Design of Optimal Midcourse Guidance Sliding-Mode Control for Missiles with TVC

FU-KUANG YEH
HSIUAN-HAU CHIEN
LI-CHEN FU
National Taiwan University

This work discusses a nonlinear midcourse missile controller with thrust vector control (TVC) inputs for the interception of a theater ballistic missile, including autopilot system and guidance system. First, a three degree-of-freedom (DOF) optimal midcourse guidance law is designed to minimize the control effort and the distance between the missile and the target. Then, converting the acceleration command from guidance law into attitude command, a quaternion-based sliding-mode attitude controller is proposed to track the attitude command and to cope with the effects from variations of missile’s inertia, aerodynamic force, and wind gusts. The exponential stability of the overall system is thoroughly analyzed via Lyapunov stability theory. Extensive simulations are conducted to validate the effectiveness of the proposed guidance law and the associated TVC.

I. NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Acceleration vector</td>
</tr>
<tr>
<td>d</td>
<td>Disturbances vector</td>
</tr>
<tr>
<td>d_p</td>
<td>Pitch angle of propellant</td>
</tr>
<tr>
<td>d_y</td>
<td>Yaw angle of propellant</td>
</tr>
<tr>
<td>F</td>
<td>Thrust vector</td>
</tr>
<tr>
<td>g</td>
<td>Gravitational acceleration vector</td>
</tr>
<tr>
<td>J</td>
<td>Moment of inertial matrix</td>
</tr>
<tr>
<td>J_0</td>
<td>Nominal parts of J</td>
</tr>
<tr>
<td>ΔJ</td>
<td>Variation of J</td>
</tr>
<tr>
<td>ℓ</td>
<td>Distance between nozzle and center of gravity</td>
</tr>
<tr>
<td>L_b</td>
<td>Displacement vector</td>
</tr>
<tr>
<td>m</td>
<td>Mass of the missile</td>
</tr>
<tr>
<td>N</td>
<td>Magnitude of thrust</td>
</tr>
<tr>
<td>q</td>
<td>Quaternion</td>
</tr>
<tr>
<td>r</td>
<td>Position vector</td>
</tr>
<tr>
<td>̂r</td>
<td>Unit vector of r</td>
</tr>
<tr>
<td></td>
<td>Magnitude of r</td>
</tr>
<tr>
<td>t</td>
<td>Present time</td>
</tr>
<tr>
<td>τ</td>
<td>Intercepting time</td>
</tr>
<tr>
<td>t_0 = τ - t</td>
<td>Time-to-go until intercept</td>
</tr>
<tr>
<td>̂T</td>
<td>Torque</td>
</tr>
<tr>
<td>v</td>
<td>Velocity vector</td>
</tr>
<tr>
<td>̇v</td>
<td>Angular velocity vector</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subscripts</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>Body coordinate frame</td>
</tr>
<tr>
<td>d</td>
<td>Desired</td>
</tr>
<tr>
<td>e</td>
<td>Error</td>
</tr>
<tr>
<td>i</td>
<td>Inertial coordinate frame</td>
</tr>
<tr>
<td>M</td>
<td>Missile</td>
</tr>
<tr>
<td>p</td>
<td>Perpendicular to line of sight (LOS)</td>
</tr>
<tr>
<td>T</td>
<td>Target</td>
</tr>
</tbody>
</table>

II. INTRODUCTION

The midcourse missile guidance concerns the stage before the missile can lock onto the target using its own sensor. Its task is to deliver the missile somewhere near the target with some additional condition, such as suitable velocity or appropriate attitude. Based on the concept of the PN guidance law, constant bearing guidance is often employed on the bank-to-turn (BTT) missiles [1, 2], whereas a different kind of guidance law, namely the zero-sliding guidance law, aims at eliminating the sliding velocity between the missile and the target in the direction normal to line of sight (LOS) [3]. Ha and Chong derived a new command to line-of-sight (CLOS) guidance law for short-range surface-to-air missile via feedback linearization [4] and its modified version [5] with improved performance. In order to utilize the prior information on the future target maneuvers or on the autopilot lags, the optimal guidance law based on the optimal control theory [6–8] has been
investigated since the 1960s, although that guidance law requires more measurements than the PN guidance law [10–12]. A new optimal guidance law without estimation of the interception time is proposed to deal with the situation where accurate time-to-go is unavailable [13].

On the other hand, attitude control is another important issue to be addressed for successful missile operation. Quaternion representation has often been adopted to describe the attitude of a spacecraft [14, 15], because it is recognized as a kind of global attitude representation. To account for the nonideal factors of the spacecraft under attitude control and to strengthen the robust property of the controller, the sliding-mode control has been employed by Chen and Lo [17], which is then followed by a smooth version [18] incorporating a sliding layer, as has been proposed by [9] to avoid the chattering phenomenon, but at the price of slightly degrading the accuracy of the tracking system. To achieve the same goal, a different approach, called “adaptive control,” has been adopted by Slotine [20] and Lian [16]. They incorporate a parameter estimation mechanism so as to solve the problems of accurate attitude tracking under large unknown loads, and of orientation control for general nonlinear mechanical systems, respectively. All the above research works address the issue of attitude control mainly to achieve the goal of attitude tracking.

A missile equipped with thrust vector control (TVC) can effectively control its acceleration direction [3, 23, 24] when the missile built with fins fails, which in turn implies that the maneuverability/controllability of the missile can be greatly enhanced at the stage of low missile velocity and/or low air density surrounding the missile. Thus, midcourse guidance employing the TVC is common in missile applications and there are also a number of other applications which employ TVC; for instance, Lichtsinder et al. [25] improved the flying qualities at high angle-of-attack and high sideslip angle of a fighter aircraft, whereas Spencer [26] dealt with the spacecraft undergoing orbital transformation where maneuver is to consume minimum power. There are also some other instances of application in the areas of launch vehicle and the transportation industry. In particular, for an upper-tier defender such as the Theater High Altitude Area Defense (THAAD) system, the midcourse phase lasts for a long period, and therefore variations in missile inertia during the travel period cannot be neglected, and the impact of aerodynamic forces and wind gusts must be compensated for in order to guarantee that missile attitude remains stable during flight. Furthermore, the midcourse guidance using TVC is subject to the limitation that the control force is then constrained by the TVC mechanical structure, which further complicates the controller design. The above issues need to be pursued in the midcourse guidance and control system.

In the work presented here, we investigate the midcourse guidance and control problem for a missile equipped with TVC so that the missile is able to reach somewhere near the target for the purpose of successful interception of an inbound target in the follow-up homing phase. At first, a 6 degree-of-freedom (DOF) model of the missile system which considers the aerodynamic force and wind force, fluctuation of missile’s mass and moment of inertia, and the 3 DOF TVC is derived. Next, a 3 DOF optimal guidance law which tries to minimize both the control effort and the distance between the missile and the target location is proposed. To realize such guidance in a realistic situation, a nonlinear robust attitude controller is also developed. This is based on the sliding-mode control principle. A general analysis is then performed to investigate the stability property of the entire missile system. Several numerical simulations have been provided to validate the excellent target-reaching property.

