Dual-Polarization Frequency Reuse with Frequency Band Shifting Allocation

Jingshown Wu, *Senior Member, IEEE*, and Meng-Che Wu

Abstract—Spectrum is a limited and precious resource in wireless communications. Effective spectrum utilization becomes more and more important because of the expeditious increase of demand for wireless communications. In principle, the dual-polarization frequency reuse system, which employs two orthogonally polarized electromagnetic waves to carry information, can double the system capacity. However, the system performance degrades seriously and sometimes becomes unacceptable due to cross polarization and fading. In this paper, a new linear-polarized dual-polarization frequency reuse system that utilizes the concept of orthogonal frequency allocation to increase spectrum utilization is proposed. We analyze the performance of the dual-polarization system modulated by the quadrature phase-shift keying in the multipath fading channel with cross-polarization interference and Doppler effect. Because of the orthogonality of the polarized carriers and frequency spacing, the effect of cross polarization, which defeats the performance of the dual-polarization system severely, can be ignored. The proposed system has a bit error rate around 10^{-7} at the bit energy-to-noise ratio of 14 dB, while the bit error rates of the dual-polarization cancellation with bootstrap system and the conventional orthogonal frequency-division multiplexing system are about 10^{-4} .

Index Terms—Dual-polarization, orthogonal frequency spacing.

I. INTRODUCTION

IN WIRELESS communications, spectrum is one of the most important resources. To increase spectrum utilization efficiency, many schemes, such as orthogonal frequency-division multiplexing (OFDM) [1]–[3], cellular frequency reuse, and dual-polarization frequency reuse [4], [5], were proposed and deployed. OFDM is an attractive technique for digital transmission. Spectrum utilization efficiency can be increased by the orthogonal frequency spacing and signal bands' overlapping. But the effects of frequency offset, multipath fading, and delay deeply affect the effectiveness of OFDM. It is impractical to use the OFDM scheme in wireless transmission without any modification and improvement. And the dual-polarization frequency reuse system, which uses orthogonally polarized electromagnetic (EM) waves, has been demonstrated in satellite communications. The linear and circular types of polarization are the most frequently used.

There are two directions of polarization (DoPs), vertical and horizontal, in the linear-polarized system, and two DoPs, clockwise and counterclockwise, in the circular-polarized system. In principle, transmission through orthogonal polarization car-

riers doubles the system capacity. But practically, that is hard to achieve because of propagation impairments and antenna imperfections. The propagation impairments, such as rainfall attenuation, depolarization, and cross-polarization interference (CPI) deteriorate the signal transmission in the satellite–earth station links. Some compensation methods were reported in the literature [6]–[9]. Furthermore, the problems will become more complicated by the multipath fading effect when transmitting in the terrestrial environment.

Although some schemes, such as equalization and diversity, are proposed to overcome the problems, they are too complicated to be practical. In this paper, we adopt the concept of orthogonal frequency spacing used in OFDM and apply to the linear dual-polarization system. The signal band allocation and orthogonal frequency spacing between the vertical and horizontal DoPs are introduced. Therefore, the performance degradation problem can be easily solved. The proposed system may employ pilot symbols for channel coefficients estimation, carrier recovery, etc. Furthermore, band-edged pilot tones can be added in the proposed system, which is impossible for a conventional OFDM system. These issues will be discussed later.

The remainder of this paper is organized as follows. The system architecture and channel model are described in Section II. In Section III, the performance analysis is presented. We show that the performance of this system is much better than that of OFDM, even without using any complicated channel imperfection compensation mechanism. In Section IV, the influence of various factors on the system performance is discussed. Section V gives the conclusion.

II. SYSTEM ARCHITECTURE AND CHANNEL MODELS

In the traditional dual-polarization frequency reuse system, the signal spectra of both DoPs completely overlap. Guard bands between signal spectra are needed to prevent adjacent channel interference due to the imperfection of bandpass filter. The difference between the proposed system and the traditional system is that the vertical polarization (i.e., y DoP) signal spectra are located at the higher frequency band relative to the horizontal polarization (i.e., x DoP) signal spectra by the amount of symbol rate B in the proposed system. The system architecture and behavior are described as next.

A. System Architecture

The schematic of the transmitter is shown in Fig. 1. The function of the serial-to-parallel converter (SPC) will be described later, and the system description begins with the two input data streams $e_{ix}(t)$ and $e_{iy}(t)$, which are modulated by

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The authors are with the Department of Electrical Engineering and Graduate Institute of Communication Engineering, National Taiwan University, Taipei, Taiwan 10617 R.O.C.

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Fig. 1. The transmitter of the proposed system.

Modulator 1 (Mod.1) and Modulator 2 (Mod.2), respectively. Any modulation scheme can be adopted. In this paper, the dual quadrature phase-shift keying (QPSK) scheme is considered. After modulation, the modulated symbols are fed into the square-rooted raised cosine band-limited (BL) filters and then transmitted through the x DoP and y DoP antennas. To maintain orthogonality and to estimate channel parameters, the narrow-band pilot tones (PTs), which are produced by the PT generators with the band-edged frequency shifter (BEFS), are added. The spectra allocation of PTs are chosen at the edges of the spectra of the corresponding in-band signals, as shown in Fig. 2(a). The number of pilot tones and their spacing interval depend on the channel complexity. Moreover, a pilot signal can be designed to carry more information about the channel in order to get better channel estimation. After adding the filtered symbols and pilot tones, the system up-converts the x DoP signal and the y DoP signal to two frequency bands f_{ix} and f_{iy} , respectively, where i denotes the i th signal channel. f_{ix} and f_{iy} differ by the amount of symbol rate, so the two signals are linearly independent theoretically. If the width of the guard band is less than the signal bandwidth, the spectra of x DoP signal in the i th channel and y DoP signal in the $(i - 1)$ th channel overlap because of the band allocation. The sketch of signal spectrum allocation is depicted in Fig. 2(b), which shows that the two different DoP signal spectra in adjacent channels are separated by the value of bit rate plus f_d . f_d can be considered as the width of guard band. Theoretically, if f_d equals zero, the two signals are orthogonally spaced with zero interference. However, the value of f_d can be adjusted to control the frequency band allocation for the spectra of PTs. On the other hand, f_d is also prepared for compensating the Doppler effect. There is a limit that f_d cannot be negative, i.e., overlapping the signal bands of the two adjacent channels in the same DoP is not allowed.

The schematic of the receiver is shown in Fig. 3. After down-converting the received signals, PT filtering and signal filtering are processed, respectively. The pilot tones are filtered out by the PT filters, and the signals are sent into the polarization recovery circuit and carrier recovery circuit, which we will not discuss further. The received symbols are filtered by the square-rooted raised cosine band-limited filters. The merit of using the filter is that we can take advantage of the properties of raised cosine filter without using other types of low-pass filters. Then the filtered symbols are fed into the demodulators with the reference

recovered from the pilot tones. The detected data streams $\hat{e}_{ix}(t)$ and $\hat{e}_{iy}(t)$ are obtained. The function of the parallel-to-serial converter (PSC) will be described later.

