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Effect of the directional blood flow on thermal dose distribution during thermal therapy: an application of a Green's function based on the porous model

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Abstract

This study presents the effects of directional blood flow and heating schemes on the distributions of temperature and thermal dose during thermal therapy. In this study, a transient bioheat transfer equation based on the porous medium property is proposed to encompass the directional effect of blood flow. A Green's function is used to obtain the temperature distribution for this modified bioheat transfer equation, and the thermal dose equivalence is used to evaluate the heating results for a set of given parameters. A $10 \times 10 \times 10 \text{ mm}^3$ tumour tissue is heated by different heating schemes to investigate the thermal dose variation with the clinical therapeutic arrangement. For a rapid heating scheme, the domain of thermal lesion can effectively cover the desired therapeutic region. However, this domain of thermal lesion may extend to the downstream normal tissue if the porosity is high and the averaged blood velocity has a larger value.

1. Introduction

Thermal therapy utilizes the cytotoxic effect of high temperature to destroy pathological tissues. Conventional hyperthermia, with target temperatures of 41–45 °C and duration ranging from 30 min to several hours, in combination with radiation, has demonstrated increasing effectiveness in thermal treatments (Overgaard *et al* 1996). In contrast, the use of temperatures above 55 °C can directly destroy tissues through thermal coagulation (Beacco *et al* 1994). This higher temperature can deliver enough energy to denature proteins and result in complete cell death in the heated region within seconds (Thomsen 1991). Moreover, thermal lesion dimensions produced by high temperature thermal therapy correlate well with prediction

based on the calculation of the transient temperature distribution (Damianou *et al* 1995, Kolios *et al* 1996). Therefore, it is important to understand the transient temperature behaviour in heated tissues during the heating process.

In general, the temperature distributions inside the heated tissues, controlled by heating modalities, are obtained by solving the bioheat transfer equation (BHTE), as proposed by Pennes in 1948 under appropriate initial and boundary conditions. A more complex framework for solving the BHTE by using a time-dependent Green's function and Fourier transform techniques was established by Gao *et al* (1995). Since the Green's function obtained from the partial differential equation is independent of the source term, the same Green's function is used to calculate temperature distributions with various heating patterns. However, the major criticism of the conventional Pennes' model focused on the assumption that the heat transfer by blood flow occurs in a non-directional, heat sink- or source-like term (Wulff 1974). Weinbaum *et al* (1984a, 1984b) and Chen and Holmes (1980) attempted to formulate a more realistic bioheat equation, which would account for the thermally convective contribution of the vasculature in addition to the heat sink- or source-like term originally proposed by Pennes (1948).

Blood flow plays an important role in determining the effectiveness of thermal therapy, both for hyperthermia alone, and in combination with either radiation or chemotherapy. In general, the effect of blood flow on temperature distribution in tissue is considered as cooling due to both thermally significant large vessels and smaller microvasculature during thermal treatments (Baish 1994). Furthermore, Kolios *et al* (1995) examined the effect of the thermally significant large blood vessels on the thermal distributions during thermal therapy. The dissipation of heat from heated tissues is carried out by convection through blood flow and also by conduction or diffusion processes. Kolios *et al* (1999) found that large vessels still have pronounced effect on the tissue temperature distribution even when the treatment time is within 3–20 s.