The midcourse control system can be separated into guidance and autopilot systems. The guidance system receives the information on the kinematic relation between the missile and the target, and via optimal guidance law determines the acceleration command to the autopilot system. The autopilot system will then convert the acceleration command into attitude command, and via the controller calculation generate the torque command to the TVC to adjust the attitude of the missile so that the forces generated from the TVC can realize the guidance command. The overall system can be represented as Fig. 1.

The rest of the paper is organized as follows. In Section III, a detailed 6 DOF motion model of the missile equipped with TVC is derived. Section IV proposes an optimal midcourse guidance law aiming at minimization of both control efforts and the distance between the missile and target. For guidance realization, an autopilot system incorporating the so-called quaternion-based sliding-mode control is developed in Section V. For sound proof, a thorough integrated analysis of the overall design is also provided in that section. To demonstrate the excellent property of the proposed integrated guidance and control, several numerical simulations have been conducted in Section VI. Finally, conclusions are drawn in Section VII.
III. EQUATIONS OF MOTION FOR MISSILES WITH TVC

The motion of a missile can be described in two parts as follows:

Translation:
\[ \ddot{v}_M = a_M + g_M, \quad \dot{r}_M = v_M \]  

Rotating:
\[ J \dot{\omega} = -\dot{J} \omega - \omega \times (J \omega) + \vec{T}_b + d. \]  

All the variables are defined in the nomenclature listing.

Assume that the nozzle is located at the center of the tail of the missile, and the distance between the nozzle center and the missile’s center of gravity is \( l \). Furthermore, we also assume that the missile is equipped with a number of sidejets or thrusters on the surface near the center of gravity that will produce a pure rolling moment whose direction is aligned with the vehicle axis \( X_b \), referred to Fig. 3. Thus, the vector \( L_b \), defined as the relative displacement from the missile’s center of gravity to the center of the nozzle, satisfies \( |L_b| = l \). Note that \( J \) is the moment of inertia matrix of the missile body with respect to the body coordinate frame as shown in Fig. 2 and hence is a \( 3 \times 3 \) symmetric matrix.

Generally speaking, for various practical reasons the rocket engines deployed on the missile body cannot vary with any flexibility the magnitude of the thrust force. Therefore, for simplicity we assume here that the missile can only gain constant thrust force during the flight. After referring to Fig. 2 and Fig. 3, the force and torque exerted on the missile can be respectively expressed in the body coordinate frame as
\[ \vec{F}_b = N \begin{bmatrix} \cos d_p \cos d_y \\ \cos d_p \sin d_y \\ \sin d_p \end{bmatrix} \]  

and
\[ \vec{T}_b = L_b \times \vec{F}_b + M_b = lN \begin{bmatrix} M_{b_x}/lN \\ \sin d_p \\ -\cos d_p \sin d_y \end{bmatrix} \]  

where \( N \) is the magnitude of thrust, \( d_p \) and \( d_y \) are respectively the pitch angle and yaw angle of the propellant, and \( M_b = [M_{b_x}, 0, 0]^T \) is the aforementioned variable moment in the axial direction of the missile.

Let the rotation matrix \( B_b \) denote the transformation from the body coordinate frame to the inertial coordinate frame. Thus, the force exerted on the missile observed in the inertial coordinate system is as follows:
\[ \vec{F}_i = B_b \vec{F}_b. \]  

From (1)–(5), the motion model of the missile can then be derived as
\[ \dot{v}_M = \vec{F}_i/m + g_M = (B_b \vec{F}_b)/m + g_M \]  

\[ J \omega = -\dot{J} \omega - \omega \times (J \omega) + lN \begin{bmatrix} M_{b_x}/lN \\ \sin d_p \\ -\cos d_p \sin d_y \end{bmatrix} + d. \]  

IV. GUIDANCE SYSTEM DESIGN

There are several midcourse guidance laws which have been proposed in the past. In particular, Lin [21] presented an analytical solution of the guidance law formulated in a feedback form, with the feedback gain being optimized to give the maximum end velocity of the missile. But the acceleration command of the proposed guidance law was derived in a continuous form, i.e., the magnitude of the acceleration command can be any value within the capability range of the missile actuators. This, however, may not be valid in the general situation.

The equations of relative motion in terms of the relative position \( r = r_T - r_M \) and the relative velocity \( v = v_T - v_M \) are as follows:
\[ \dot{v}(t) = -a_M(t) \quad \text{and} \quad \dot{r}(t) = v(t) \]  

where we assume the target is not maneuvering (i.e., \( a_T = 0 \)) and the direction of \( r \) is along the LOS from the missile to the target. The optimal control theory [6–8] is then adopted for design of the guidance law in the aforementioned interception problem, where our objective is to compute the necessary missile acceleration \( a_M \) at the present time \( t \) in terms of \( r(t) \) and \( v(t) \) so that a minimum-effort interception occurs at some terminal time \( \tau \geq t \). To solve this problem, the acceleration command is derived based on minimization of the following cost function
\[ J = \int_0^\tau v_T^2r_T^2 + \frac{1}{2} \int_0^\tau a_M(t)a_M(t)dt \]
where $\gamma > 0$ is the appropriate weighting and $t_0$ is some starting time. The first term on the right-hand side of (9) is the weighted squared miss-distance. As a consequence, for very large values of $\gamma$, we should expect that the terminal miss-distance $|r(\tau)|$ will be very small so that a practical interception will occur at time $\tau$.

Via the optimal control theory, the optimal acceleration command $a_M$ can be found to be of a form of state feedback [6], as

$$a_M(t) = \frac{3}{t_s}[r(t) + t_s v(t)]$$

where $t_s$ denotes the time-to-go from the current time $t$ to the intercepting time $\tau$. Here $t_s$ is assumed to be a known variable, but in fact it is unknown, and hence a procedure for estimating $t_s$, whose accuracy will affect the performance of the optimal guidance law significantly, is required.

Since the magnitude of the thrust is assumed to be a constant, this means that it is impossible to maintain the acceleration along the LOS as a constant value when $a_p$ (shown in Fig. 4), to be defined shortly, varies in magnitude. Therefore, $t_s$ cannot be accurately established using any approximation formula.

