeros

Fig. 7. Example of ambiguous diagnosis.

Fig. 8. Proposed method to find S^{MD} .

in the i th row and the j th column of S^{MD} . A column flag (CF) vector and a row flag (RF) vector are initialized to zeros. The CF row vector is of size l and the RF column vector is of size m . A one in the CF vector indicates that the corresponding column parity has been used in a column equation. A one in the RF vector indicates that the corresponding row parity has been used in a row equation. Second, find a row with the smallest nonzero weight from the U matrix and pick an unsolved error variable $E_{i,j}$ from that row. Third, check the RF vector and the CF vector of that selected error variable $E_{i,j}$. If both CF_j and RF_i are ones, which means both the i th row parity and the j th column parity have been used, the error variable $E_{i,j}$ is *unsolvable* in this deterministic diagnosis. An unsolvable error variable does not mean that it is unsolvable forever. It may become solvable in future deterministic diagnosis. If CF_j and RF_i are not both ones, the error variable $E_{i,j}$ is then *solvable*. The corresponding element $S^{MD}_{i,j}$ is set to one. Either CF_j or RF_i is turned from zero to one. Setting RF_i or CF_j to one prevents the corresponding row or column parity from being reused by the subsequent unsolved error variables. In the fifth step, label $E_{i,j}$ is solved by setting all bits in the $E_{i,j}$ column of the U matrix to 0s. If there is any 1 in the U matrix, go back to Step 2; otherwise, the greedy algorithm finishes with an S^{MD} .

Fig. 9 demonstrates an example to find S^{MD} . Initially, all five unsolved error variables are initialized to 0s in S^{MD} . The error variable $E_{2,5}$ is first picked because the second

Fig. 9. Example to find S^{MD} . (a) Initial. (b) After first iteration. (c) After third iteration.

row of U has the smallest nonzero row weight. Since neither RF_2 nor CF_5 is 1, this error variable is solvable. The corresponding $S^{MD}_{2,5}$ is set to one and the RF_2 is also set to one. The whole column six in the U matrix is cleared to 0s. The second error variable picked is $E_{2,3}$. Since RF_2 has already been one, CF_3 is now set to one. The fourth column in the U matrix is cleared. In the last iteration, $E_{3,1}$ is set to 1 and RF_3 is set to 1. After three iterations, there are no more 1s in the U matrix and the final S^{MD} is obtained.

Once the S^{MD} matrix is obtained, the corresponding RS-LFSR seed can be solved. Solving LFSR seeds has been published in previous papers, such as [27]. An example to solve the RS-LFSR is provided in Fig. 10. The black cells are to be masked (corresponding to $E_{3,3}$ and $E_{3,5}$ in S^{MD}) and the white scan cells are to be selected. The other gray cells are don't cares. Suppose that the initial contents in the four-stage RS-LFSR are S_0, S_1, S_2 , and S_3 . Based on the feedback of this LFSR, a system of linear equations can be derived and the RS-LFSR seed is solved: $[S_0 S_1 S_2 S_3] = [1 0 1 1]$.

It has been shown that the chance of success is very high as long as the number of care bits is smaller than the number of LFSR stages by 20 [28]. In other words, the chance of finding an RS-LFSR seed is high as long as enough random sessions are applied with very few unsolved error variables left. In case no RS-LFSR seed is found to solve all error variables, two solutions are available: adding more deterministic sessions or adding more random sessions. For the former solution, unsolved error variables are divided into groups, which are solved one group by one group in multiple deterministic diagnosis

Fig. 10. Example of finding RS-LFSR seed.

Fig. 11. Clustered errors and ROI.

sessions. The drawback of multiple deterministic sessions is that it requires iterative data collection. To avoid the iterative data collection, it is desirable to have a guideline to determine the number of random sessions, which is presented in the next session.

4 PROBABILITY ANALYSIS

Two models are proposed to analyze the effectiveness of the CPRS diagnosis: the clustered model and the scattered model. The former assumes that errors are spatially confined within a small number of rows and columns; the latter assumes that errors are uniformly distributed over all rows and columns.

4.1 Clustered Model

4.1.1 Assumptions

Real silicon data have shown that defects can cluster together on the die [29]. These clustered defects can manifest themselves as clustered errors which are physically very close to each other. The clustered model is proposed here to diagnose circuits with clustered defects. Fig. 11 shows an example error matrix of clustered defects. Errors are clustered in a *region of interest* (ROI) sized $r \times c$, where r is the number of erroneous rows and c is the number of erroneous columns. The error multiplicity, e , represents the total number of errors in the ROI. For clustered defects, it is reasonable to assume that the number r is much smaller than the number c , which means that errors are located very close to each other. A detailed discussion of the defect distribution is out of the scope of this paper. Interested readers can refer to [29] for more information.