B. Channel Model

In this paper, the multipath Rayleigh-fading channel with Doppler effect [10], [11] is considered. Taking the inherent limitation of dual-polarization transmission into account, we only analyze the two-ray case. The impulse responses of the dual polarized channel are given by

$$h_x(t) = \alpha_x \delta(t) + \gamma_x A_x \delta(t - \tau_x) e^{j\varphi_x} e^{j2\pi f_D t}, \quad \text{for } x\text{DoP} \quad (1)$$

and

$$h_y(t) = \alpha_y \delta(t) + \gamma_y A_y \delta(t - \tau_y) e^{j\varphi_y} e^{j2\pi f_D t}, \quad \text{for } y\text{DoP} \quad (2)$$

where

- α_j attenuation coefficient;
- $\delta(\cdot)$ delta function;
- f_D Doppler shift;
- τ_j delay time of the second path relative to the line-of-sight;
- γ_j square-rooted power ratio with respect to the first path;
- φ_j initial phase of the second path;
- A_j Rayleigh random variable with unity second moment;
- $j = \begin{cases} x, & x\text{DoP} \\ y, & y\text{DoP} \end{cases}$.

Because Doppler effect results from the relative velocity between the transmitter and receiver, it is reasonable to set the same Doppler shift f_D in both x DoP and y DoP.

Pilot tones are conducive to the estimation of the channel parameters. There is at least one pilot tone in each coherent bandwidth. The theoretical functions of reflection coefficients of horizontal-polarized wave and vertical-polarized wave were discussed in the literature [10]. For horizontal polarization, the function indicates that the relative phase of the incident and reflected waves is nearly 180° for all angles of incidence. But for vertical polarization, the results are quite different. When the angle of incidence is small, the behaviors of horizontal and vertical polarization are almost the same. But as the angle of incidence increases, the magnitude and relative phase of the reflected wave decrease rapidly. It is a torment to recover the vertical polarization wave. However, with the existence of pilot tones in the system, this problem can be solved. Both reflection coefficients are frequency dependent. Therefore, the relative phase changes of both pilot tone and in-band signal are almost the same. We can recover the carrier phase by the pilot tones easily.

C. Data Flow Issue

In the traditional dual-polarized systems, each DoP is used as an independent transmission channel. It means that the data streams transmitted in both DoPs are uncorrelated. For better transmission performance, the error correction code (e.g., convolutional code) can be used by expanding the code word by some redundant bits. The training sequence, which estimates the channel characteristic, can also be used. To save the transmission payload, the concept of parallel transmission in OFDM [1] can be introduced to reduce the code redundancy. In Fig. 1, the

(a) The overall signal spectrum include pilot tones.

(b) Diagram of CPI due to frequency overlap.

Fig. 2. Sketch of frequency band allocation and frequency shifting.

Fig. 3. The receiver of the proposed system.

coded data stream is fed into the SPC and split into two data sequences $c_{ix}(t)$ and $c_{iy}(t)$. The consequent processes were indicated in the previous sections. The receiving processes in the receiver are also illustrated in Fig. 3. Finally, the desired coded data stream is obtained by sending the detected data sequences $\hat{c}_{ix}(t)$ and $\hat{c}_{iy}(t)$ through the PSC.

III. SYSTEM ANALYSIS

Here we consider the dual-QPSK system and express the input data sequence to the signal space in complex domain. The

symbols of x and y DoP in the k th duration are denoted by

$$u_{ix}(t - kT) = a_i(t - kT) + jb_i(t - kT) \quad (3)$$

$$u_{iy}(t - kT) = c_i(t - kT) + jd_i(t - kT) \quad (4)$$

where $a_i(t)$ and $c_i(t)$ are the QPSK signals in the I-branch of x and y DoPs in the i th channel, respectively. $b_i(t)$ and $d_i(t)$ are the QPSK signals in the Q-branch of both DoPs in the i th channel. Thereafter, a_k^i will represent $a_i(t - kT)$ for convenience. T denotes the symbol duration of the QPSK modulators. The outputs of the modulators are sampled every T seconds and then passed through the square-root raised cosine filter with the impulse response $p(t)$. After up converting the mapped signals to the carrier frequencies, we get the transmitting signals as

$$S_{ix}(t) = \sum_{k=-\infty}^{\infty} \sqrt{2}u_{ix}(t - kT)p(t - kT)\exp(j2\pi f_{ix}t) \quad (5)$$

$$S_{iy}(t) = \sum_{k=-\infty}^{\infty} \sqrt{2}u_{iy}(t - kT)p(t - kT)\exp(j2\pi f_{iy}t). \quad (6)$$

Because of the orthogonality between $S_{ix}(t)$ and $S_{iy}(t)$, they do not interfere with each other. However, $S_{ix}(t)$ will be interfered by the adjacent channel signals $S_{(i-1)y}(t)$ and the delayed signal due to propagation imperfection. Similarly, $S_{iy}(t)$ will be interfered with by $S_{(i+1)x}(t)$ and the delayed signal. Here we only present the x DoP case because the other is exactly the same.

At the receiver, the received signal of x DoP is given by

$$r_{ix}(t) = y_{ix}(t) + n_{ix}(t) \quad (7)$$

where $n_{ix}(t)$ is the additive white Gaussian random process with zero mean and the variance $N_0/2$. $y_{ix}(t)$ is the x DoP channel output with the inputs $S_{ix}(t)$ and $S_{(i-1)y}(t)$. It can be expressed as

$$\begin{aligned} y_{ix}(t) = & \text{Re} \{ \alpha_x (S_{ix}(t) + RS_{(i-1)y}(t)) \\ & + \beta_1 (\gamma_x A_x S_{ix}(t - \tau_x) e^{j\varphi_x} \\ & + R\gamma_y A_y S_{(i-1)y}(t - \tau_y) e^{j\varphi_y}) \\ & \cdot e^{j2\pi f_D t} \}. \end{aligned} \quad (8)$$

The first term is the line-of-sight signal. The second term is the delayed signal and interference with delay time τ_x and τ_y of x and y DoPs, respectively. α_x is the attenuation coefficient. Coefficient R is the cross-polarization coefficient, satisfying $0 \leq R \leq 1$. β_1 is the polarization-mismatch coefficient of delayed symbol of the second path. It lies between zero and one. The frequency f_D is the Doppler shift in the second path. In the following analysis, we will take values all normalized to α_x , i.e., $\alpha_x = 1$.