However, a modified optimal guidance law without estimation of time-to-go can be designed, based on the component of the relative velocity normal to the LOS, i.e., $v_p = v - (v^T \hat{r}) \hat{r}$. To proceed, we first derive the equation of the relative motion perpendicular to the LOS as follows:

$$\dot{v}_p = -a_p - \frac{d}{dt}(v^T \hat{r}) \hat{r} - (v^T \hat{r}) \dot{\hat{r}}$$
$$= -a_p - \frac{1}{|r|} |v_p|^2 \hat{r} - \frac{v^T \hat{r}}{|r|} v_p$$

where $a_p = a_M - (a_M^T \hat{r}) \hat{r}$ denote the missile’s acceleration component perpendicular to the LOS. Then our principal objective concerning the optimal guidance law is to receive the perpendicular acceleration command $a_p$, at the present time $t$ in terms of $v_p$, $v$, $r$, and the cost function parameters in order to fulfill the optimization principle after some appropriate feedback linearization. Specifically, the perpendicular acceleration component $a_p$ is set as

$$a_p = -u - \frac{v^T \hat{r}}{|r|} v_p$$

which then leads to the equation of the normal component of the relative motion as $\dot{v}_p = u - (1/|r|) |v_p|^2 \hat{r}$, where $a_p$, $u$, and $v_p$ are all in the normal direction of LOS. And hence the governing equation considering normal direction of LOS is $\dot{v}_p = u$, where $u$ is calculated by minimizing the quadratic cost function, defined as

$$J = \frac{1}{2}\gamma(T)v_p^T(T)v_p(T) + \frac{1}{2}\int_0^T (\sigma v_p^T(t)v_p(t) + \rho u^T(t)u(t))dt$$

where $\gamma(T) \geq 0$, $\sigma \geq 0$, $\rho > 0$, and $[t_0, T]$ is the time interval in the behavior of the plant in which we are interested. Using optimal control theory, the Riccati equation [6] can be derived as

$$\dot{\gamma}(t) = \frac{\gamma^2(t)}{\rho} - \sigma$$

where $\gamma(t)$ is the solution, subject to the final condition $\gamma(T)$. Using separation of variables and setting $\gamma(T)$ at a very large value allows us to compute $\gamma(t)$ via backward integration of the Riccati equation, so that we have [6]

$$\gamma(t) = \sqrt{\rho \sigma} \left(1 + \frac{2}{e^{2\sqrt{\sigma / \rho (T-t)}} - 1}\right)$$

(see Appendix A) which leads to the optimal control $u$ as follows:

$$u(t) = -\sqrt{\frac{\sigma}{\rho}} \left(1 + \frac{2}{e^{2\sqrt{\sigma / \rho (T-t)}} - 1}\right) v_p$$

Thus, the acceleration component perpendicular to the LOS is

$$a_p(t) = \sqrt{\frac{\sigma}{\rho}} \left(1 + \frac{2}{e^{2\sqrt{\sigma / \rho (T-t)}} - 1}\right) - \frac{v^T \hat{r}}{|r|} v_p(t)$$

so that the equation of relative motion in (15) becomes

$$\dot{v}_p = -\sqrt{\frac{\sigma}{\rho}} \left(1 + \frac{2}{e^{2\sqrt{\sigma / \rho (T-t)}} - 1}\right) v_p - \frac{1}{|r|} |v_p|^2 \hat{r}.$$  

(18)

REMARK 1 From (17), which is well defined unless $r = 0$, the midcourse guidance law will switch to the terminal guidance law when the sensor affixed to the missile’s body can lock onto the target, so that $|r| \neq 0$ is always true throughout the whole midcourse phase. The modified optimal guidance law does not consider the estimation of time-to-go, and hence $T$
can be freely selected as sufficiently large to avoid the singularity problem of (15), i.e., the $T$ can be greater than the time $t$ during the entire midcourse phase. Since the present work is focused on midcourse guidance and control, the situation where $T = t$ will not occur, thereby obviating the problem of infinity input magnitude.

In order to verify that the modified optimal midcourse guidance law will cause the system to be exponentially stable, Lemma 1 is proposed to serve purposes of verification.

**Lemma 1** Let the equation of relative motion perpendicular to the LOS and the modified optimal guidance law be given by (11) and (17), respectively. If $v$ has no component in the normal direction of the LOS, and $v^r < \beta < 0$ with $v$ being bounded away from zero, then the ideal midcourse guidance system will ensure that the target is reached.

**Proof** See Appendix B.

**V. AUTOPILOT SYSTEM DESIGN**

The autopilot system rotates the missile so that its thrust is aligned with the desired direction. It can be deduced that the acceleration derived from (17) has limited magnitude, i.e.,

$$|a_p| \leq N/m.$$  

Hence, the desired direction of the thrust vector is aligned with the composed vector, i.e.,

$$a_p + \left\{ \sqrt{(N/m)^2 - |a_p|^2} \right\} \hat{r}$$  

where $\hat{r}$ is the unit vector of $r$.

For a missile propelled using a TVC input device, the force and the torque exerted on the missile are closely related. As mentioned above, to realize the composed acceleration given in (20), some desirable force and the torque exerted on the missile are closely related. As mentioned above, to realize the composed acceleration given in (20), some desirable

**Fig. 2.** Then, by substituting (21) with (22), the other components of $q_e$ can be solved as

$$q_{c2} = \frac{-F_{dbz}}{2Nq_{e4}}, \quad q_{c3} = \frac{F_{dbv}}{2Nq_{e4}}, \quad q_{c4} = \sqrt{\frac{F_{dbh}}{2N} + \frac{1}{2}}.$$  

Accordingly, the desired quaternion $q_d$ can be derived via substitution of the error quaternion $q_e$ in (23) and the measured current quaternion $q$, rendering

$$\begin{bmatrix} q_{e1} \\ q_{e2} \\ q_{e3} \\ q_{e4} \end{bmatrix} = \begin{bmatrix} q_{d4} & q_{d3} & -q_{d2} & -q_{d1} \\ -q_{d3} & q_{d4} & q_{d1} & -q_{d2} \\ q_{d2} & -q_{d1} & q_{d4} & -q_{d3} \\ q_{d1} & q_{d2} & q_{d3} & q_{d4} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$  

thereby establishing the desired attitude command to the autopilot system. For the other, the desired angular velocity and its time derivative can be expressed as

$$\omega_d = 2E^T(q_d)\dot{q}_d$$  

$$\ddot{\omega}_d = 2E^T(q_d)\ddot{q}_d$$  

which has been proposed by [18, 22], where

$$E(q_d) = \begin{bmatrix} (\dot{q}_d \times) + q_{d4}I_{3 \times 3} \\ -\dot{q}_d \end{bmatrix} \in R^{4 \times 3}.$$  

If the desired quaternion $q_d$, $\dot{q}_d$ and $\ddot{q}_d$ are given with unit-norm property of $q_d$, then the main goal of the attitude control is to let the quaternion $q$ approach $q_d$ and angular velocity $\omega$ approach $\omega_d$. In this paper, in order to avoid singularity of $(\dot{q}_d \times) + q_{d4}I_{3 \times 3}$, we must limit $q_{d4}$ to a constant sign, say positive, i.e.,
$q_{dA} > 0$, throughout the midcourse, so that the Euler
rotation angle is between $\pm \cos^{-1} q_{dA}$.