To simplify the probability analysis, every error variable is assumed to be selected by an independent probability of $1/2$. It follows that the probability of identifying an erroneous row and the probability of identifying an erroneous column are both $1/2$. The number of rows and columns included in E^R up to the s th session are represented by random variables x and y , respectively. Because an erroneous row is included in E^R only if its row parity fails at least once during the s diagnosis sessions, the random variable x obeys the binomial distribution with parameters $n_x = r$ and $p = (1 - 2^{-s})$. Similarly, random variable y obeys the binomial distribution with parameters $n_y = c$ and $p = (1 - 2^{-s})$. The random variables x and y are assumed to be independent.

4.1.2 Expected Number of Correctly Diagnosed Bits

For the clustered model, the expected number of correctly diagnosed bits up to the s th session is given by

$$EC_{cluster}(s) = m \times l - v(s) + u(s) - \varepsilon(s), \quad (7)$$

where $m \times l$ is the total number of bits in the whole error matrix. $v(s)$ represents the expected number of error variables included in E^R , out of which $u(s)$ are uniquely solved. $\varepsilon(s)$ is the expected number of errors that escape all s diagnosis sessions.

The expected value of x is $E[x] = n_x p = r(1 - 2^{-s})$, which represents the average number of rows in E^R up to the s th session. (Please distinguish the symbol of expected value from the symbol of errors.) Similarly, the expected value of y is $E[y] = n_y p = c(1 - 2^{-s})$, which represents the average number of columns in E^R . The expected number of error variables included in E^R up to the s th session is given by

$$v(s) = E[xy] = E[x]E[y] = cr(1 - 2^{-s})^2. \quad (8)$$

The expected number of row equations and column equations up to the s th session is $q(s)$, which is $sE[x + y] = s(c + r)[1 - 2^{-s}]$. Out of these $q(s)$ equations, $q_i(s)$ of them are linearly independent. The exact value of $q_i(s)$ is difficult to calculate in an analytical form, so the following linear approximation is made:

$$q_i(s) \approx \frac{q(s)}{\alpha}, \quad (9)$$

where α is a fitting parameter between 2 and 3 in a typical CPRS diagnosis environment. The linear approximate will be validated by experiments in the coming subsection. Finally, the number of uniquely solved error variables, $u(s)$, is approximately equal to the number of linear independent equations $q_i(s)$ or the number of error variables $v(s)$, whichever is the minimum.

$$u(s) \approx \min[q_i(s), v(s)]. \quad (10)$$

An error is not included in E^R if its row parity or its column parity never fails in any of the s sessions. Assuming two events are independent, the probability of each event is 2^{-s} and the probability of both events is 4^{-s} . The expected number of errors escaping the diagnosis is, therefore, as follows:

$$\varepsilon(s) = e[2 \times 2^{-s} - 4^{-s}]. \quad (11)$$

To have a good diagnosis, the last three terms in (7) are close to zero. Assuming that s is large and the 2^{-s} term is ignored, the following condition is required for a good CPRS diagnosis:

$$s_{cluster} \geq \alpha \times \left(\frac{cr}{c + r} \right). \quad (12)$$

Equation (12) provides a criterion to estimate the number of random diagnosis sessions needed. It indicates that the effectiveness of CPRS depends only on the error distribution, not on the CUT size. This is a very promising result because the number of CPRS diagnosis sessions needed does not increase with CUT size. It is also observed that the effectiveness of CPRS does not change with the RS-LFSR weight. This is because, as the weight factor increases, the decrease in $\varepsilon(s)$ is cancelled by the increase in $v(s)$. Tuning the RS-LFSR weight higher than $1/2$, on the one hand, gives the benefit of obtaining more equations, but, on the other

Fig. 12. Validation of linear assumption.

hand, has the drawback of involving more error variables. Eventually, the number of sessions needed does not change with the biased RS-LFSR.