Detailed descriptions of the development and morphological characteristics of tumour vasculature and tumour blood flow have been extensively investigated by Folkman (1986) and Shubik (1982). The growth of a larger solid tumour requires nutrition from adequate vascular distribution and blood flow. By morphometric analysis of the microvasculature of tumours (C3H/Bi mammary carcinomas), Hilmas and Gillette (1974) found that the changes in vascular parameters during tumour growth ranged from 4 mm average diameter (approximately 35 mm³) to 14 mm average diameter (approximately 1500 mm³). Furthermore, Hirst (1989) found that the blood supply to tumours can be classified into three broad categories with important implications for the effects of vasoactive agents. He modelled the vascular beds assuming that the relationship between the tumour vascular bed and the normal tissue vascular bed is 'in parallel', 'in series' or 'combined'. Hirst *et al* (1992) studied the tumour vascular physiology to gain a better understanding of the rules that determine how blood will flow through tumours of different types, sizes and growth sites in response to changes in blood pressure, vascular tone and blood viscosity. Lacking smooth muscle layers and innervations, the thin capillary-like tumour vessels are unable to autoregulate (Molls and Vaupel 1998). According to Hirst's tumour vascular modelling, a heating target volume containing the capillary-like tumour vascular bed with directional blood flow is considered as shown in figure 1. In some studies (Wan *et al* 1996, Fan and Hynynen 1996, Daum and Hynynen 1999, Wu and Sherar 2002) on ultrasound surgery, they could successfully treat a 10 × 10 × 10 mm³ tumour tissue. Baish *et al* (1997) used an isotropic porous medium concept to describe the transmural coupling of fluid flow in a microcirculatory network and interstitium in tumours. They assumed that the vascular bed of tumour tissue can be modelled as a network of vessels connecting the arterial (upstream) and venous (downstream) ends. In this study we chose a 10 × 10 × 10 mm³ target

Figure 1. Schematic diagram of the geometry studied for the distributions of the temperature and thermal dose equivalence (EM_{43}) in the tissue. (a) Geometric relationship between the tumour vascular bed and the directional blood flow (modelled after Hirst (1989)), presenting a schematic diagram of the directional characteristics of the vascular bed. (b) The large treatment tumour (volume enclosed by the coarse line) with one-way directional blood flow was divided into several small heating cubes. A simulated heated cube was assumed to be of dimensions $10\text{ mm} \times 10\text{ mm} \times 10\text{ mm}$. The initial temperatures of the selected heating cube and the surrounding tissue were set to 37°C .

tissue volume with directional tumour blood flow to analyse the thermal distributions during thermal therapies.

Since there is little information concerning the effects of blood flow direction on the region of the thermal lesion, especially for transient and three-dimensional calculations, the purpose of this study is to investigate the effect of blood flow direction and porosity on thermal distributions within the heated tumour volume, and to use the Green's function to solve the analytical solution of the modified bioheat transfer equation based on the porous medium that encompasses the directional effect of the blood flow.

2. Methods

2.1. Derivation of heat transfer in the blood-perfused tumour tissue

Heat transfer in living tissue can be modelled by solving a conjugate problem with the porous medium concept. This model contains a solid sub-domain for the tissue and a liquid sub-domain for the blood. Herein the vascular volume in solid tissues is fully occupied with blood flow. That is, we assume that the vascular volume is fully equal to the blood volume. Heat transfer in blood-perfused tumour tissue volume can be modelled by solving a conjugate problem containing a solid sub-volume (the tissue domain) and a liquid sub-volume (the blood domain). The energy transport equations of the tissue and the blood flow domains are used

Table 1. Parameters used for the numerical simulations.

Symbol	Definition	Value	Units
ρ_t, ρ_b	Density of tissue and blood	1050	kg m ⁻³
c_t, c_b	Specific heat of tissue and blood	3770 ^a	J kg ⁻¹ °C ⁻¹
t	Time	Variable	s
T	Temperature	Calculated	°C
k_t, k_b	Thermal conductivity of tissue and blood	0.5 ^b	W m ⁻¹ °C ⁻¹
u	Average blood velocity	0.3–2 ^c	mm s ⁻¹
T_a	Arterial temperature	37	°C
q	Heating power	Variable	W cm ⁻³

^a Damianou and Hynynen (1993).^b Fan and Hynynen (1996).^c Guyton (1986), Nilsson (1996), Keller and Seiler (1971), Crezee and Lagendijk (1992) and Hobbie (1997).