From (2) and quaternion dynamic equation, the
dynamic model of a missile, treated as a rigid body,
can be derived by differentiation of the associated
quaternion as a function of the corresponding angular
velocity and the quaternion itself, i.e.,
\[
\begin{align*}
\ddot{q}_e &= -\frac{1}{2}(\hat{q}_e \times \omega)_e + \frac{1}{2}q_{ed}\omega_e \\
q_{ed} &= -\frac{1}{8}\omega^2 q_e \\
J\dot{\omega} &= -J\omega - \omega \times (J\omega) + \tilde{T}_b + d
\end{align*}
\] (27)
where $\omega_e = \omega - \omega_d$ is the error between angular
velocities at the present attitude and the desired
attitude, and $\tilde{T}_b$ is the torque exerted on the missile
due to TVC and the rolling moment.

In the controller design, the required feedback
signals $\omega$ and $q$ are assumed to be measurable.
Besides, to demonstrate the robustness of the
controller, we allow the dynamic equation (27) to
possess bounded input disturbances $d$ and bounded
induced 2-norm of $J$, $\Delta J$, and $J = J_0 + \Delta J$. The
objective of the tracking control here is to drive the
missile such that $q_e = 0$, i.e., the quaternion $q(t)$ is
controlled to follow the given reference trajectory
$q_{d}(t)$. Note that if the vector $q_{d}(t)$ is constant, it means
that there is an attitude orientation problem.

To tackle such a robust attitude tracking control
problem, the well-known sliding-mode control
technique is adopted here, which generally involves
two fundamental steps. The first step is to choose a
sliding manifold such that in the sliding-mode the goal
of sliding condition is achieved. The second step is to
design control laws such that the reaching condition is
satisfied, and thus the system is strictly constrained on
the sliding manifold. In the following, the procedure
of designing the sliding-mode controller is given in
detail.

**Step 1** Choose a sliding manifold such that the
sliding condition will be satisfied and hence the error
origin is exponentially stable.

Let us choose the sliding manifold as
\[
S_a = P\hat{q}_e + \omega_e
\] (28)
where $P = \text{diag}[p_1, p_2, p_3]$ is a positive definite
diagonal matrix. From the sliding-mode theory, once
the reaching condition is satisfied, the system is
eventually forced to stay on the sliding manifold,
i.e., $S_a = P\hat{q}_e + \omega_e = 0$. The system dynamics are then
constrained by the following differential equations
\[
\begin{align*}
\dot{\hat{q}}_e &= -\frac{1}{2}(\hat{q}_e \times \omega)_e - \frac{1}{2}q_{ed}\omega_e \\
\dot{q}_{ed} &= \frac{1}{2}\omega^2 q_e
\end{align*}
\] (29)
It has been shown by [3] that the system origin
$(\hat{q}_e, \omega_e) = (0_{3x1}, 0_{3x1})$ of the ideal system (29) is
indeed exponentially stable.

**Step 2** Design the control laws such that the
reaching condition is satisfied.

Assume that $J$ is symmetric and positive definite,
and let the candidate of a Lyapunov function be set as
\[
V_s = \frac{1}{2}S_a^T J S_a \geq 0
\] (30)
where $V_s = 0$ only when $S_a = 0$. Taking the first
derivative of $V_s$, we have
\[
\dot{V}_s = S_a^T [ -J\dot{\omega} - \omega \times (J\omega) + \tilde{T}_b + d - J\dot{\omega}_d + JP(\frac{1}{2}(\hat{q}_e \times \omega)_e + \frac{1}{2}q_{ed}\omega_e) + \frac{1}{2}J\dot{S}_a ].
\] (31)
Let the control law be proposed as
\[
\tilde{T}_b = -J_0 P(\frac{1}{2}(\hat{q}_e \times \omega)_e + \frac{1}{2}q_{ed}\omega_e) + \omega \times (J\omega) + J_0\dot{\omega}_d + \tau
\] (32)
where $\tau = [\tau_1 \tau_2 \tau_3]^T$, $\tau_i = -k_i(q_i, \dot{q}_i, \dot{q}_d, \dot{\dot{q}}_d)$,
\[
\text{sgn}(S_a) = \begin{cases} 
1 & S_a > 0 \\
0 & S_a = 0 \\
-1 & S_a < 0
\end{cases}
\]
and $S_a = [S_{a1} S_{a2} S_{a3}]^T$.

Equation (31) then becomes
\[
\dot{V}_s = S_a^T [\delta + \tau] = 3 \sum_{i=1}^{3} S_{ai} (\delta_i + \tau_i),
\] (33)
where
\[
\delta = -J\omega - \omega \times (\Delta J\omega) + d - \Delta J\dot{\omega}_d + \Delta JP(\frac{1}{2}(\hat{q}_e \times \omega)_e + \frac{1}{2}q_{ed}\omega_e) + \frac{1}{2}J\dot{S}_a.
\] (34)

Assume that the external disturbances $d$ and the
induced 2-norm of $J$ and $\Delta J$ are all bounded, then
the bounding function on $|\delta_i|$, which obviously is a
function of $q_i, \dot{q}_i, \dot{q}_d, \dot{\dot{q}}_d$, and $\dot{\dot{q}}_d$, can be found and
represented as $\delta_{i}^{\text{max}}(q_i, \dot{q}_i, \dot{q}_d, \dot{\dot{q}}_d, \dot{\dot{q}}_d) \geq |\delta_i|$, as can clearly
be seen from (34), (25). It is evident that if we choose
$k_i(q_i, \dot{q}_i, \dot{q}_d, \dot{\dot{q}}_d) > \delta_{i}^{\text{max}}(q_i, \dot{q}_i, \dot{q}_d, \dot{\dot{q}}_d, \dot{\dot{q}}_d)$ for $i = 1, 2, 3$,
(33) then becomes
\[
\dot{V}_s = -\sum_{i=1}^{3} |S_{ai}| [k_i - \delta_{i}^{\text{max}}(S_{ai})] \\
\leq -\sum_{i=1}^{3} |S_{ai}| [k_i - \delta_{i}^{\text{max}}] < 0
\] (35)
for $S_{ai} \neq 0$. Therefore, the reaching and sliding
conditions of the sliding-mode $S_a = 0$ are guaranteed.

**Remark 2** However, since the practical
implementation of the sign function $\text{sgn}(S_{ai})$ is
anything but ideal, the control law $\tilde{T}_b$ in (32) always
suffers from the chattering problem. To alleviate such
an undesirable phenomenon, the sign function can be
simply replaced by the saturation function. The system
is now no longer forced to stay on the slidingmode.
REMARK 3 Recall that previously.
The cost of such substitution is a reduction in the accuracy of the desired performance.