4.1.3 Validation of Linear Approximation

Experiments are performed to validate the linear approximation in (9). In each experiment, a selection matrix is generated by an RS-LFSR with random seeds. The number of independent equations, $q_i(s)$, is accumulated as a function of s . Fig. 12 shows the average number of $q_i(s)$ over 10,000 experiments. The size of the ROI is 5×10 , so the maximum value of $q_i(s)$ is 50. The expected number of error multiplicity is 40 (i.e., 80 percent of the ROI is randomly filled with errors). Although $q_i(s)$ is not actually linear with respect to $q(s)$, the linear assumption of parameters $\alpha = 2$ or 3 gives a reasonably close estimation. $\alpha = 2$ is better for a small number of sessions and $\alpha = 3$ is better for a large number of sessions.

4.2 Scattered Model

4.2.1 Assumptions

Real diagnosis data have shown that multiple faults (possibly caused by multiple defects) do occur on defective chips [30]. The scattered model is proposed to take into account the effect of multiple defects that are not clustered together. Analysis of the scattered model is more complicated than that of the clustered model because little error location information is available. To simplify the analysis, it

$$\begin{aligned} m &= 5 \\ l &= 6 \\ e &= 5 \end{aligned}$$

1					
			1		
		1			
	1				
				1	

Fig. 13. Nonsharing spatial distribution.

is assumed that no error shares a column. This assumption is legitimate because l is much greater than m and the probability of multiple errors sharing a column is much smaller than the probability of multiple errors sharing a row. It is also assumed that the error multiplicity is small enough (say, $e < 2m$) that the possibility of more than four errors sharing a row is very low.

4.2.2 Expected Number of Correctly Diagnosed Bits

For the scattered model, the expected number of correctly diagnosed bits up to the s th session is given by

$$EC_{\text{scatter}}(s) = m \times l - \sum_{\text{all } d} SP(d) \times [v(d, s) - u(d, s) + \varepsilon(d, s)], \quad (13)$$

where d represents the spatial distribution of errors and $SP(d)$ is the *spatial probability* of d . $SP(d)$ stands for the probability that a specific spatial distribution d appears. Equation (13) is similar to (7) except that the $v(d, s)$, $u(d, s)$, and $\varepsilon(d, s)$ are now dependent on the spatial distribution d as well as s .

Fig. 13 shows an example *nonsharing* spatial distribution. In the nonsharing spatial distribution, every error occupies a column and every error occupies a row—that is, no error shares a row or a column. The spatial probability of the nonoverlapping distribution is shown in the first row in Table 1. In this specific spatial distribution, random variables x and y are identical binomial random variables with parameters $n = e$ and $p = [1 - 2^{-s}]$. x and y are identical because, when an error is selected, both its CP and RP are erroneous. The expected numbers of error

TABLE 1
Seven Most Frequent Spatial Distributions

d	description	$SP(d)$	$v(d, s)$	$q(d, s)$	$\varepsilon(d, s)$
1	Non-sharing (No error shares a row.)	$C_e^m e! / m^e$	$e[1 - 2^{-s}]2^{-s} + e^2[1 - 2^{-s}]^2$	$2se[1 - 2^{-s}]$	$e2^{-s}$
2	Only one pair of errors shares one row.	$C_1^m C_2^e C_{e-2}^{m-1} (e-2)! / m^e$	$< e(e-1)[1 - 2^{-s}]$	$> 2s(e-1)[1 - 2^{-s}]$	$\approx e2^{-s}$
3	Only two pairs of errors share two rows.	$C_2^m C_4^e C_{e-4}^{m-2} (e-4)! / m^e$	$< e(e-2)[1 - 2^{-s}]$	$> 2s(e-2)[1 - 2^{-s}]$	$\approx e2^{-s}$
4	Only three errors share one row.	$C_1^m C_3^e C_{e-3}^{m-1} (e-3)! / m^e$	$< e(e-2)[1 - 2^{-s}]$	$> 2s(e-2)[1 - 2^{-s}]$	$\approx e2^{-s}$
5	Only four errors share one row.	$C_1^m C_4^e C_{e-4}^{m-1} (e-4)! / m^e$	$< e(e-3)[1 - 2^{-s}]$	$> 2s(e-3)[1 - 2^{-s}]$	$\approx e2^{-s}$
6	Two errors share a row and three errors share another row.	$C_1^m C_3^e C_1^{m-1} C_2^{e-3} C_{e-5}^{m-2} (e-5)! / m^e$	$< e(e-3)[1 - 2^{-s}]$	$> 2s(e-3)[1 - 2^{-s}]$	$\approx e2^{-s}$
7	Only three pairs of errors share a row.	$C_3^m C_6^e C_2^3 C_{e-6}^{m-3} (e-6)! / m^e$	$< e(e-3)[1 - 2^{-s}]$	$> 2s(e-3)[1 - 2^{-s}]$	$\approx e2^{-s}$

variables included in the reduced error matrix— $v(d, s)$ —is $E[x^2]$. The value of $v(d, s)$ is calculated by the fact that the second moment of a binomial random variable is $np(1-p) + n^2p^2$. The expected number of row and column equations— $q(d, s)$ —is $2s$ times $E[x]$. The expected number of escaped errors is $\varepsilon(d, s) = n(1-p) = e2^{-s}$.