At the BL filter output, we get the sampled signal Z_{ix} , which can be expressed as

$$\begin{aligned} Z_{ix}(nT) = & r_{ix}(t) \sqrt{2} \cos(2\pi f_{ix} t) * p(t) \Big|_{t=nT} \\ = & \sum_{k=-\infty}^{\infty} a_k^i p(t - kT) * p(t) \\ & + R \sum_{k=-\infty}^{\infty} (c_k^{i-1} p(t - kT) \\ & \cdot \cos(2\pi(f_{(i-1)y} - f_{ix})t) \\ & - d_k^{i-1} p(t - kT) \\ & \cdot \sin(2\pi(f_{(i-1)y} - f_{ix})t)) * p(t) \\ & + \beta_1 \gamma_x A_x \cos \alpha_1 \sum_{k=-\infty}^{\infty} (a_k^i p(t - kT - \tau_x) \\ & \cdot \cos 2\pi f_D t - b_k^i p(t - kT - \tau_x) \sin 2\pi f_D t) * p(t) \\ & + \beta_1 \gamma_x A_x \sin \alpha_1 \sum_{k=-\infty}^{\infty} (a_k^i p(t - kT - \tau_x) \\ & \cdot \sin 2\pi f_D t + b_k^i p(t - kT - \tau_x) \cos 2\pi f_D t) * p(t) \\ & + R\beta_1 \gamma_y A_y \cos \alpha_2 \sum_{k=-\infty}^{\infty} (c_k^{i-1} p(t - kT - \tau_y) \\ & \cdot \cos(2\pi(f_{(i-1)y} - f_{ix} + f_D)t) \\ & - d_k^{i-1} p(t - kT - \tau_y) \\ & \cdot \sin(2\pi(f_{(i-1)y} - f_{ix} + f_D)t)) * p(t) \\ & + R\beta_1 \gamma_y A_y \sin \alpha_2 \sum_{k=-\infty}^{\infty} (c_k^{i-1} p(t - kT - \tau_y) \end{aligned}$$

$$\begin{aligned} & \cdot \sin(2\pi(f_{(i-1)y} - f_{ix} + f_D)t) \\ & + d_k^{i-1} p(t - kT - \tau_y) \\ & \cdot \cos(2\pi(f_{(i-1)y} - f_{ix} + f_D)t)) * p(t) + N \end{aligned} \quad (9)$$

and

$$N = n_{ix}(t) \sqrt{2} \cos(2\pi f_{ix} t) * p(t) \Big|_{t=nT}$$

where $\alpha_1 = 2\pi f_{ix} \tau_x - \varphi_x$, and $\alpha_2 = 2\pi f_{(i-1)y} \tau_y - \varphi_y$. N is the filtered Gaussian noise with zero mean and the variance $N_0/2$.

Let

$$q_1(t - kT, x) = \left(p(t - kT) \cos\left(\frac{2\pi x t}{T}\right) \right) * p(t) \quad (10)$$

$$q_2(t - kT, x) = \left(p(t - kT) \sin\left(\frac{2\pi x t}{T}\right) \right) * p(t). \quad (11)$$

After some manipulation, we get

$$q_1(t - kT, x) = \cos\left(\frac{\pi x}{T}(t + kT)\right) m(t - kT, x) \quad (12)$$

and

$$q_2(t - kT, x) = \sin\left(\frac{\pi x}{T}(t + kT)\right) m(t - kT, x) \quad (13)$$

where $m(t, x)$ is given in Table I. The detailed derivation is presented in Appendix I.

In the following analysis, we consider the two kinds of channel characteristics.

A. Mutually Dependent Rayleigh Fading Channel

In this case, the amplitude coefficients of x - and y -directions of polarization are dependent, i.e., A_x is a function of A_y . For simplicity, we choose $A_x = A_y$, which indicates that the amplitude coefficients of two DoPs are positively correlated. At the output of the correlator in the receiver, we get

$$\begin{aligned} Z_{ix}(k=0) = & \left(r_{ix}(t) \sqrt{2} \cos(2\pi f_{ix} t) \right) * p(t) \Big|_{t=0} \\ = & a_0 + X_1 + AX_2 + N \end{aligned} \quad (14)$$

where $A = A_x = A_y$. X_1 is cross-polarization interference of the current symbol. X_2 is the sum of the delayed-symbol interferences (DSIs) of the current and the last symbol corresponding to x - and y -directions of polarization. N is the low-pass additive white Gaussian noise (AWGN) with variance $N_0/2$. By the assumption of $\tau_x < T$ and $\tau_y < T$, only the interferences from the current symbol and the last symbol have to be considered. According to (9), X_1 and X_2 can be expressed as

$$\begin{aligned} X_1 = & R \left(c_0^{i-1} q_1(0, \delta + 1) + c_{-1}^{i-1} q_1(-T, \delta + 1) \right. \\ & \left. - d_0^{i-1} q_2(0, \delta + 1) - d_{-1}^{i-1} q_2(-T, \delta + 1) \right) \end{aligned} \quad (15)$$