here as shown in equations (1) and (2), respectively,

$$\rho_t c_t \frac{\partial T_t}{\partial t} = k_t \left(\frac{\partial^2 T_t}{\partial x^2} + \frac{\partial^2 T_t}{\partial y^2} + \frac{\partial^2 T_t}{\partial z^2} \right) - W_b c_b (T_t - T_b) + q_t(x, y, z, t) \quad (1)$$

$$\rho_b c_b \left(\frac{\partial T_b}{\partial t} + u \frac{\partial T_b}{\partial x} + v \frac{\partial T_b}{\partial y} + w \frac{\partial T_b}{\partial z} \right) = k_b \left(\frac{\partial^2 T_b}{\partial x^2} + \frac{\partial^2 T_b}{\partial y^2} + \frac{\partial^2 T_b}{\partial z^2} \right) + q_b(x, y, z, t) \quad (2)$$

where ρ is the density, c is the specific heat, k is the thermal conductivity, T is the temperature, $q(x, y, z, t)$ is the absorbed power density with the subscripts t and b referring to tissue and blood domain, respectively, and W_b is the blood perfusion rate. All units and values of the above parameters are given in table 1. Similar equations had been found in Kolios *et al* (1995) that were valid for cylindrical coordinates and axial flow. In the report of Hilmas and Gillette (1974), it is observed that the tumour vessel diameter in a 1000 mm³ tumour volume is less than 30 μ m. This means that thermal equilibrium occurs in this region and thereafter we also choose a 10 $\times$ 10 $\times$ 10 mm³ heated cube as the target volume. In addition, the thermal equilibration length of a vessel is a useful tool for examining which of the vessels are the equilibrating ones. The equilibration length is defined as the length after which the blood temperature in the vessel has approached the tissue temperature within e^{-1} . For vessel types of capillaries, venules and arterioles, their equilibration length ranges are of the order of microns (Chen and Holmes 1980, Crezee and Lagendijk 1992). This implies that blood flowing in these small vessels will be completely equilibrated with the surrounding tissue. In this study we assume the blood temperature equals the tissue temperature in heated tissue so that the heat source/sink term $W_b c_b (T_t - T_b)$ will vanish. Based on this assumption, the volumetric averaging of the energy conservation principle can be achieved by combining and rearranging equation (1) multiplied by $(1 - \phi)$ and equation (2) multiplied by ϕ with the porous structure regarded as a homogeneous medium to obtain

$$\begin{aligned} & [\phi \rho_b c_b + (1 - \phi) \rho_t c_t] \frac{\partial T}{\partial t} + \phi \rho_b c_b \left[u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right] \\ & = [\phi k_b + (1 - \phi) k_t] \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + (1 - \phi) q_t + \phi q_b. \end{aligned} \quad (3)$$

The porosity, ϕ , is defined as the ratio of blood volume to the total (solid tissue + liquid blood) volume. During the computer simulation, the ultrasound power was uniformly deposited within the desired heating volume and the absorption of ultrasound power by the blood is

assumed to be 10%, $q_b = 0.1q_t$, of that for the tissue (Duck *et al* 1998). Hence, the total absorbed power density q_a is equal to $(1 - 0.9\phi)q_t$. We assumed that the initial and boundary temperatures of a $10 \times 10 \times 10 \text{ mm}^3$ heating target cube and of its outside tissue volume were given as $T(x, y, z, 0) = 37^\circ\text{C}$, as shown in figure 1(b).

2.2. Dimensionless parameters

In order to derive a compact representation, the following parameters are defined:

$$\begin{aligned} k_{\text{eff}} &= \phi k_b + (1 - \phi)k_t & \alpha^* &= \frac{k_{\text{eff}}}{\phi \rho_b c_b + (1 - \phi)\rho_t c_t} \\ \alpha_{\text{eff}} &= \frac{k_{\text{eff}}}{\rho_b c_b} & q^*(x, y, z, t) &= \phi q_b(x, y, z, t) + (1 - \phi)q_t(x, y, z, t). \end{aligned} \quad (4)$$

The dimensionless parameters have the following forms:

$$X = \frac{x}{l} \quad Y = \frac{y}{l} \quad Z = \frac{z}{l} \quad T^* = \frac{T - T_a}{T_a} \quad \tau = \frac{\alpha^* t}{l^2} \quad (5)$$

where T_a is the arterial blood temperature, which is set to 37°C .