Following the same procedure as the clustered model, the minimum number of sessions needed for a good diagnosis is as follows:

$$s_{\text{nonsharing}} \geq \alpha \times \left(\frac{e^2}{2e}\right) = \alpha \times \frac{e}{2}. \quad (14)$$

The linear assumption states that $q_i(d, s) = q(d, s)/\alpha$. The fitting parameter α of the scattered model is also between 2 and 3 according to our experiments. Equation (14) indicates that, in the scatter model, the number of sessions needed is about 1 to 1.5 times as many as the error multiplicity.

In addition to the nonsharing spatial distribution, there are many other *sharing spatial distributions*. The sharing spatial distributions have at least two errors that share one row. Table 1 lists the six most frequent sharing spatial distributions ($d = 2$ to 7). These spatial distributions are mutually exclusive and their spatial probabilities sum up to nearly one as long as e is smaller than $2m$. For the sharing spatial distribution, x and y are not identical, so the exact values of $v(d, s) = E[xy]$ are hard to calculate. Nevertheless, the upper bound of $E[xy]$ is e times $E[x]$ because y is always smaller than e . The $q(d, s) = sE[x + y]$ is lower bounded by $2s$ times $E[x]$ because x is always smaller than y . $\varepsilon(d, s)$ of the sharing spatial distributions is approximated by that of the nonsharing spatial distribution.

4.3 Considering Unknowns

An unknown that shares the same row with at least one error is called a *row sharing unknown*. An unknown that shares the same column with at least one error is called a *column sharing unknown*. An unknown that shares neither row nor column with any error does not cause diagnosis problems because its associated row/column equations can be discarded without penalty. Only the row sharing unknowns or column sharing unknowns cause diagnosis degradation. The distribution of unknowns and errors can be classified into four cases: scattered errors scattered unknowns, clustered errors scattered unknowns, scattered errors clustered unknowns, and clustered errors clustered unknowns.

In the first and the second cases, it is assumed that the frequency of row sharing unknowns is much higher than the frequency of column sharing unknowns. Similarly to the scattered errors, scattered unknowns also have various spatial distributions. Two modifications in $EC(s)$ are therefore proposed to consider the effect of X . The number of variables involved in the presence of unknowns, $v_X(s)$, can be approximated by

$$v_X(s) \approx v(s) - \frac{r_X e 2^{-s}}{2}, \quad (15)$$

where r_X is the average number of row sharing unknowns. The last term in (15) represents the number of error variables that are not included due to the unknowns. The

Fig. 14. Correctly diagnosed bits (clustered errors).

term is divided by two because the probability of selecting an unknown is a half. In addition to $v(s)$, the number of equations in the presence of unknowns, $q_X(s)$, is approximated by

$$q_X(d, s) \approx q(d, s) - sr_X[1 - 2^{-s}], \quad (16)$$

in which the last term represents the number of row equations discarded due to the selected unknowns. Besides $v_X(s)$ and $q_X(s)$, the other parts of $EC(s)$ remain unchanged.

Consider the third and fourth cases in which unknowns are clustered in a few scan chains. If no error is located in the same row as clustered unknowns, the diagnosis is not affected. If there are one or more errors in the same row as clustered unknowns, the row equation is useful only when none of the unknowns is selected. The probability of observing this row is 2^{-N_X} , where N_X is the number of unknowns clustered in the same row.

5 EXPERIMENTAL RESULTS

Experimental results are shown to demonstrate the effectiveness of the CPRS technique. The following experiments are performed on 1,000 scan cells in 10 scan chains ($m = 10$ and $l = 100$) unless specified otherwise. Every one of the following experiments is performed on 10,000 randomly generated error matrices with various values of r , c , and e parameters. Although 10×100 does not represent a big CUT, it is already shown by previous analysis that the effectiveness of CPRS does not depend on the CUT size.