Generally, the stability of an integrated system cannot be guaranteed by the stability of each individual subsystem of the integrated system, and thus the closed-loop stability of the overall system must be reevaluated. The guidance system design in the previous section is based on the assumption that the autopilot system is perfect. That is, we can get the desired attitude at any arbitrary speed, and therefore the acceleration exerted on the missile is always as desired. But, in fact, there is an error between the actual acceleration and the desired acceleration. In other words, if the desired acceleration is \( a_p + \tilde{r} \), and we assume that the flying direction will be along the axial direction of the missile, then the relationship between the actual acceleration \( a_M \) applied on the missile and the desired acceleration \( a_p + \tilde{r} \) is the following:

\[
a_M = B_b(q)B_T(q_a)B_b^I(q)(a_p + \tilde{r})
\]

referring to Fig. 5, where \( B_b(\cdot) \) and \( B(\cdot) \) are as defined previously.

REMARK 3 Recall that \( B_b(q) \) is the transformation from the current body coordinate to the inertial coordinate, and \( B(q_a) \) is the transformation from the current body coordinate to the desired body coordinate. From Fig. 5, \( B = [X_b \; Y_b \; Z_b] \) is some inertial coordinate with its origin coincident with the missile’s center of gravity and \( B_b = [X_b \; Y_b \; Z_b] \) is the current body coordinate. By definition, the axial direction of the missile is along the \( X_b \) direction, and the actual acceleration \( a_M \) is also coincident with the \( X_b \) axis. On the other hand, \( X_d = [X_d \; Y_d \; Z_d] \) is the desired force coordinate, and the desired acceleration \( a_p + \tilde{r} \) should be aligned with the \( X_d \) axis. Finally, \( \tilde{r} = \sqrt{\frac{1}{\rho} - |a_p|^2} \) is the acceleration exerted by thrust along the LOS.

Since the actual acceleration exerted on the missile is \( a_M \), the component of the actual acceleration perpendicular to the LOS is

\[
a_{M_F} = \frac{\{B_b(q)B_T(q_a)B_b^I(q)(a_p + \tilde{r})\} a_p}{|a_p|^2}
\]

where \( a_{M_F} \) in (17) is the desired acceleration perpendicular to the LOS, as previously mentioned. Substitute (11) with (37), and we get a new state equation as follows:

\[
\dot{v}_p = -\frac{\{B_b(q)B_T(q_a)B_b^I(q)(a_p + \tilde{r})\} a_p}{|a_p|^2} - \frac{1}{|r|^3}|r|^2 \tilde{r} - \frac{v^T \tilde{r}}{|r|} v_p.
\]

Let the Lyapunov function candidate be \( V_G = \frac{1}{2} v_p^T v_p \), as has been shown in Appendix B. Then, the time derivative of the Lyapunov function can be derived as

\[
\dot{V}_G = v_p^T \dot{v}_p = -\frac{2}{\rho^2} \left( 1 + \frac{2}{e^2 \sqrt{|r|}} \right) \left( v_p^T \tilde{r} - \frac{v_p^T v_p}{|r|} v_p \right)
\]

where \( a_p \) is given as in (17), \( F_b = B_b^I(q)(a_p + \tilde{r}) \), \( B_T^I = B_p^{-1} \), \( \tilde{a}_p = B_p a_p \), and \( B(q_a) = \frac{\tilde{q}_e \times (\tilde{q}_e \times) + q_e q_e \tilde{q}_e \times}{|q_e|^2} \).

Note that we use the fact that \( v_p^T \tilde{r} = \tilde{r} a_p = 0 \), and \( B(q_a) = I + 2(\tilde{q}_e \times) + 2q_e \tilde{q}_e \times \). If the error quaternion is zero, that is \( \tilde{q}_e = 0 \), the stability is apparently valid.

To verify the stability of the overall system, we define the Lyapunov function candidate of the overall system as

\[
V = V_i + V_G.
\]

The time derivative of the Lyapunov function can be derived as

\[
\dot{V} = S_i [-J \omega - \omega \times (J \omega) + \tilde{T}_b] + d - J \ddot{\omega}_d
\]

\[
+ JP \left( \frac{\tilde{q}_e \times \omega}{|q_e|^2} + \frac{q_e \omega_e}{|q_e|^2} \right) + \frac{1}{2} J S_a
\]

\[
- K_1(v_p, T, t) - K_2(v_p, v, r, T, t) \frac{2\{B(q)F_b T a_p\}}{|a_p|^2}
\]

referring to (31) and (39), where

\[
K_1(v_p, T, t) = \frac{\sigma}{\rho} \left( 1 + \frac{2}{e^2 \sqrt{|r|}} \right) v_p^T v_p
\]

\[
= K_1(T, t) v_p^T v_p > \sqrt{\frac{\sigma}{\rho}} v_p^T v_p \geq 0
\]

\[
K_2(v_p, v, r, T, t) = \left[ \frac{\sigma}{\rho} \left( 1 + \frac{2}{e^2 \sqrt{|r|}} \right) v_p^T v_p \right] v_p^T v_p
\]

\[
= K_2(v, r, T, t) v_p^T v_p > \sqrt{\frac{\sigma}{\rho}} v_p^T v_p \geq 0
\]
for all cases where $t < T$ and $v^T \dot{r} < 0$. Apparently, both $\dot{K}_1$ and $\dot{K}_2$ are greater than $\sqrt{\sigma_p}$ for all time $t$.

To simplify (41), we first investigate the last term of (41) as follows:

$$-K_3(v_p, v, r, T, t) \frac{2[\hat{\beta} F_i(a_p) T_i a_p]}{|a_p|^2}$$

$$= \frac{2K_2(v, r, T, t) F_i T_i (\hat{\beta}_x \times) + q_{ae} l_3 s_3 (\hat{\beta}_y \times) S_a}{|a_p|^2} - \frac{2K_2(v, r, T, t) F_i T_i (\hat{\beta}_x \times) + q_{ae} l_3 s_3 (\hat{\beta}_y \times) S_a}{|a_p|^2}$$

which is then substituted into (41) so that

$$\dot{V} = S_a^T \left[ -J \omega - \omega \times (J \omega) + \hat{B}_d + d - J \omega_d \right.$$

$$J \frac{2K_2(v, r, T, t) F_i T_i (\hat{\beta}_x \times) + q_{ae} l_3 s_3 (\hat{\beta}_y \times) S_a}{|a_p|^2} + \frac{2K_2(v, r, T, t) F_i T_i (\hat{\beta}_x \times) + q_{ae} l_3 s_3 (\hat{\beta}_y \times) S_a}{|a_p|^2} - K_3 v_p \dot{v}_p$$

where $K_3 = K_3(v, r, q_e, \omega, a_p, T, t)$ is defined as

$$K_3 = K_1 + \frac{2K_2(v, r, T, t) F_i T_i (\hat{\beta}_x \times) + q_{ae} l_3 s_3 (\hat{\beta}_y \times) S_a}{|a_p|^2}$$

and the matrix $P$ is chosen to be $P = p \times I_3$, $s_3$.

Now, we are ready to state the following theorem which provides conditions under which the proposed overall midcourse optimal guidance and TVC autopilot guarantees the stability of the entire system, and the target-reaching objective is achieved.