Substituting equations (4) and (5) into equation (3), equation (3) can be expressed in dimensionless form as

$$\frac{\partial T^*}{\partial \tau} = \frac{\partial^2 T^*}{\partial X^2} + \frac{\partial^2 T^*}{\partial Y^2} + \frac{\partial^2 T^*}{\partial Z^2} - 2c_1 \frac{\partial T^*}{\partial X} - 2c_2 \frac{\partial T^*}{\partial Y} - 2c_3 \frac{\partial T^*}{\partial Z} + q_1(X, Y, Z, \tau) \quad (6)$$

where

$$\begin{aligned} c_1 &= \frac{u^* l}{2\alpha_{\text{eff}}} & c_2 &= \frac{v^* l}{2\alpha_{\text{eff}}} & c_3 &= \frac{w^* l}{2\alpha_{\text{eff}}} & u^* &= \phi u & v^* &= \phi v & w^* &= \phi w \\ q_1(X, Y, Z, \tau) &= \frac{q^*(X, Y, Z, \tau) l^2}{T_a k_{\text{eff}}}. \end{aligned} \quad (7)$$

By setting the product form as

$$T^*(X, Y, Z, \tau) = \bar{T}(X, Y, Z, \tau) \exp(c_1 X + c_2 Y + c_3 Z - c_4 \tau) \quad (8)$$

where

$$c_4 = c_1^2 + c_2^2 + c_3^2 + \gamma \quad (9)$$

and substituting it into equation (6), another simple form can be obtained as:

$$\frac{\partial \bar{T}}{\partial \tau} = \frac{\partial^2 \bar{T}}{\partial X^2} + \frac{\partial^2 \bar{T}}{\partial Y^2} + \frac{\partial^2 \bar{T}}{\partial Z^2} + q_2(X, Y, Z, \tau) \quad (10)$$

where

$$q_2(X, Y, Z, \tau) = q_1(X, Y, Z, \tau) \exp[-(c_1 X + c_2 Y + c_3 Z - c_4 \tau)]. \quad (11)$$

Since the Green's function obtained for the differential equation is independent of the source term, the same Green's function can be used to calculate the temperature distribution for various sources with different spatial and temporal profiles. Therefore, once the Green's function is known, the temperature distribution in the medium is readily calculated. For an infinite medium ($-\infty < X < \infty$, $-\infty < Y < \infty$, $-\infty < Z < \infty$) and for time $0 < \tau$, the desired Green's function is not difficult to find (Özisik 1993) and is given by

$$G(X, Y, Z, \tau | X_p, Y_p, Z_p, \tau_p) = [4\pi(\tau - \tau_p)]^{-\frac{3}{2}} \exp\left[-\frac{(X - X_p)^2 + (Y - Y_p)^2 + (Z - Z_p)^2}{4(\tau - \tau_p)}\right] \quad (12)$$

where the positions and time with the subscript p refer to a heating power location. The Green's function $G(X, Y, Z, \tau | X_p, Y_p, Z_p, \tau_p)$ represents the dimensionless temperature $\bar{T}(X, Y, Z, \tau)$, at the location (X, Y, Z) , at time τ , due to an instantaneous heat source, located at (X_p, Y_p, Z_p) , releasing its energy spontaneously at time τ_p . The solution of equation (10) can be written in terms of the above Green's function, according to equation (11), as

$$\bar{T}(X, Y, Z, \tau | X_p, Y_p, Z_p, \tau_p) = \int_{\tau_s}^{\tau_f} \int_{X_s}^{X_f} \int_{Y_s}^{Y_f} \int_{Z_s}^{Z_f} G(X, Y, Z, \tau | X_p, Y_p, Z_p, \tau_p) \times q_2(X_p, Y_p, Z_p, \tau_p) dZ_p dY_p dX_p d\tau_p \quad (13)$$