5.1 Diagnosis without X

5.1.1 Clustered Errors

Fig. 14 shows the expected number of correctly diagnosed bits versus the number of sessions. The thick line is the experimental data with ROI size $r = 5$ and $c = 10$. The error multiplicity is 40—that is, 80 percent of the ROI is randomly filled with errors. The thin lines are calculated by (7) with parameters $\alpha = 2$ and $\alpha = 3$. The experimental results fit well with the analytical curve of $\alpha = 2$ when s is small. CPRS achieves nearly perfect diagnosis after 10 sessions, which is predicted by the analytical curve of $\alpha = 3$.

Table 2 shows the number of sessions to achieve a 99.9 percent correct diagnosis. The analytical results are calculated by (12) with $\alpha = 3$. The analytical results match well with the experimental results. The results show that

TABLE 2
Sessions Needed for 99.9 Percent Correct Diagnosis

r	c	e	Experiment	Analytical $\alpha=3$
2	10	15	7	5
3	10	25	7	7
4	10	35	8	9
5	10	40	9	10

CPRS provides good diagnosis results within 10 diagnosis sessions for an ROI of 5×10 .

One notable feature of the $EC(s)$ curve in Fig. 14 is that the $EC(s)$ increases slowly when the number of sessions is small. In certain scenarios, the number of correctly diagnosed bits even decreases in the first few sessions. This phenomenon is because the number of error variables $v(s)$ increases faster than the number of equations $q(s)$ when s is small. The session that suffers from the worst diagnosis results is obtained by taking the derivative of $EC_{cluster}(s)$ with respect to s . The $EC_{cluster}(s)$ derivative can be approximated by (17) with the 4^{-s} terms ignored. Assuming that the s term is greater than the $(1 - s \ln 2)$ term, the session that produces the worst $EC_{cluster}(s)$ is further simplified to (18).

$$\frac{dEC_{cluster}(s)}{ds} \cong -2cr2^{-s} \ln 2 + \frac{c+r}{\alpha} (1 + 2^{-s} - s2^{-s} \ln 2) + 2e2^{-s} \ln 2, \quad (17)$$

$$s_{worst_EC} \cong \log_2 \left[\frac{\alpha(cr - e)}{c + r} \right] + 0.47, \quad s_{worst_EC} \geq 0. \quad (18)$$

When errors are fully clustered ($e/cr = 1$), the s_{worst_EC} is equal to zero. When the errors are half clustered, ($e/rc = 0.5$), the s_{worst_EC} is around 3. Equation (18) indicates an encouraging fact that the s_{worst_EC} is not strongly dependent on the parameters c , r , and e because of the $\log_2$ operator. For typical values of r , c , and e , the s_{worst_EC} is no larger than four. Table 3 shows the diagnosis sessions that produce the worst $EC(s)$. The experimental results are performed in a 5×10 ROI with various error density (e/rc). The s_{worst_EC} slowly increases as the error density decreases, but the worst EC session does not exceed four. The analytical results calculated from (18) match well with the experimental results.

TABLE 3
 s_{worst_EC} ($r = 5, c = 10$)

e/cr	Experiment	Analytical	
		$\alpha=2$	$\alpha=3$
1	1	0	0
0.8	1	1	1
0.5	2	2	3
0.25	2	3	3
0.2	2	3	3

Fig. 15. Correctly diagnosed bits (scattered errors).

5.1.2 Scattered Errors

Fig. 15 shows the number of correctly diagnosed bits versus the number of sessions. The thick lines are the experimental results and the thin lines are calculated by (13). Seven spatial distributions in Table 1 are used. The error multiplicities are 2 and 7. As explained in the last section, a “hook” is observed when the number of sessions is small. The experimental data show that the worst diagnosis result occurs in the second session. Similarly to the clustered model, the worst-case diagnosis of the scattered model can be estimated by taking the derivative of $EC_{scatter}(s)$. The s_{worst_EC} of the scattered model, however, cannot be expressed in simple analytical form due to the variety in spatial distributions.

Table 4 shows the average number of correctly diagnosed bits, wrong bits, and ambiguous bits. If a bit is uniquely solved, its diagnosis result is either correct or wrong. If a bit is not uniquely solved, its diagnosis result is ambiguous. The left three columns are obtained from the random diagnosis only and the right three columns are from the random diagnosis followed by one deterministic diagnosis session. After 15 random diagnosis sessions, every bit is correctly diagnosed, even in the presence of 15 errors (1.5 percent of total scan cells). The same experiments are performed using the previous technique [22]. The results show that only 993, 971, and 928 bits are correctly diagnosed for 2, 7, and 15 errors, respectively. In comparison, CPRS is more effective because of the row selection hardware.