**THEOREM 1** Let the modified optimal guidance law be proposed as in (17), (37), so that the torque input of the autopilot is given as follows:

$$\hat{F}_p = -J_0 p \left( \frac{1}{2} \hat{\beta}_x \times \omega + \frac{1}{2} q_{ae} \omega \right) + \omega \times (J_0 \omega) + J_0 \omega_d + \tau'$$

where $\tau' = [\tau'_1 \, \tau'_2 \, \tau'_3]^T$, $\tau'_i = -K_i(q, \omega, q_d, \hat{\beta}_x, \hat{\beta}_y, \hat{\beta}_z, v, r, T, t) \left[ \text{sgn}(S_{a_i}) \right]$ for some existing stabilizing gains $k_i$, $i = 1, 2, 3$ and $p$ is chosen to be large enough. If $v$ is such that $v^T \hat{r}(t_0) \dot{v}(t_0) < 0$, where $t_0$ is the starting time and $v$ is bounded away from zero, then the integrated overall midcourse guidance and autopilot system will be stable and the target reaching property is achieved.

**PROOF** After substitution of the torque input (45) by hypothesis, the expression of $\dot{V}$ can be readily simplified as

$$\dot{V} = S_a^T [\delta + \tau'] - K_3 v_p \dot{v}_p$$

where

$$\delta = \frac{d}{\Delta \omega + p \Delta J} \left( \frac{1}{2} \hat{\beta}_x \times \omega \right) + \frac{1}{2} q_{ae} \omega + \frac{2K_2(v, r, T, t) F_i T_i (\hat{\beta}_x \times) + q_{ae} l_3 s_3 (\hat{\beta}_y \times) S_a}{|a_p|^2}$$

Assuming that external disturbance $d$ and uncertainties $\Delta J$ are all bounded, we conclude that the upper bounded $\delta_i^{\max}$, $\delta_i^{\min} \geq [\delta]$ and $i = 1, 2, 3$ are functions of $q_e, \omega, q_d, \hat{\beta}_x, \hat{\beta}_y, v_p, v, r, T, t$. It is evident that if we choose functional gains $K_i(q, \omega, q_d, \hat{\beta}_x, \hat{\beta}_y, v_p, v, r, T, t) > \max \{\delta_i^{\max}, \delta_i^{\min}\}$ then, $i = 1, 2, 3$, for $\eta_i > 0$, referring to (34) and (47), (35) apparently holds and the expression (46) can be further explored as

$$\dot{V} = - \sum_{i=1}^{3} |S_{a_i}| [\delta_i - \delta_i \text{sgn}(S_{a_i})] - K_3 v_p \dot{v}_p$$

The former result implies that $S_{a_i}$ and, hence, $q_e, \omega \rightarrow 0$ when $t \rightarrow \infty$. About the latter, the following is an important working lemma revealing that $K_3$ will always be bounded below by a positive constant.

**LEMMA 2** Through the entire midcourse phase, if $v^T \dot{r} < 0$, with $v$ being bounded away from zero, then we can always find appropriate gain $p$ and the adjustable convergent time parameter $T > t$, such that

$$K_3 \geq K_30 > 0$$

whenever $t \geq 0$.

**PROOF** See Appendix B.

As a result, (48) can be expressed as

$$\dot{V} \leq - \sum_{i=1}^{3} \eta_i |S_{a_i}| - K_3 v_p \dot{v}_p$$

which means that $-\dot{V}$ is positive definite, and hence $S_{a_i} \rightarrow 0$, $v_p \rightarrow 0$ as $t \rightarrow \infty$ via use of Lyapunov stability theory. In another words, not only the attitude and the component of the relative velocity perpendicular to LOS, $v_p$, are both stabilized all the time, but also the objectives of attitude tracking and LOS velocity alignment are achieved.

Up to this point, we have provided an integrated stability analysis of the overall system. Finally, to show that Lemma 1 is satisfied, i.e. target reaching, we need to show that $v^T \dot{r} \leq \beta < 0$ at all times. First, $v_p$ has to be verified as exponentially decaying.

Although $V$ and $V_c$ can be shown to be negative definite via Theorem 1 via torque input (45), by definition $\dot{V} = V_G + V_s$ in (41) $V_G$ cannot be proved.
we can conclude that $\dot{V}_G$ explicitly from (39) and (41)–(43), i.e.,

$$
\dot{V}_G = -\left[ \tilde{K}_1 - \frac{2K_p F_p^T(\tilde{a}_{\tilde{r}} \times) + q_d I_{3 \times 3} \tilde{a}_{\tilde{r}} \times (S_u - \omega_c)}{|\tilde{a}_{\tilde{r}}|^2} \right] v_r^T v_r.
$$

(50)

Using the proof of Lemma 2, the maximum value $\tilde{P}_{\text{max}}$ in (50) can also be obtained as

$$
\tilde{P}_{\text{max}} \geq \frac{F_p^T(\tilde{a}_{\tilde{r}} \times) + q_d I_{3 \times 3} \tilde{a}_{\tilde{r}} \times (S_u - \omega_c)}{|\tilde{a}_{\tilde{r}}|^2}
$$

where we already showed that with reference to (28) and (35), $S_u \to 0$ and $\omega_c \to 0$ as $t \to \infty$ due to the autopilot system design in Section V. Thus, if the inequality (56) in Lemma 2 can be modified as

$$
P > 2 \max \{P_{\text{max}}, \tilde{P}_{\text{max}} \} \cdot \tilde{K}_{\text{max}}
$$

then $\dot{V}_G$, $\dot{V}$, and $\dot{V}_r$ are all negative definite, meaning $v_p$ will be attenuated exponentially at all times due to the definition of $V_G = \frac{1}{2} v_p^T v_p$, which is always positive definite. Given this much, we are now ready to prove the above claim as follows.

The relative velocity between the target and the missile is depicted in Fig. 6, where $v$, $v_p$, and $v_r (v_r = (v^T \tilde{r}) \tilde{r})$ are the present relative velocity between the missile and the target, the component of $v$ perpendicular to the LOS, and the component of $v$ in the LOS direction, respectively. Assume that missile thrust is sufficient during the midcourse phase to overcome aerodynamic effects, gravity, and wind force such that the magnitude of the relative velocity $|v|$ will be a nondecreasing function with respect to time. By defining the angle $\theta$ between $v$ and $v_r$ as

$$
\theta = \tan^{-1} \left( \frac{|v_p|}{|v_r|} \right)
$$

we can conclude that $|v_p|$ will decay exponentially, with reference to (50)–(52), and $|v| = |v - v_p| \geq |v| - |v_p|$ due to $v = v_p + v_r$, with reference to Fig. 6. Therefore, $|v_r|$ will be an increasing function with respect to time, implying in turn that the angle $\theta$ will be monotonically decreasing as time proceeds. Hence, we can conclude that $v^T \tilde{r} < 0$ for all $t \geq 0$, since $v^T(t_0) \tilde{r}(t_0) < 0$, which justifies our assumption in Lemma 1. Therefore, the target tracking objective can be achieved as claimed by the aforementioned theorem.