TABLE I
SOLUTION FUNCTIONS OF CONVOLUTION PROCESS WITH DIFFERENT FREQUENCY BAND ALLOCATION

	$0 < \beta \leq 0.5$	$0.5 < \beta \leq 1$
	$0 < x \leq \beta$	$0 < x \leq 1 - \beta$
$m(t, x)$	<div>(I)</div> $\begin{aligned} &-(x-1+\beta)\operatorname{sinc}\left(\frac{\pi(x-1+\beta)}{T}\right) + (\beta-x)\cos\left(\frac{\pi x}{2\beta}\right)\cos\left(\frac{\pi}{T}\right)\operatorname{sinc}\left(\frac{\pi(\beta-x)}{T}\right) \\ &+ x\left[\cos\left(\frac{\pi(\beta-1)}{T}-\frac{\pi(\beta+1)}{4\beta}\right)\operatorname{sinc}\left(\frac{\pi x}{T}+\frac{\pi x}{4\beta}\right) + \cos\left(\frac{\pi(\beta-1)}{T}+\frac{\pi(\beta+1)}{4\beta}\right)\operatorname{sinc}\left(\frac{\pi x}{T}-\frac{\pi x}{4\beta}\right)\right] \\ &+ \frac{x-\beta}{2}\left[\sin\left(\frac{\pi}{T}\right)\operatorname{sinc}\left(\frac{\pi(x-\beta)}{2\beta}+\frac{\pi(x-\beta)}{T}\right) - \sin\left(\frac{\pi}{T}\right)\operatorname{sinc}\left(\frac{\pi(x-\beta)}{2\beta}-\frac{\pi(x-\beta)}{T}\right)\right] \end{aligned}$	
	$\beta < x \leq 1 - \beta$	$1 - \beta < x \leq \beta$
	<div>(II)</div> $\begin{aligned} &-(x-1+\beta)\operatorname{sinc}\left(\frac{\pi(x-1+\beta)}{T}\right) \\ &+ \frac{T}{\left(\frac{\pi T}{2\beta}\right)^2 - (2\pi t)^2} \left(\frac{\pi T}{\beta} \cos\left(\frac{\pi t(x-1-\beta)}{T}\right) - 4\pi t \sin\left(\frac{\pi t(x-1-\beta)}{T}\right) \right) \end{aligned}$	<div>(III)</div> $\begin{aligned} &\frac{x-1+\beta}{2}\cos\left(\frac{\pi(x-1+\beta)}{2\beta}\right)\operatorname{sinc}\left(\frac{\pi(x-1+\beta)}{T}\right) \\ &+ \frac{x-1+\beta}{4}\operatorname{sinc}\left(\frac{\pi(x-1+\beta)}{2\beta}-\frac{\pi(x-1+\beta)}{T}\right) \\ &+ \frac{x-1+\beta}{4}\operatorname{sinc}\left(\frac{\pi(x-1+\beta)}{2\beta}+\frac{\pi(x-1+\beta)}{T}\right) \\ &+ (1-\beta)\cos\left(\frac{\pi(2x+\beta-1)}{4\beta}+\frac{\pi x}{T}\right)\operatorname{sinc}\left(\frac{\pi(1-\beta)}{4\beta}+\frac{\pi(1-\beta)}{T}\right) \\ &+ (1-\beta)\cos\left(\frac{\pi(2x+\beta-1)}{4\beta}-\frac{\pi x}{T}\right)\operatorname{sinc}\left(\frac{\pi(1-\beta)}{4\beta}-\frac{\pi(1-\beta)}{T}\right) \\ &+ \cos\left(\frac{\pi x}{2\beta}\right)\cos\left(\frac{\pi(x-\beta)}{T}\right)\operatorname{sinc}\left(\frac{\pi}{T}\right) \\ &- \frac{x-\beta}{2}\sin\left(\frac{\pi}{T}\right)\operatorname{sinc}\left(\frac{\pi(x-\beta)}{2\beta}+\frac{\pi(x-\beta)}{T}\right) \\ &+ \frac{x-\beta}{2}\sin\left(\frac{\pi}{T}\right)\operatorname{sinc}\left(\frac{\pi(x-\beta)}{2\beta}-\frac{\pi(x-\beta)}{T}\right) \end{aligned}$
	$1 - \beta < x \leq 1$	$\beta < x \leq 1$
	<div>(IV)</div> $\begin{aligned} &\frac{x-1+\beta}{4}\left[2\cos\left(\frac{\pi(x-1+\beta)}{2\beta}\right)\operatorname{sinc}\left(\frac{\pi(x-1+\beta)}{T}\right) + \sin\left(\frac{\pi(x-1+\beta)}{2\beta}-\frac{\pi(x-1+\beta)}{T}\right) + \sin\left(\frac{\pi(x-1+\beta)}{2\beta}+\frac{\pi(x-1+\beta)}{T}\right)\right] \\ &+ \frac{x-1}{2}\left[\sin\left(\frac{\pi\beta}{T}-\frac{\pi(x-1)}{4\beta}\right)\operatorname{sinc}\left(\frac{\pi(x-1)}{T}-\frac{\pi(x-1)}{4\beta}\right) + \sin\left(\frac{\pi\beta}{T}+\frac{\pi(x-1)}{4\beta}\right)\operatorname{sinc}\left(\frac{\pi(x-1)}{T}+\frac{\pi(x-1)}{4\beta}\right)\right] \end{aligned}$	
	$1 < x \leq \beta + 1$	$1 < x \leq \beta + 1$
	<div>(V)</div> $\begin{aligned} &-\frac{x-1-\beta}{2}\cos\left(\frac{\pi(x-1+\beta)}{2\beta}\right)\operatorname{sinc}\left(\frac{\pi(x-1-\beta)}{T}\right) \\ &- \frac{x-1-\beta}{4}\left[\sin\left(\frac{\pi(x-1-\beta)}{2\beta}-\frac{\pi(x-1-\beta)}{T}\right) + \sin\left(\frac{\pi(x-1-\beta)}{2\beta}+\frac{\pi(x-1-\beta)}{T}\right)\right] \end{aligned}$	
	$x > \beta + 1$	$x > \beta + 1$
	<div>(VI)</div> <div>0</div>	

and

$$\begin{aligned}
X_2 = & \beta_1 \gamma \left\{ (a_0^i \cos \alpha_1 + b_0^i \sin \alpha_1) q_1(-\tau_x, \Delta) \right. \\
& + (a_{-1}^i \cos \alpha_1 + b_{-1}^i \sin \alpha_1) q_1(-T - \tau_x, \Delta) \\
& + (a_0^i \sin \alpha_1 - b_0^i \cos \alpha_1) q_2(-\tau_x, \Delta) \\
& + (a_{-1}^i \sin \alpha_1 - b_{-1}^i \cos \alpha_1) q_2(-T - \tau_x, \Delta) \\
& + R \left[(c_0^{i-1} \cos \alpha_2 + d_0^{i-1} \sin \alpha_2) \right. \\
& \quad \cdot q_1(-\tau_y, \delta + \Delta + 1) \\
& \quad \left. + (c_{-1}^{i-1} \cos \alpha_2 + d_{-1}^{i-1} \sin \alpha_2) \right.
\end{aligned}$$

$$\begin{aligned}
& \cdot q_1(-T - \tau_y, \delta + \Delta + 1) \\
& + (c_0^{i-1} \sin \alpha_2 - d_0^{i-1} \cos \alpha_2) \\
& q_2(-\tau_y, \delta + \Delta + 1) \\
& + (c_{-1}^{i-1} \sin \alpha_2 - d_{-1}^{i-1} \cos \alpha_2) \\
& \left. \cdot q_2(-T - \tau_y, \delta + \Delta + 1) \right\} \quad (16)
\end{aligned}$$

where a_0^i , b_0^i , c_0^{i-1} , and d_0^{i-1} are the bits of the current symbol. a_{-1}^i , b_{-1}^i , c_{-1}^{i-1} , and d_{-1}^{i-1} are that of the last symbol. $\delta = f_d \cdot T$ is the normalized shifting frequency and $\Delta = f_D \cdot T$ is the normalized Doppler shift frequency.

Then the bit error probability is given by

$$\begin{aligned}
 Pe_{dep} &= \frac{1}{2} (\Pr \{Z_{ix} < 0 | a_0^i = 1\} + \Pr \{Z_{ix} > 0 | a_0^i = -1\}) \\
 &= \Pr \{Z_{ix} < 0 | a_0^i = 1\} \\
 &= \left(\frac{1}{2\pi}\right)^2 \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \left(Q\left(\sqrt{2z}(1+X_1)\right) \right. \\
 &\quad \left. - x \sqrt{\frac{2z}{1+2zx^2}} \exp\left[-\frac{z(1+X_1)^2}{1+2zx^2}\right] \right. \\
 &\quad \left. \cdot Q\left(\frac{2zx(1+X_1)}{\sqrt{1+2zx^2}}\right) \right)_{x=X_2} d\alpha_1 d\alpha_2. \quad (17)
 \end{aligned}$$

The detailed derivation is given in Appendix II.