The numbers of random sessions needed to achieve higher than 99.9 percent correct diagnosis for 2, 7, and 15 errors are 3, 8, and 15, respectively. The experimental results match the analytical equation (14), which predicts that 1 to 1.5 times e sessions are required. The sessions that have the worst diagnosis for 2, 7, and 15 errors are 1, 2, and 4, respectively. The results agree with the analysis, which states that the worst session is no more than four as long as the error multiplicity is reasonably small.

It is observed that the deterministic diagnosis helps to improve the diagnosis results, especially when s is small. For the example of 15 errors, the number of correctly diagnosed bits is 914 if three random sessions are performed. The number can be improved to 930 if two random sessions plus one deterministic session are performed. The deterministic diagnosis, however, becomes less helpful when s is large because the random diagnosis

TABLE 4
Diagnosis Results of 10×100 (no X)

# of random diagnosis sessions	error multiplicity	random only			random+ 1 deterministic		
		correct	wrong	ambiguous	correct	wrong	ambiguous
1	2	997.7	1.2	1.1	998.1	1.2	0.7
	7	986.1	5.4	8.4	987.5	6.2	6.3
	15	960.8	11.9	27.4	955.4	11.9	32.7
2	2	998.6	0.8	0.6	998.8	0.8	0.5
	7	981.7	5.3	13.0	986.9	6.8	6.3
	15	931.8	7.5	60.7	930.3	7.8	62.0
3	2	999.3	0.4	0.3	999.3	0.4	0.2
	7	984.5	6.6	8.9	990.4	6.9	2.7
	15	914.6	5.2	80.2	915.5	6.1	78.4
4	2	999.6	0.2	0.1	999.6	0.2	0.1
	7	991.4	6.4	2.2	992.6	5.9	1.5
	15	909.3	5.3	85.4	917.8	8.3	73.9
5	2	999.8	0.1	0.1	999.8	0.1	0.1
	7	995.1	4.0	0.9	995.5	3.8	0.8
	15	929.2	11.5	59.2	956.7	14.2	29.1
8	2	1000.0	0.0	0.0	1000.0	0.0	0.0
	7	999.3	0.6	0.1	999.4	0.6	0.1
	15	994.2	5.2	0.5	994.5	5.0	0.5
10	2	1000.0	0.0	0.0	1000.0	0.0	0.0
	7	999.9	0.1	0.0	999.9	0.1	0.0
	15	998.8	1.1	0.1	998.8	1.1	0.1
15	2	1000.0	0.0	0.0	1000.0	0.0	0.0
	7	1000.0	0.0	0.0	1000.0	0.0	0.0
	15	1000.0	0.0	0.0	1000.0	0.0	0.0

already provides very good diagnosis results, leaving very small room for improvement.

Fig. 16 compares the diagnosis data volume of CPRS and the direct dumping technique. The X-axis shows the total number of patterns applied (P_{total}) and the Y-axis shows the data volume. This plot is drawn according to (1) and (2) with $m = 10$ and $l = 100$. Two bottom lines indicate the CPRS data volume with two different f parameters ($f = P_{failing}/P_{total}$): 1 percent and 10 percent. The number of random diagnosis sessions is 10, which is chosen based

on the results of Table 4. The top line represents the data volume of direct dumping. It can be seen that the CPRS data volume is only about one-fifth of the direct dumping, even when 10 percent of 5,000 patterns applied fail.

5.1.3 Experiments on Benchmark Circuits

To further validate the CPRS diagnosis technique, experiments are performed on three large ISCAS'89 and ITC'99 benchmark circuits. Fourteen randomly selected single stuck-at faults (f_1 to f_{14}) are fault simulated in diagnosis mode and their complete faulty outputs are obtained (without stop on first failure). Regular SSF test patterns generated by a commercial ATPG are applied and the number of total patterns (P_{total}) is shown in Table 5. The number of failing patterns ($P_{failing}$) and the number of errors are listed in the second and the third columns, respectively. All errors are correctly diagnosed in eight random sessions, except one error of f_6 , which needs a deterministic diagnosis session. The last column is the percentage of CPRS data volume with respect to that of direct dumping. CPRS effectively reduced the data volume except for fault f_{12} , in which every failing pattern has a single error. As predicted by (1), the CPRS data volume is close to $1/m$ when the number of errors is small (such as f_4 , f_{10} , f_{11}).