where the positions and time with the subscripts s and f refer to the start and finish of the computational domain, respectively. The approximating solution of equation (13) can be solved by the Gauss–Legendre integration method. Since Legendre polynomials are defined only in the interval $[-1, 1]$, it is necessary to change the interval of integration for any arbitrary integral by making the substitution form

$$\begin{aligned} X_p &= \frac{(X_f - X_s)X'_p + X_f + X_s}{2} & Y_p &= \frac{(Y_f - Y_s)Y'_p + Y_f + Y_s}{2} \\ Z_p &= \frac{(Z_f - Z_s)Z'_p + Z_f + Z_s}{2} & \tau_p &= \frac{(\tau_f - \tau_s)\tau'_p + \tau_f + \tau_s}{2}. \end{aligned} \quad (14)$$

Applying the variable transformations that are expressed as equation (14) to equation (13), we can obtain another form:

$$\begin{aligned} \bar{T}(X, Y, Z, \tau | X'_p, Y'_p, Z'_p, \tau'_p) &= \frac{(X_f - X_s)}{2} \frac{(Y_f - Y_s)}{2} \frac{(Z_f - Z_s)}{2} \frac{(\tau_f - \tau_s)}{2} \\ &\times \int_{-1}^1 \int_{-1}^1 \int_{-1}^1 \int_{-1}^1 G(X, Y, Z, \tau | X'_p, Y'_p, Z'_p, \tau'_p) \\ &\times q_2(X'_p, Y'_p, Z'_p, \tau'_p) dZ'_p dY'_p dX'_p d\tau'_p. \end{aligned} \quad (15)$$

Finally, it can be considered to define the transient and three-dimensional analytical solution form of equation (3) as

$$\begin{aligned} T(x, y, z, t) &= T_a \left\{ 1 + \frac{(X_f - X_s)(Y_f - Y_s)(Z_f - Z_s)(\tau_f - \tau_s)}{16} \right. \\ &\times \int_{-1}^1 \int_{-1}^1 \int_{-1}^1 \int_{-1}^1 [4\pi(\tau - \tau'_p)]^{\frac{-3}{2}} q_1(X, Y, Z, \tau) \\ &\times \exp \left[c_1(X - X'_p) + c_2(Y - Y'_p) + c_3(Z - Z'_p) - c_4(\tau - \tau'_p) \right. \\ &\left. \left. - \frac{(X - X'_p)^2 + (Y - Y'_p)^2 + (Z - Z'_p)^2}{4(\tau - \tau'_p)} \right] dZ'_p dY'_p dX'_p d\tau'_p \right\}. \end{aligned} \quad (16)$$

An analytic formulation has been derived for the time-dependent and three-dimensional spatial-dependent porous medium bioheat transfer equation. The exact solution can be used to predict the effect of blood flow on the transient temperature distribution of heated tissues without thermally significant vessels during thermal therapies. The approximate solution of equation (16) can be evaluated by the Gauss-quadrature integration scheme (Abramowitz and Stegun 1965).

Figure 2. Effects of the heating schemes in table 2 on the transient profiles of the temperature at the central point within a heated cube with the nine heating schemes and for the three different blood averaged velocities $u = 0, 1$ and 2 mm s^{-1} , were presented. The dotted, dashed and solid lines represented the blood averaged velocities $u = 0, 1$ and 2 mm s^{-1} , respectively.

2.3. Thermal dose calculations

The thermal dose or equivalent minutes at 43°C defined by Sapareto and Dewey (1984) can describe the extent of thermal damage or destruction of tissue, which is a function of the temperature elevation and time. The Sapareto–Dewey expression can be written as

$$\text{EM}_{43}(\text{in min}) = \int R^{(T-43)} dt \quad (17)$$

where $R = 2$ for $T \geq 43^\circ\text{C}$ and $R = 4$ for $37^\circ\text{C} < T < 43^\circ\text{C}$, T is the temperature, t is the time and EM_{43} is the thermal dose equivalence in minutes. The threshold values of thermal dose for necrosis are $\text{EM}_{43} = 25 \text{ min}$ and 240 min for brain and muscle tissues, respectively (Sapareto and Dewey 1984, Damianou and Hynynen 1993). In this simulation study, a conservative value ($\text{EM}_{43} = 300 \text{ min}$) has been taken, which is similar to the analysis of Damianou and Hynynen (1993) for focused ultrasound surgery and Diederich and Burdette (1996) for prostate ultrasound thermal therapy.