B. Independent Rayleigh Fading Channel

In the most random case, we assume that A_x and A_y are independent and identically Rayleigh distributed with unity second moment. There may be different power ratios for each direction of polarization. For simplicity, we set the power ratios for both directions of polarization to the same value.

At the output of the correlator in the receiver, we get

$$\begin{aligned}
 Z_{ix}(k=0) &= \left(r_{ix}(t) \sqrt{2} \cos(2\pi f_{ix} t) \right) * p(t) \Big|_{t=0} \\
 &= a_0 + Y_1 + A_x Y_2 + A_y Y_3 + N \quad (18)
 \end{aligned}$$

where Y_1 is the cross-polarization interference of the current symbol. Y_2 is the sum of the DSI of the current and the last symbol at the x -direction of polarization. Similarly, Y_3 is the DSI at the y -direction polarization. N is the low-pass AWGN with variance $N_0/2$. Based on the assumption of $\tau_x < T$ and $\tau_y < T$, only the interference from the current symbol and the last symbol are considered. According to (9), Y_1 , Y_2 , and Y_3 can be expressed as follows:

$$\begin{aligned}
 Y_1 &= R \left(c_0^{i-1} q_1(0, \delta + 1) + c_{-1}^{i-1} q_1(-T, \delta + 1) \right. \\
 &\quad \left. - d_0^{i-1} q_2(0, \delta + 1) - d_{-1}^{i-1} q_2(-T, \delta + 1) \right) \quad (19)
 \end{aligned}$$

$$\begin{aligned}
 Y_2 &= \beta_1 \gamma \left[(a_0^i \cos \alpha_1 + b_0^i \sin \alpha_1) q_1(-\tau_x, \Delta) \right. \\
 &\quad + (a_{-1}^i \cos \alpha_1 + b_{-1}^i \sin \alpha_1) q_1(-T - \tau_x, \Delta) \\
 &\quad + (a_0^i \sin \alpha_1 - b_0^i \cos \alpha_1) q_2(-\tau_x, \Delta) \\
 &\quad \left. + (a_{-1}^i \sin \alpha_1 - b_{-1}^i \cos \alpha_1) q_2(-T - \tau_x, \Delta) \right] \quad (20)
 \end{aligned}$$

$$\begin{aligned}
 Y_3 &= \beta_1 \gamma R \left[(c_0^{i-1} \cos \alpha_2 + d_0^{i-1} \sin \alpha_2) \right. \\
 &\quad \cdot q_1(-\tau_y, \delta + \Delta + 1) \\
 &\quad + (c_{-1}^{i-1} \cos \alpha_2 + d_{-1}^{i-1} \sin \alpha_2) \\
 &\quad \cdot q_1(-T - \tau_y, \delta + \Delta + 1) \\
 &\quad + (c_0^{i-1} \sin \alpha_2 - d_0^{i-1} \cos \alpha_2) \\
 &\quad \cdot q_2(-\tau_y, \delta + \Delta + 1) \\
 &\quad \left. + (c_{-1}^{i-1} \sin \alpha_2 - d_{-1}^{i-1} \cos \alpha_2) \right. \\
 &\quad \left. \cdot q_2(-T - \tau_y, \delta + \Delta + 1) \right]. \quad (21)
 \end{aligned}$$

Now, we can compute the bit error probability of the system using dual-QPSK. The bit error rate (BER) is given by

$$\begin{aligned}
 Pe &= \frac{1}{2} (\Pr \{Z_{ix} < 0 | a_0^i = 1\} \\
 &\quad + \Pr \{Z_{ix} > 0 | a_0^i = -1\}) \\
 &= \Pr \{Z_{ix} < 0 | a_0^i = 1\} \\
 &= Pe_{dep}(Y_3) - \left(\frac{1}{2\pi}\right)^2 \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \\
 &\quad \cdot \left\{ \frac{Y_2 \sqrt{2z} \sqrt{1+2zY_2^2}}{1+2zY_2^2+2zY_3^2} \right. \\
 &\quad \cdot \left[e^{-((z(1+Y_1)^2)/(1+2zY_2^2))} \right. \\
 &\quad \cdot Q\left(\frac{2zY_2(1+Y_1)}{\sqrt{1+2zY_2^2}}\right) - \frac{2zY_2Y_3}{\sqrt{(1+2zY_2^2)(1+2zY_3^2)}} \\
 &\quad \cdot e^{-((z(1+Y_1)^2)/(1+2zY_3^2))} Q\left(\frac{2zY_3(1+Y_1)}{\sqrt{1+2zY_3^2}}\right) \Big] \\
 &\quad - \frac{Y_2Y_3(1+Y_1)}{\sqrt{\pi}((2z)^{-1}+Y_2^2+Y_3^2)^{3/2}} \\
 &\quad \left. \cdot e^{-((z(1+Y_1)^2)/(1+2zY_2^2+2zY_3^2))} \cdot \text{Res} \right\} d\alpha_1 d\alpha_2 \quad (22)
 \end{aligned}$$

where Res is the residual part of the equation. It can be expressed as

$$\text{Res} = \begin{cases} \int_{B_1}^{\infty} \int_{B_2}^{B_3} e^{-A_y^2} e^{-n^2/2} dA_y dn, & Y_2Y_3 > 0 \\ \int_{-\infty}^{B_1} \int_{B_3}^{\infty} e^{-A_y^2} e^{-n^2/2} dA_y dn \\ + \int_{B_1}^{\infty} \int_{B_2}^{\infty} e^{-A_y^2} e^{-n^2/2} dA_y dn, & Y_2Y_3 \leq 0 \end{cases} \quad (23)$$

and

$$\begin{aligned}
 B_1 &= \frac{2zY_2(1+Y_1)}{\sqrt{1+2zY_2^2}} \\
 B_2 &= \frac{2zY_3(1+Y_1)}{\sqrt{(1+2zY_2^2+2zY_3^2)(2+4zY_2^2)}} \\
 B_3 &= \frac{1}{2zY_2Y_3} \sqrt{\frac{1+2zY_2^2+2zY_3^2}{2}} (n - B_1) + B_2. \quad (24)
 \end{aligned}$$

The detailed derivation is shown in Appendix III.

Now we take the intersymbol interference (ISI) effect into consideration. Because of different symbol combinations, there are different BER performances. We have to average the performance including the ISI effect over all cases given for the current and the last bits. By the assumption of equally probable bit combination, the averaged BER is

$$\begin{aligned}
 Pe_{ave} &= \frac{1}{256} \sum_{a_0^i=-1}^1 \sum_{b_0^i=-1}^1 \cdots \sum_{d_{-1}^{i-1}=-1}^1 Pe(a_0^i, b_0^i, \dots, d_{-1}^{i-1}) \\
 &= Pe(a_0^i, b_0^i, \dots, d_{-1}^{i-1}). \quad (25)
 \end{aligned}$$

Fig. 4. Performance with various values of γ and β_1 for $\Delta_1 = 0.2$, $\Delta_2 = 0.3$, $\delta = 0.3$, $R = 0.2$, $\beta = 0$, $\Delta = 0.2$, and $\beta_1 = 0.95$.