Fig. 16. Data volume versus P_{total} .

TABLE 5
Diagnosis Results of Benchmark Circuits

Circuit	injected fault	$P_{failing}$	number of errors	correctly diagnosed	Data Volume		
					direct dump	CPRS	CPRS/DD
b19 230K gates $m = 21$ $l = 328$ $P_{total} = 931$	f_1	395	395	100%	6,560,000	1447,680	22.1%
	f_2	16	23	100%	6,560,000	392,544	6.0%
	f_3	77	112	100%	6,560,000	562,368	8.6%
	f_4	2	2	100%	6,560,000	353,568	5.4%
	f_5	31	31	100%	6,560,000	434,304	6.6%
s9234 5.6K gates $m = 5$ $l = 42$ $P_{total} = 614$	f_6	104	245	99.6%	215,000	87,936	40.9%
	f_7	96	104	100%	215,000	84,864	39.5%
	f_8	24	32	100%	215,000	57,216	26.6%
	f_9	39	47	100%	215,000	62,976	39.3%
	f_{10}	4	12	100%	215,000	49,536	23.0%
s38584 19K gates $m = 5, l = 285$ $P_{total} = 1,070$	f_{11}	2	3	100%	1,430,000	295,656	20.7%
	f_{12}	549	549	100%	1,430,000	1569,072	109.7%
	f_{13}	78	78	100%	1,430,000	472,584	33.0%
	f_{14}	207	414	100%	1,430,000	772,896	54.0%

TABLE 6
Diagnosis Results of 10×100 (Seven Errors, with X)

random sessions	unknown multiplicity	Scattered X			Clustered X		
		correct	wrong	Amb.	correct	wrong	Amb.
1	1	985.6	5.4	9.0	985.7	5.5	8.9
	5	986.7	6.0	7.3	986.6	5.9	7.5
	10	986.4	6.3	7.4	986.6	6.0	7.4
2	1	982.1	5.5	12.4	981.6	5.3	13.0
	5	984.2	6.5	9.3	984.0	6.3	9.7
	10	985.8	7.1	7.1	984.5	6.6	8.9
3	1	984.8	7.0	8.1	984.1	6.8	9.2
	5	985.4	7.9	6.7	985.7	8.0	6.3
	10	986.7	8.5	4.8	986.4	8.6	5.0
4	1	990.8	7.0	2.2	991.0	6.8	2.1
	5	990.2	7.9	1.9	990.2	8.1	1.8
	10	989.4	8.7	1.9	989.7	8.7	1.7
5	1	994.4	4.7	0.9	994.3	4.8	0.9
	5	993.2	6.0	0.8	992.7	6.5	0.9
	10	991.6	7.5	0.9	991.7	7.5	0.8
8	1	999.2	0.8	0.1	998.5	1.3	0.2
	5	997.6	2.2	0.2	996.8	3.0	0.2
	10	995.4	4.3	0.3	995.5	4.4	0.1
10	1	999.7	0.3	0.0	999.5	0.5	0.1
	5	998.2	1.7	0.1	997.8	2.1	0.1
	10	996.2	3.5	0.2	996.2	3.8	0.1
15	1	999.8	0.2	0.0	999.8	0.2	0.0
	5	998.8	1.1	0.1	998.2	1.7	0.1
	10	997.2	2.6	0.2	996.6	3.3	0.1

5.2 Diagnosis with Unknowns

Table 6 shows the diagnosis results with seven errors and various *unknown multiplicities*. Unknown multiplicity means the number of unknowns injected. Two types of experiments are performed: scattered X and clustered X. In the former experiment, the locations of Xs are randomly generated, assuming a scattered spatial distribution. In the latter experiment, the unknowns are assumed to be clustered into two scan chains; the other eight scan chains are free of unknowns. This is to model the real circuit scenario in which only a few scan chains generate unknowns [1]. It can be seen from the results that CPRS is equally effective for both experiments. More than 99 percent of the 10,000 scan cells are correctly diagnosed, even in the

Fig. 17. Correctly diagnosed bits (seven errors, five X).

TABLE 7
Solving RS-LFSR Seeds (1,000,000 Cases per Row)

RS-LFSR length	Care bits	CPU time(sec)	Solved cases
16	6	21.7	999,832
16	12	28.8	971,489
32	6	31.2	999,995
32	12	40.5	999,203

presence of 1 percent unknowns. Unfortunately, previous BIST diagnosis techniques, such as [3], [8], [15], [16], do not consider so many unknowns, so there is no direct comparison available so far.