VI. SIMULATIONS

To validate the proposed optimal guidance and the autopilot of the missile systems presented in Section IV and Section V, we provide realistic computer simulations in this section. We assume that the target is launched from somewhere 600 km distant. The missile has a sampling period of 10 ms. The bandwidth of the TVC is 20 Hz and the two angular displacements are both limited to $5^\circ$. Here, we consider the variation of the missile’s moment of inertia. Thus, the inertia matrix including the nominal part $J_0$ and the uncertain part $\Delta J$ used here is

$$
J = J_0 + \Delta J (kg m^2)
$$

where

$$
J_0 = \begin{bmatrix} 1000 & 100 & 200 \\ 100 & 2000 & 200 \\ 200 & 2000 & 200 \end{bmatrix}, \quad \Delta J = \begin{bmatrix} 100 & 200 & 200 \\ 200 & 200 & 200 \end{bmatrix}
$$

and the variation of the inertial matrix is

$$
\dot{J} = \begin{bmatrix} -1 & -1 & -2 \\ -2 & -2 & -2 \end{bmatrix}.
$$

The initial conditions are set at $q = [0 \ 0 \ -0.707 \ 0 \ 0.707]^T$ and $\omega(0) = [0 \ 0 \ 0]^T$, and the variation in missile mass is $m = -1$ kg/s for the initial mass $m = 600$ (kg). In simulation, the propulsion of the TVC is $N = 30000(Nt)$, so that the acceleration limit will be constrained by the inequality of $|a_p(t)| \leq N/m(t)$, where $m(t)$ is a decreasing function with respect to time. Further, we also consider the aerodynamic force and wind gusts exerted on the missile by $d_{i}(t) = \sin(t) + 10(u(t - 20) - u(t - 21))$ (Nt-m) for $i = 1, 2, 3$, where $u(t)$ is the step function. Besides that, we also check the error angle, which is the angle between the axial direction and the LOS, to see whether prior conditions for possible intercept by the terminal phase guidance and control [3] will be met.

In scenario one, the error angle is constrained within the limit for successful subsequent interception, and the simulation time of scenario one is 91.85 s. The feasibility of the presented approach is satisfactorily demonstrated by the simulation results of scenario one presented in Fig. 7.

Finally, we use the terminating condition in scenario one as the initial condition for the subsequent terminal phase guidance and control, and then check whether the final interception as established by Fu et al. [3] may be successful. Scenario two is listed below.

In scenario two, the missile can intercept the target in a very short period of time. Thus the midcourse phase offers applicable terminating conditions to ensure the subsequent interception of the missile. The
Fig. 7. Simulation results of scenario one.
success of integrating midcourse and terminal phase guidance laws is verified in Fig. 8.

VII. CONCLUSIONS

Overall procedures for intercepting a ballistic missile comprise two phases: midcourse and terminal. In this paper, we focus on the midcourse phase, which is a period of time lasting until the missile is close enough to the target such that the sensor located on the missile can lock onto the target. Considering the properties of the TVC and the nonideal conditions during the midcourse phase, we employ a controller incorporating the modified optimal guidance law, where the time-to-go of the missile does not have to be estimated, and the sliding-mode autopilot system, which can robustly adjust the missile attitude even under conditions of model uncertainty, such as...
variation of missile inertia, changing aerodynamic forces, and unpredictable wind gusts. We prove the stability of the individual guidance, autopilot, and overall systems, respectively, via the Lyapunov stability theory.

A simulation has been conducted to verify the feasibility of the integrated midcourse guidance and control system using TVC. To demonstrate the superior property of the midcourse integrated design from the viewpoint of the subsequent terminal phase, simulations based on the terminal guidance law proposed by Fu et al. [3] have also been provided. The results are quite satisfactory and encouraging.

APPENDIX A

From (14) via using separation of variables we have
\[ \int_{(t)}^{(T)} \frac{1}{\rho - \sigma} = \int_{t}^{T} 1 dt \]
\[ \Rightarrow \frac{1}{2} \sqrt{\frac{1}{\rho - \sigma} + \frac{1}{\gamma - \sqrt{\rho \sigma}}} d\gamma = (T - t) \]
\[ \Rightarrow \gamma(T) - \sqrt{\rho \sigma} \gamma(t) + \sqrt{\rho \sigma} \gamma(t) - \sqrt{\rho \sigma} = e^{2\sqrt{\sigma/\rho(T-t)}}. \]

If we want to ensure that the optimal control derives the component of the terminal velocity perpendicular to LOS, \( v_p(T) \), exactly to zero, we can let \( \gamma(T) \to \infty \) to weight \( v_p(T) \) more heavily in (13). Under this limit, we have
\[ \gamma(t) = \sqrt{\rho \sigma} \left( 1 + \frac{2}{e^{2\sqrt{\sigma/\rho(T-t)}}} - 1 \right). \]

APPENDIX B

PROOF OF LEMA 1. Taking \( V_G = \frac{1}{2} v_p^T v_p \) as a Lyapunov function candidate, it can be easily seen that
\[ \frac{1}{2} v_p^T v_p \leq V_G \leq v_p^T v_p. \]

Hence, \( V_G \) is positive definite, decrescent, and radially unbounded. The time derivative of \( V_G \) along the trajectories of the system is given by
\[ \dot{V}_G = -v_p^T \left( \sqrt{\frac{\sigma}{\rho}} \left( 1 + \frac{2}{e^{2\sqrt{\sigma/\rho(T-t)}}} - 1 \right) \right) v_p \\
- \frac{1}{|r|} |v_p|^2 v_p |\dot{r}| \leq - \frac{\sigma}{\rho} v_p^T v_p |\dot{r}| \]
where \( v_p^T \dot{r} = 0 \) and \( \sqrt{\sigma/\rho} \) is a positive constant. Thus, \( \dot{V}_G \) is apparently negative definite, and hence via Lyapunov stability theory, we can conclude that the origin of \( v_p \) is globally exponentially stable.

Accordingly, in order to verify that the intercepting missile will gradually approach to the target, we take \( V_r = \frac{1}{2} v_r^T v_r \) as another Lyapunov function candidate. Thus, it can be easily seen that \( V_r \) is positive definite, decrescent, and radially unbounded, and then the time derivative is as follows:
\[ \dot{V}_r = v_r^T \dot{r} = (v - v_p)^T r = v^T r < 0 \]
where \( v_r = v - v_p \), so that \( \dot{V}_r \) is negative definite. Therefore, via the Lyapunov stability theory and constant bearing condition [3], the ideal midcourse guidance will render the origin of the missile interceptions system globally exponentially stable.