IV. DISCUSSION

The BER performance is influenced by several factors. In this section, the performance affected by these factors is discussed and a comparison with other systems is made.

A. Power Strength of the Second Path

The influence of the square-root power ratio of the first and the second paths γ and the correlation between the polarizations of line-of-sight and the second path β_1 is shown in Fig. 4. When γ increases from 0.15 to 0.25 at $\beta_1 = 0.95$, the system performance is degraded about 2 dB at the BER of 10^{-4} . For fixed $\gamma = 0.2$, the performance degrades about 1.5 dB when β_1 increases from 0.35 to 0.95. It implies that the performance is sensitive to the power strength of the second path signal. Because of the delay of the second path, the effect of ISI has to be considered. The BER expressed in (25) is the averaged performance of all bit-combination patterns in a symbol duration interval. In general, the averaged BER is dominant by the worst case. Therefore, the performance will be better with smaller β_1 , i.e., the lower polarization correlation between the line-of-sight signal and the second-path signal yields better performance.

B. Effect of Cross-Polarization Interference (CPI)

The performance for various values of the CPI coefficient R is shown in Fig. 5. It indicates that CPI does not affect the performance at all. The reason is that the orthogonal frequency shifting has eliminated the damage of cross-polarization interference. This is the merit of the system, and we can save the effort to compensate cross-polarization problems. The anti-CPI characteristic can be observed more clearly in Fig. 6. When $\delta \geq 0$ (i.e., frequency shift is larger than the orthogonal frequency), the BER is constant no matter what the value of R is. When $\delta = -0.3$ (i.e., frequency shift is smaller than the orthogonal frequency), the performance is getting worse with larger R . Furthermore, comparison of the two delay times of x and y DoPs is shown in Fig. 7, where $\Delta_1 = \tau_x/T$ and $\Delta_2 = \tau_y/T$

are the normalized delay times with respect to the symbol duration. It shows that the performance is significantly affected by the values of parameter Δ_1 but is not influenced by the values of Δ_2 . Therefore, the delay interference produced by the second-path signal of the other DoP can be completely ignored.

C. Guard Band

Because of the existence of guard band (as shown in Fig. 2), the spectrum of $S_{ix}(t)$ is not orthogonally allocated with $S_{(i-1)y}(t)$ (i.e., the normalized shifting frequency δ has to be larger than zero). Fig. 8 illustrates the relation between the BER and δ . It shows that the BER is almost constant when δ is larger than zero. So the system performance does not change when the guard band exists.

D. Effect of Doppler Shift

The relation between the BER and the rolloff factor β is shown in Fig. 9. The BER for $\Delta = 0.2$ has a peak when β is about 0.2. Similarly, for $\Delta = 0.3$, the BER has a peak when β is about 0.3. It implies that the BER is degraded significantly when β and Δ have about the same value. This phenomenon is caused by the Doppler shift. Generally speaking, a slight frequency shift caused by the channel impairments will dominate the system performance. When a slight frequency shift occurs, $m(t, x)$ has the form listed in Region I of Table I, and it has huge interference. Moreover, there is a second peak around $\beta = 0.8$ in Fig. 9. In this case, $m(t, x)$ has the form listed in Region III of Table I, which will also result in large interference. Therefore, it is important to estimate the most probable f_D and to choose an appropriate value of β to avoid BER degradation.

E. Performance Comparison

Comparison with other systems is shown in Fig. 10. The performances of the proposed, conventional OFDM and dual-polarization canceler with bootstrap [12] systems are presented. The performance improvement of the proposed system over the

Fig. 5. Performance with various values of R for $\Delta_1 = 0.2$, $\Delta_2 = 0.3$, $\delta = 0.3$, $\gamma = 0.2$, $\beta = 0$, $\Delta = 0.2$, and $\beta_1 = 0.95$.

Fig. 6. Performance versus R with various values of δ for $\Delta_1 = 0.2$, $\Delta_2 = 0.3$, $\gamma = 0.2$, $\beta = 0$, $\Delta = 0.2$, $\beta_1 = 0.95$, and $E_b/N_0 = 10$ (dB).

dual-polarization canceler with bootstrap system is about three orders of magnitude at $E_b/N_0 = 14$ dB. The performance improvement over the conventional OFDM system is about 3.5 orders of magnitude at the same value of E_b/N_0 . It is believed that the proposed system outperforms the other.

V. CONCLUSIONS

A new dual-polarization frequency reuse scheme with frequency allocation between x and y DoPs is presented without any complex compensation circuits. The system performance

is improved significantly with little alteration of conventional dual-polarization systems. Unlike the conventional dual-polarization system, cross-polarization interference hardly influences the proposed system performance, and CPI can be totally ignored. The delay interference caused by the other DoP hardly impairs the desired symbol, and it simplifies the design for further improvement of the proposed system (e.g., equalization). Furthermore, unlike the OFDM system, multiple pilot tones can be added. The polarization recovery, carrier recovery, and channel estimation from the pilot tones are easier to implement.

Fig. 7. Performance with various values of Δ_1 and Δ_2 for $\gamma = 0.2$, $\delta = 0.3$, $R = 0.2$, $\beta = 0$, $\Delta = 0.2$, and $\beta_1 = 0.95$.

Fig. 8. Performance versus δ with various values of β and Δ for $\Delta_1 = 0.2$, $\Delta_2 = 0.3$, $\gamma = 0.2$, $R = 0.2$, $\beta_1 = 0.95$, and $E_b/N_o = 10$ (dB).

APPENDIX I

Let $Q_1(f)$ be the Fourier transform of $q_1(t - kT, x)$ in (10) and $P(f)$ be the Fourier transform of the square-root raised cosine impulse response $p(t)$. $Q_1(f)$ is given by

$$\begin{aligned} Q_1(f) &= F\{q_1(t, x)\} \\ &= \frac{1}{2} \left[P\left(f - \frac{x}{T}\right) P(f) e^{-j2\pi(f - (x/T))kT} \right. \\ &\quad \left. + P\left(f + \frac{x}{T}\right) P(f) e^{-j2\pi(f + (x/T))kT} \right] \\ &= \frac{1}{2} \left[M'(f) e^{j2\pi kx} + M'\left(f + \frac{x}{T}\right) e^{-j2\pi kx} \right] e^{-j2\pi f kT} \end{aligned} \quad (A1-1)$$

where

$$M'(f) = P\left(f - \frac{x}{T}\right) P(f). \quad (A1-2)$$

Taking the inverse Fourier transform, we obtain $q_1(t - kT, x)$ as

$$\begin{aligned} q_1(t - kT, x) &= F^{-1}\{Q_1(f)\} \\ &= \frac{1}{2} \left[m'(t - kT, x) e^{j2\pi kx} \right. \\ &\quad \left. + m'(t - kT, x) e^{-j(2\pi x t/T)} \right] \\ &= \frac{1}{2} \left[m(t - kT, x) e^{j(\pi x(t+kT))/T} \right. \\ &\quad \left. + m(t - kT, x) e^{-j(\pi x(t+kT))/T} \right] \\ &= \cos\left(\frac{\pi x}{T}(t + kT)\right) m(t - kT, x). \end{aligned} \quad (A1-3)$$

Equation (12) is proved. Equation (13) can be proved by the same fashion. In (A1-3), the functions $m'(t, x)$ and $m(t, x)$ are

Fig. 9. Performance versus β with various values of Δ for $\Delta_1 = 0.2$, $\Delta_2 = 0.3$, $\delta = 0.3$, $\gamma = 0.2$, $R = 0.2$, $\beta_1 = 0.95$, and $E_b/N_o = 10$ (dB).