Fig. 17 plots the number of correctly diagnosed bits in the presence of seven errors and five unknowns. The thick dots are from experiments and the thin line is calculated with the modifications in (15) and (16) taken into account. The parameter r_X used in the figure is three because approximately half of the seven errors can share a row with unknowns. The analytical result is pessimistic when s is small, but optimistic when s is large. Compared with the $EC(s)$ curve without X, the $EC(s)$ curve with X deviates farther away from the experimental result. This is understandable since the location of unknowns is simply modeled by a single parameter r_X .

6 DISCUSSIONS

6.1 RS-LFSR Seed Solver

The deterministic RS-LFSR seed can be solved from the S^{MD} by a mod-2 linear equation solver. To demonstrate the efficiency of the seed solving, a Gauss-Jordan elimination algorithm is implemented in about 500 lines of C++ program. Table 7 shows the data from four experiments performed on a 32-bit and a 16-bit RS-LFSR with 1,000 error variables in S^{MD} . In every experiment, 1,000,000 S^{MD} with a specified number of care bits (6 or 12) are randomly generated. The number of care bits represents the number of unsolved variables after random diagnosis. The number of care bits is based on the results of Table 4, in which the number of unsolved bits is less than six after eight random sessions. The unspecified bits are filled with don't cares. On a 2.66 GHz Intel PC, the total CPU time for 1,000,000 cases is less than a minute, which shows that the deterministic RS-LFSR seeds can be easily solved in a short time. The last column shows the number of solved cases (out of 1,000,000).

TABLE 9
RS-LFSR Area

Stages	Scannable FF	XOR	Total Area (NAND)
8	8	2	57
16	16	2	110
32	32	2	213

A case is solved if an RS-LFSR seed is found to satisfy the care bits of the S^{MD} . The success rate is very high as long as the RS-LFSR stage number is much larger than the number of care bits.

6.2 Hardware Overhead

Table 8 compares the diagnosis circuitry area overhead of several techniques. The number of flip-flops, XOR, and AND gates is derived from the original papers. The total area is expressed in the number of equivalent NAND gates as a function of m (number of scan chains). The conversion of a cell area is based on the numbers in the TSMC 0.18 μm standard cell library. The CPRS costs about 20 NAND gates per scan chain, which is approximately on the same order as the other techniques. For convenience of comparison, some typical numbers are assumed for the previous techniques.

The area overhead of 8, 16, and 32-stage RS-LFSR is shown in Table 9. Because the seeds have to be loaded into the RS-LFSR, the flip-flops are scannable. Also, RS-LFSR needs two 2-input XOR for the feedback. The total equivalent gate count is calculated in the last column. According to the probability analysis in [28], the 32-stage RS-LFSR is able to handle 12 unsolved error variables in the deterministic diagnosis, which is good enough for most diagnosis scenarios. From this table, it can be seen that RS-LFSR is an inexpensive solution for diagnosis in terms of area, especially for large design under test.

7 SUMMARY

The CPRS is an effective X-tolerant BIST diagnosis technique. The row selection LFSR randomly selects scan outputs from multiple scan chains. The column parity and the row parity of the selected scan outputs are observed after every scan cycle and every scan unload, respectively. Experimental data show that CPRS is very effective, even in the presence of 1 percent unknowns. Two models are proposed to evaluate the performance of CPRS analytically. The probability analysis shows that the CPRS effectiveness

TABLE 8
Comparison of Area Overhead

Techniques	FF	XOR	NAND	Total Area (NAND)
Random [16]	$\text{Log}_2 b + m + k$	$K \log_2 b + m + k$	$m(\log_2 b + 1)$	$32m + 23^*$
CC3 [23]	32	$3m$	1	$182 + 8m$
CC7 [23]	32	$7m$	1	$182 + 19m$
Parity [22]	m	$2m$	0	$11m$
CPRS	$2m$	$3m$	m	$20m$

*assume $k=m$, $b=16$

does not depend on the CUT size. The analysis provides good insight into the diagnosis effectiveness and the analytical results match well with the experimental data.

ACKNOWLEDGMENTS

This research is partly sponsored by the National Science Council of Taiwan under contract 95-2220-E-002 and by the Ministry of Economy of Taiwan under contract 94-EC-17-A-S1-00 (HOY project). The idea of CPRS was first proposed at the International Test Conference in 2005.

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