3. Results and discussion

Figure 2 shows that the temperature at the centre of the heating target keeps increasing during the heating and then decays immediately if the heating power is turned off. Due to the assumption that the absorption of ultrasound power density by the blood is only 10% of that

Figure 3. Effects of the heating schemes in table 2 on the transient profiles of the thermal dose equivalence (EM_{43}) corresponding to figure 2 are presented. The dotted, dashed and solid lines represent the blood averaged velocities $u = 0, 1$ and 2 mm s^{-1} , respectively.

Table 2. Parameters of the different heating schemes used in simulation.

	Case	I	II	III	IV	V	VI	VII	VIII	IX
Porosity			$\phi = 0.005$			$\phi = 0.05$			$\phi = 0.1$	
Absorbed power density for soft tissue q_t (W cm^{-3})		50	10	2	50	10	2	50	10	2
Total absorbed power density $q_a = (1 - 0.9\phi)q_t$ (W cm^{-3})		49.775	9.955	1.991	47.75	9.55	1.91	45.5	9.1	1.82
Heating duration (s)		2	10	50	2	10	50	2	10	50
Total absorbed energy density (J cm^{-3})			99.55			95.5			91	

by the tissue, a higher porosity leads to a lower total absorbed energy density which is also shown in table 2. Hence, the time-varying temperature distributions of higher porosity are lower than those of lower porosity when the heating duration and blood velocity maintain the same input. Under the same porosity and total absorbed power density, a more rapid heating results in a higher peak temperature. This implies that the peak temperature strongly depends upon the heating duration. For instance, the peak temperatures are about 62°C in case I and 56°C in case III shown in figures 2(a) and (c). When the porosity is less than 0.005, figures 2(a)–(c) and figures 3(a)–(c) reveal that the results of time-varying temperature distributions and thermal dose level are independent of the blood velocity. Except for the cases VI and IX, the peak temperatures shown in figure 2 and thermal dose levels shown in

Figure 4. Temperature distributions and their corresponding thermal dose distributions on the x - y plane with the heating case VIII and for the averaged blood velocity $u = 1 \text{ mm s}^{-1}$ along the x -axis direction at $t = 5, 10, 60$ and 180 s , respectively, are presented. Power was turned off at 10 s . The porosity ϕ is 0.1 (Hilmas and Gillette 1974).

figure 3 are nearly independent of the blood velocity. In other words, the peak temperature and thermal dose level are significantly influenced by the blood velocity only if both longer heating duration and higher porosity exist. Billard *et al* (1990) also mentioned that the

Figure 5. Effects of the different heating schemes on thermal dose equivalence $EM_{43} = 300$ contours at $t = 180$ s for averaged blood velocity 1 mm s^{-1} . The dashed lines are the boundary of the heating cube.

influence of blood perfusion on thermal distributions is less sensitive for a short ultrasound pulse. In addition, figure 3 illustrates that the accumulation of EM_{43} reaches its plateau faster for a rapid heating duration, and the level of the plateau is proportional to the peak temperature.

Figure 4 demonstrates the transient behaviour of temperature distribution and thermal dose profile on the x - y plane for case VIII. During the heating process, the elevation of temperature distribution is proportional to the heating time ranging from 0 to 10 s. The peak temperature reaches about 59°C when the power is off. Then, the plateau of temperature distribution shifts towards the downstream region of blood flow and decays with time. The elevation of thermal dose profile (i.e., EM_{43}) mainly depends on the peak temperature. Consequently, the above results indicate that the temperature variation and thermal dose accumulation within the heating tissue depend on the heating duration, the porosity, the averaged blood velocity and the direction of blood flow during thermal therapy.