PROOF OF LEMA 2. Since \( a_p/|a_p|, q_e, \) and \( \omega_e \) are bounded, and \( \omega_e \to 0 \) as \( t \to \infty \), we have both
\[ \frac{F_p}{|a_p|} = \frac{B_p^T(a_p + \dot{r})}{|a_p|} \quad \text{and} \quad \frac{\tilde{a}_p \times}{|a_p|} = \frac{(B_p^T a_p) \times}{|a_p|^2} \]
are bounded, so that the value of
\[ F_p ((\tilde{a}_e \times) + q_e \bar{I}_{3 \times 3}) (\tilde{a}_p \times) \omega_e / |a_p|^2 \]
can be concluded to be bounded and converge to zero as \( t \to \infty \). Therefore, the maximum value \( p_{\max} \) can be obtained as
\[ p_{\max} \geq \left| \frac{F_p ((\tilde{a}_e \times) + q_e \bar{I}_{3 \times 3}) (\tilde{a}_p \times) \omega_e}{|a_p|^2} \right|. \]

Moreover, \( \bar{K}_1 \) and \( \bar{K}_2 \) are both positive, and
\[ K_3 = \bar{K}_1 \left( 1 + \frac{2 \bar{K}_2 F_p ((\tilde{a}_e \times) + q_e \bar{I}_{3 \times 3}) (\tilde{a}_p \times) \omega_e}{|a_p|^2} \right) \]
due to the fact of (42). Hence, the ratio of \( \bar{K}_2 \) to \( \bar{K}_1 \) can be expressed as
\[ \bar{K}_2 = \bar{K}_1^2 \left( 1 + \frac{2}{e^{2\sqrt{\sigma/\rho(T-t)}}} - 1 \right) \frac{|\tau|}{\sqrt{\sigma/\rho}} \]
where the assumptions that
\[ v_r^T \frac{\dot{r}}{|\tau|} \quad \text{and} \quad - \frac{v_r^T |\dot{r}|}{|\tau|} = - \frac{1}{|\tau|} \frac{d}{dt} |\tau| = - \frac{d}{dt} \ln |\tau| \]
is bounded for \( |\tau| \neq 0 \) are satisfied. Therefore, the maximum \( \bar{K}_{\max} \) of \( \bar{K} \) in (54) can be found and is denoted as \( \bar{K} \leq \bar{K}_{\max} \). Then, (49) can be reexpressed as
\[ \bar{K}_3 = \bar{K}_1 \left( 1 + \frac{2 \bar{K}_{\max} F_p ((\tilde{a}_e \times) + q_e \bar{I}_{3 \times 3}) (\tilde{a}_p \times) \omega_e}{|a_p|^2} \right) \]
\[ \geq \bar{K}_1 \left( 1 - \frac{2 \bar{K}_{\max}}{p_{\max}} \right). \]
If we let
\[ p > 2p_{\text{max}} \left( \hat{K}^{\text{max}} \right) \tag{56} \]
which together with that fact that \( \hat{K}_1 \) is lower-bounded by a positive constant immediately implies the inequality conclusion (49).

REFERENCES

[1] Lian, K.-Y., and Fu, L.-C. (1994)
Nonlinear autopilot and guidance for a highly maneuverable missile.
A nonlinear constant bearing guidance and adaptive autopilot design for BTT missiles.
A nonlinear missile guidance controller with pulse type input devices.
Design of a CLOS guidance law via feedback linearization.
A modified CLOS guidance law via right inversion.
Optimal Control.
Inverse Optimal stabilization of a rigid spacecraft.
Survey of numerical methods for trajectory optimization.
Sliding controller design for nonlinear systems.
Optimal guidance for acceleration constrained missile and maneuvering target.
An optimal mid-course guidance law for fixed-interval propulsive maneuvers.
A quaternion formulation for boost phase attitude estimation, guidance and control of exoatmospheric interceptors.
Modern guidance laws via receding horizon control without the time-to-go.
Quaternion feedback for spacecraft large angle maneuvers.
Quaternion feedback regulator for spacecraft eigenaxis rotations.
Globally valid adaptive controllers of mechanical systems.
Sliding-mode controller design for spacecraft attitude tracking maneuvers.
Smooth sliding-mode control for spacecraft attitude tracking maneuvers.
The attitude control problem.
Hamiltonian adaptive control of spacecraft.
Analytical solution of optimal trajectory-shaping guidance.
Quaternion kinematic and dynamic differential equations.
Agile missile dynamics and control.
An optimal composite guidance strategy for dogfight air-to-air IR missiles.
Minimum-time maneuvers of thrust-vectored aircraft.
Designing continuous-thrust low-earth-orbit to geosynchronous-earth-orbit transfers.
Fu-Kuang Yeh was born in Taoyuan, Taiwan, ROC in 1961. He received his B.S. and M.S. degrees in electronic engineering and automatic control engineering from Feng Chia University, Taichung, Taiwan in 1985 and 1988, respectively. He is currently pursuing the Ph.D. degree in electrical engineering from National Taiwan University.

In 1988 he was an assistant scientist at Chung-Shan Institute of Science and Technology. His research interests include the guidance and autopilot systems design using the variable structure system and optimal control theory, the adaptive controller design, the electromechanical system analysis and implementation, and the control circuit design as well as the micro controller design for the servo control system.

Hsiuan-Hau Chien received the B.S. and M.S. degrees in electrical engineering from National Taiwan University in 1998 and 2000, respectively.

He is currently with Ali C. His research interests are in nonlinear control theory and PLL circuits design for consumers.

Li-Chen Fu was born in Taipei, Taiwan, ROC in 1959. He received the B.S. degree from National Taiwan University in 1981, and the M.S. and Ph.D. degrees from the University of California, Berkeley, in 1985 and 1987, respectively.

Since 1987 he has been on the faculty and currently is a professor of both the Department of Electrical Engineering and Department of Computer Science and Information Engineering of National Taiwan University. He now also serves as the deputy director of Tjing Ling Industrial Research Institute of National Taiwan University. His areas of research interest include adaptive control, nonlinear control, induction motor control, visual tracking, control of robots, FMS scheduling, and shop floor control.

He is now a senior member in both Robotics and Automation Society and Automatic Control Society of IEEE, and is also a board member of Chinese Automatic Control Society and Chinese Institute of Automation Engineers. During 1996–1998 and 2000, he was appointed a member of AdCom of IEEE Robotics and Automation Society, and will serve as the program chair of 2003 IEEE International Conference on Robotics and Automation and program chair of 2004 IEEE Conference on Control Applications. He has been the editor of Journal of Control and Systems Technology and an associate editor of the prestigious control journal, Automatica. In 1999 he became an editor-in-chief of Asian Journal of Control.

He received the Excellent Research Award in the period of 1990–1993 and Outstanding Research Awards in the years of 1995, 1998, and 2000 from National Science Council, ROC, respectively, the Outstanding Youth Medal in 1991, the Outstanding Engineering Professor Award in 1995, the Best Teaching Award in 1994 from Ministry of Education, The Ten Outstanding Young Persons Award in 1999 of ROC, the Outstanding Control Engineering Award from Chinese Automatic Control Society in 2000, and the Lee Kuo-Ding Medal from Chinese Institute of Information and Computing Machinery in 2000.