Fig. 10. Performance comparison with the dual-polarization cancellation system and the OFDM system.

given as

$$m'(t, x) = F^{-1}\{M(f)\} \quad (\text{A1-4})$$

and

$$m(t, x) = m'(t, x)e^{-j(\pi xt/T)} \\ = F^{-1}\left\{P\left(f - \frac{x}{2T}\right)P\left(f + \frac{x}{2T}\right)\right\}. \quad (\text{A1-5})$$

The function of $m(t, x)$ can be obtained from (A1-5), where

$P(f)$ is given by

$$P(f) = \begin{cases} \sqrt{T}, & 0 \leq |f| \leq \frac{1-\beta}{2T} \\ \sqrt{T} \cos\left(\frac{\pi T|f|}{2\beta} - \frac{\pi(1-\beta)}{4\beta}\right), & \frac{1-\beta}{2T} \leq |f| \leq \frac{1+\beta}{2T} \\ 0, & \text{otherwise.} \end{cases} \quad (\text{A1-6})$$

Based on the characteristic of $P(f)$, we have to divide our analysis according to $0 \leq \beta < 0.5$ and $0.5 \leq \beta < 1$, where β is the roll off factor. The result of $m(t, x)$ is listed in Table I.

APPENDIX II

The detailed derivation of (17) is shown in the following. First, let the phase terms α_1 and α_2 be constant. The conditional error probability is given by

$$\begin{aligned}
 P_e &= \Pr \{Z_{ix} < 0 | a_0^i = 1, \alpha_1, \alpha_2\} \\
 &= \int_0^\infty \Pr \{N < -(1 + X_1 + AX_2) | \alpha_1, \alpha_2\} \\
 &\quad \cdot Ae^{-(A^2/2)} dA \\
 &= \int_0^\infty Q\left(\sqrt{\frac{2}{N_0}}(1 + X_1 + AX_2)\right) Ae^{-(A^2/2)} dA \\
 &= \int_0^\infty \int_{\sqrt{2/N_0}(1+X_1+AX_2)}^\infty \frac{1}{\sqrt{2\pi}} \\
 &\quad \cdot e^{-(n^2/2)} Ae^{-(A^2/2)} dn dA \quad (A2-1)
 \end{aligned}$$

where Z_{ix} is shown in (14) and N is a zero-mean AWGN with variance of $N_0/2$. The bit energy E_b has been normalized to unity. Let z be the energy-to-noise ratio $1/N_0$. After exchanging the inner integral and outer integral, we have P_e as

$$P_e = \begin{cases} \int_{\sqrt{2z}(1+X_1)}^\infty \int_0^{((n/\sqrt{2z})-1-X_1)/X_2} \frac{1}{\sqrt{2\pi}} e^{-(n^2/2)} Ae^{-(A^2/2)} dA dn, & X_2 > 0 \\ \left(\int_{-\infty}^{\sqrt{2z}(1+X_1)} \int_{((n/\sqrt{2z})-1-X_1)/X_2}^\infty \right. \\ \quad \left. + \int_{\sqrt{2z}(1+X_1)}^\infty \int_0^\infty \right) \frac{1}{\sqrt{2\pi}} e^{-(n^2/2)} Ae^{-(A^2/2)} dA dn, & X_2 < 0. \end{cases} \quad (A2-2)$$

After finishing integration, we get P_e as

$$\begin{aligned}
 P_e &= \begin{cases} Q(\sqrt{2z}(1+X_1)) - \sqrt{\frac{2zX_2^2}{1+2zX_2^2}} \\ \exp\left[-\frac{z(1+X_1)^2}{1+2zX_2^2}\right] \\ Q\left(\frac{2zX_2(1+X_1)}{\sqrt{1+2zX_2^2}}\right), & X_2 > 0 \\ Q(\sqrt{2z}(1+X_1)) + \sqrt{\frac{2zX_2^2}{1+2zX_2^2}} \\ \exp\left[-\frac{z(1+X_1)^2}{1+2zX_2^2}\right] \\ Q\left(-\frac{2zX_2(1+X_1)}{\sqrt{1+2zX_2^2}}\right), & X_2 < 0 \end{cases} \\
 &= Q(\sqrt{2z}(1+X_1)) - x\sqrt{\frac{2z}{1+2zx^2}} \\
 &\quad \cdot \exp\left[-\frac{z(1+X_1)^2}{1+2zx^2}\right] Q\left(\frac{2zx(1+X_1)}{\sqrt{1+2zx^2}}\right) \Big|_{x=X_2} \quad (A2-3)
 \end{aligned}$$

So the overall bit error probability given in (17) can be obtained by

$$P_{dep} = \left(\frac{1}{2\pi}\right)^2 \int_{-\pi}^\pi \int_{-\pi}^\pi P_e(\alpha_1, \alpha_2) d\alpha_1 d\alpha_2. \quad (A2-4)$$

APPENDIX III

The derivation of (22) can be undertaken from (A2-3). Because there are two Rayleigh random variables A_x and A_y in Eq. (18), the integration has to be done one more time than that in Appendix II. Let α_1 and α_2 be constant. The conditional error probability is given by

$$\begin{aligned}
 P'_e &= \Pr \{Z_{ix} < 0 | a_0^i = 1, \alpha_1, \alpha_2\} \\
 &= \int_0^\infty \left[Q\left(\sqrt{2z}(1+Y_1+Y_3)\right) \right. \\
 &\quad \left. - Y_2 \sqrt{\frac{2z}{1+2zY_2^2}} \exp\left[-\frac{z(1+Y_1+Y_3)^2}{1+2zY_2^2}\right] \right. \\
 &\quad \left. \cdot Q\left(\frac{2zY_2(1+Y_1+Y_3)}{\sqrt{1+2zY_2^2}}\right) \right] A_y e^{-A_y^2/2} dA_y. \\
 &= P_{dep}(x=Y_3) - Y_2 \sqrt{\frac{2z}{1+2zY_2^2}} \\
 &\quad \cdot \int_0^\infty \exp\left[-\frac{z(1+Y_1+Y_3)^2}{1+2zY_2^2}\right] \\
 &\quad \cdot Q\left(\frac{2zY_2(1+Y_1+Y_3)}{\sqrt{1+2zY_2^2}}\right) A_y e^{-A_y^2/2} dA_y. \quad (A3-1)
 \end{aligned}$$