In order to further investigate the effect of the heating duration on the domain of thermal lesion ($EM_{43} = 300 \text{ min}$), the averaged blood velocity was set at 1 mm s^{-1} . Figure 5 depicts that the contours of the $EM_{43} = 300 \text{ min}$ for cases VII and VIII almost coincide with each other downstream. This is due to the blood flow effect that transports the input thermal energy from upstream to downstream along the blood flow direction. Furthermore, the effective lesion domain for case IX is located on the right half portion of the heating target. Meanwhile, this area is obviously below 50% of the treatment target which we want to cure. Under the same porosity which induces the same amount of total absorbed energy density, the domain of thermal lesion that covers the desired therapeutic region for the shorter heating duration is larger than that for the longer heating duration. The effects with different heating schemes on thermal prediction have shown that a rapid heating scheme produces both higher peak temperature and larger thermal dose level. This phenomenon was also validated by the results from Torvi and Dale (1994) and Chen *et al* (1993). It is also found that the thermal dose level on the front portion of the treatment target cannot approach the therapeutic level, especially for a longer heating duration. Comparison of the heating schemes under the same porosity (i.e., the same total absorbed energy density) shows that a rapid heating scheme can produce a higher thermal dose level on the front portion of the heating target. Therefore, rapid heating can obtain more useful therapeutic results in terms of thermal dose level, and the blood flow

Figure 6. Effects of the porosity and averaged blood velocity on the thermal dose equivalence $EM_{43} = 300$ min contours at $t = 180$ s with different porosity values of 0.005, 0.05, 0.1, averaged blood velocities of 0.3, 0.7, 1.0, 1.5, 2 mm s⁻¹, two heating patterns. Heating durations were 2 s for (a), (b) and (c), and 10 s for (d), (e) and (f). (a) and (d) with total absorbed energy density 99.55 J cm⁻³; (b) and (e) with total absorbed energy density 95.5 J cm⁻³ and (c) and (f) with the total absorbed energy density 91 J cm⁻³.

direction can drastically affect the domain and location of the thermal lesion during thermal therapy.

To study the effects of the porosity values ranging from 0.005 to 0.1 and the averaged blood flow velocities ranging from 0.3 to 2 mm s⁻¹ on the thermal dose distributions, two

groups of rapid heating schemes (cases I, IV and VII and cases II, V and VIII in table 2) are presented for comparison. When the porosity is 0.005, figures 6(a) and (d) demonstrate that the contour of thermal lesion of $EM_{43} = 300$ min is nearly independent of the magnitude of the blood velocity. When the averaged blood velocity is 0.3 mm s^{-1} , figures 6(a)–(c) show that the thermal dose equivalence contours of $EM_{43} = 300$ min inside the heating target are almost identical with various inputs of porosity and heating duration. Although the domain of thermal lesion can effectively cover the wanted therapeutic region by using rapid heating, figures 6(b) and (c) show that thermal lesion on normal tissue will occur when the blood velocity and the porosity are increasing. Higher values of the porosity and the blood velocity in the rapid heating treatment can not only fully cover the therapeutic region, but also induce larger areas of overheating on normal tissue downstream of the flow direction. Similar results appear in figures 6(e) and (f).

4. Conclusion

By applying the porous medium model to describe the heat transfer in living tissue, an analytical solution can be obtained by means of a Green's function. The exact solution can be used to predict the effect of directional blood flow on the transient three-dimensional temperature distribution and thermal dose quantity of heated tissues during thermal therapy. The blood flow has the potential of transporting the deposition energy from the heating target to the normal tissue along the flow direction, especially with higher values of velocity and porosity. A higher porosity causes a lower total absorbed energy density. Under the same porosity, the lower peak temperature associated with its lower thermal dose level is acquired for the longer heating. Regarding rapid heating, there is no influence on temperature distribution and thermal dose profile when the averaged blood velocity is below 0.3 mm s^{-1} or the porosity is below 0.005. For higher porosity and a larger blood velocity, the domain of the thermal lesion may cover not only the wholly wanted therapeutic region but even go beyond to the normal tissue along the flow direction.

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