Let

$$\begin{aligned}
 P_b &= \int_0^\infty \exp\left[-\frac{z(1+Y_1+Y_3)^2}{1+2zY_2^2}\right] \\
 &\quad \cdot Q\left(\frac{2zY_2(1+Y_1+Y_3)}{\sqrt{1+2zY_2^2}}\right) A_y e^{-A_y^2/2} dA_y. \quad (A3-2)
 \end{aligned}$$

After exchanging the inner integral and outer integral of P_b , we have P_e as

$$P_b = \begin{cases} \frac{1}{\sqrt{2\pi}} \int_{G(1+Y_1)}^\infty \int_0^{((n-G(1+Y_1))/GY_3)} e^C A_y e^{-E(A_y+F)^1} e^{-n^2/2} dA_y dn, & Y_2Y_3 > 0 \\ \frac{1}{\sqrt{2\pi}} \left(\int_{-\infty}^{G(1+Y_1)} \int_{((n-G(1+Y_1))/GY_3)}^\infty \right. \\ \quad \left. + \int_{G(1+Y_1)}^\infty \int_0^\infty \right) e^C A_y e^{-E(A_y+F)^1} e^{-n^2/2} dA_y dn, & Y_2Y_3 < 0 \end{cases} \quad (A3-3)$$

where

$$C = -\frac{z(1+Y_1)^2}{1+2zY_2^2+2zY_3^2} \quad (A3-4)$$

$$\begin{aligned}
P_b = & \begin{cases} e^C \left\{ \begin{aligned} & \frac{1}{2E} \begin{pmatrix} e^{-EF^2} Q[G(1+Y_1)] \\ -e^{-(EH^2/(2E+G^2Y_3^2))} \sqrt{\frac{G^2Y_3^2}{2E+G^2Y_3^2}} \\ Q \left[\sqrt{\frac{2E+G^2Y_3^2}{G^2Y_3^2}} \left(G(1+Y_1) - \frac{2EH}{2E+G^2Y_3^2} \right) \right] \end{pmatrix} \\ & -\frac{F}{\sqrt{2\pi E}} \int_{G(1+Y_1)}^{\infty} \int_{\sqrt{EF}}^{\sqrt{E}((n-G(1+Y_1)+FGY_3)/GY_3)} e^{-A_y^2} e^{-n^2/2} dA_y dn \end{aligned} \right\}, & Y_2Y_3 > 0 \\ \\ e^C \left\{ \begin{aligned} & \frac{1}{2E} \begin{pmatrix} e^{-EF^2} Q[G(1+Y_1)] \\ +e^{-(EH^2/(2E+G^2Y_3^2))} \sqrt{\frac{G^2Y_3^2}{2E+G^2Y_3^2}} \\ Q \left[-\sqrt{\frac{2E+G^2Y_3^2}{G^2Y_3^2}} \left(G(1+Y_1) - \frac{2EH}{2E+G^2Y_3^2} \right) \right] \end{pmatrix} \\ & -\frac{F}{\sqrt{2\pi E}} \left(\int_{-\infty}^{G(1+Y_1)} \int_{\sqrt{E}((n-G(1+Y_1)+FGY_3)/GY_3)}^{\infty} + \int_{G(1+Y_1)}^{\infty} \int_{\sqrt{EF}}^{\infty} \right) e^{-A_y^2} e^{-n^2/2} dA_y dn \end{aligned} \right\}, & Y_2Y_3 < 0 \end{cases} \\
= & \frac{1+2zY_2^2}{1+2zY_2^2+2zY_3^2} \left[e^{-((z(1+Y_1)^2)/(1+2zY_2^2))} Q \left(\frac{2zY_2(1+Y_1)}{\sqrt{1+2zY_2^2}} \right) \right. \\
& -3 \frac{2zY_2Y}{\sqrt{(1+2zY_2^2)(1+2zY_3^2)}} e^{-((z(1+Y_1)^2)/(1+2zY_3^2))} Q \left(\frac{2zY_3(1+Y_1)}{\sqrt{1+2zY_3^2}} \right) \Big] \\
& - \frac{Y_2Y_3(1+Y_1)}{\sqrt{\pi}((2z)^{-1}+Y_2^2+Y_3^2)^{3/2}} \exp \left[-\frac{z(1+Y_1)^2}{1+2zY_2^2+2zY_3^2} \right] \cdot \text{Res} \quad (\text{A3-8})
\end{aligned}$$

$$E = \frac{1+2zY_2^2+2zY_3^2}{2+4zY_2^2} \quad (\text{A3-5})$$

$$F = \frac{2zY_3(1+Y_1)}{1+2zY_2^2+2zY_3^2} \quad (\text{A3-6})$$

$$G = \frac{2zY_2}{\sqrt{1+2zY_2^2}}. \quad (\text{A3-7})$$

After tedious manipulation, we obtain P_b as shown in (A3-8) at the top of the page, where

$$H = G(1+Y_1) - FGY_3. \quad (\text{A3-9})$$

The overall error probability can be obtained by

$$P_e = \left(\frac{1}{2\pi} \right)^2 \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} P'_e(\alpha_1, \alpha_2) d\alpha_1 d\alpha_2. \quad (\text{A3-10})$$

Equations (22)–(24) are proved.

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Jingshown Wu (S'73–M'79–SM'99) received the B.S. and M.S. degrees in electrical engineering from National Taiwan University, Taipei, Taiwan, R.O.C., in 1970 and 1972, respectively, and the Ph.D. degree from Cornell University, Ithaca, NY, in 1978.

He joined Bell Laboratories in 1978, where he worked on digital network standards and performance and optical fiber communication systems. In 1984, he joined the Department of Electrical Engineering, National Taiwan University, as a Professor, and was Chairman of the department

from 1987 to 1989. He was also Director of the university's Communication Research Center, College of Engineering, from 1992 to 1995. From 1995 to 1998, he was Director of the Division of Engineering and Applied Science, National Science Council, R.O.C., on leave from the university. Since 1999, he has become Chairman of the Commission on Research and Development and Director of the Center for Sponsor Programs of National Taiwan University. He is interested in optical fiber communications, wireless communication, and computer communication networks.

Prof. Wu is a Life Member of the Chinese Institute of Engineers, the Optical Society of China, and the Institute of Chinese Electrical Engineers. He is Chairman of the IEEE Taipei Section.

Meng-Che Wu was born in Taipei, Taiwan, R.O.C., in 1975. He received the B.S. and M.S. degrees in electrical engineering from the National Taiwan University, Taipei, in 1997 and 1999, respectively.

He joined the Wireless Communication Technology Lab, Chunghwa Telecom Laboratories, Taoyuan, Taiwan, in 1999. His research interests include modulation techniques, coding, mobile communication, spread-spectrum communication, and protocol analysis.