Biomechanical analysis of distal radius fractures using intramedullary Kirschner wires

Wen-Chi Tsai^{a,b}, Shang-Chih Lin^{c,d}, Chun-Ching Hsiao^e, Tung-Wu Lu^a,
Ching-Kong Chao^f, Hwa-Chang Liu^{a,g,*}

^a Institute of Biomedical Engineering, National Taiwan University, Taipei, Taiwan

^b Department of Orthopaedic Surgery, Shuang Ho Hospital, Taipei Medical University, Taipei, Taiwan

^c Department of Mechanical Engineering, National Central University, Taoyuan, Taiwan

^d Institute of Biomedical Engineering, National Central University, Taoyuan, Taiwan

^e Nano-Electro-Mechanical-Systems Research Center, National Taiwan University, Taipei, Taiwan

^f Department of Mechanical Engineering, National Taiwan University of Science and Technology, Taipei, Taiwan

^g Department of Orthopedic Surgery, National Taiwan University Hospital, Taipei, Taiwan, ROC

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Abstract

Colles's fracture is the most common type of distal radius fracture. Surgically, it remains a challenge to restore radial height and volar tilt in order to regain optimal wrist function. Ulson's procedure provides a dynamic effect on fixing fractured fragments and restoring joint function using two wires. However, the biomechanical influences of bone and wire remain critical issues for fracture reduction and bone union in Ulson's procedure. Based on elastic beam and foundation theory, this study formulated a closed-form mathematical model to investigate the effects of bone and wire parameters on wire deflection and bony reaction. The wire deflection and bony reaction were chosen as the indices of wrist stability and reduction within the post-operative period.

The predicted results showed that greater bone strength, higher wire stiffness, and longer wire contact length provide a more stable wire–bone construct, thus facilitating fracture reduction and bone union. The wire stiffness had a much more significant effect on the construct stability compared with bone quality and contact length. In terms of entry point and insertion angle, surgical planning for the contact length was more important than bony quality for stabilizing the whole wire–bone construct.

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1. Introduction

The comminuted distal radius fracture (AO classification: A2.2 and A3.2), particularly in osteoporotic bone, has been a difficult management problem in terms of restoring wrist anatomy and function, and currently remains a surgical challenge [1,2]. Both external fixators and internal fixation using plates and screws have been popular treatment methods for

this type of fracture [3–5]. In comparison with plating fixation, intramedullary fixation with percutaneous wires allows a shorter operation time, excellent cosmetic outcomes, minimal invasiveness and minimal soft-tissue dissection, ease of hardware removal, and early motion after removal. The Kapandji technique (Fig. 1A), initially designed to treat distal radius fractures in children, is capable of avoiding damage to the epiphyseal growth area by way of intrafocal invasion [6–8]. The modified Kapandji technique (Ulson's procedure) which uses Kirschner wires via intrafocal invasion for intramedullary insertion has been proposed as a method of achieving excellent results with few complications (Fig. 1B). For Ulson's procedure, the Kirschner wires are placed into the dorsal styloid in order to restore radial height and volar tilt [1,6].

* Corresponding author at: Department of Orthopedic Surgery, National Taiwan University Hospital, Institute of Biomedical Engineering, National Taiwan University, No. 7 Chung Shan S. Road, Taipei 100, Taiwan, ROC. Tel.: +886 2 23956988; fax: +886 2 23956988.

E-mail address: hcliu@ntuh.gov.tw (H.-C. Liu).

Fig. 1. Two views showing the traditional treatments for distal radius fractures. (A) Kapandji procedure. (B) As an alternative, an intramedullary wire was inserted into the radial medullary canal in the current study.

In radius fracture management, a satisfactory functional outcome is unlikely unless a good anatomical result is achieved [1,9,10]. For Ulson's procedure, the underlying principle of the Kirschner wires stabilizing the fractured fragments is the reaction force of the deflected wires in the medullary canal. The deflected wire exerts a reaction force directly at the wire–bone interfaces to facilitate bony union and fragment repositioning. From a biomechanical point of view, the reduction of the distal radius fracture is related to the contact length between the inserted wire and the medullary bone, the wire stiffness, the entry point and insertion angle of the wires, and the bone quality [1,11,12]. An unsuitable reaction force could lead to bony non-union or loss of reduction. Hence, it is an important issue to estimate pre-operatively the reaction force induced by the deflected Kirschner wires. However, to the best of the authors' knowledge, the load-transmission mechanics between the inserted wires and the radius fragments have not yet been studied extensively and reported in the literature.

In this study, a closed-form mathematical model based on linearly elastic beam and foundation theory was developed to investigate the influence of bone quality, wire stiffness, and contact length on the wire deflection and bone reaction force. The wire–bone reaction force was hypothesized to be closely related to fracture reduction and bony union using Ulson's procedure. The objective of the current study was to estimate the wire–bone reaction force that could be beneficial for pre-operative planning and post-operative outcome when treating a distal radius fracture with intramedullary Kirschner wires.

2. Methods

2.1. Assumptions of mathematical model

Using the AO classification, the model of a distal radius fracture in this study simulated an A2.2–A3.2 fracture. For reduction of the fractured distal radius, the intramedullary

wire is gradually bent while the wire tip makes contact with the medullary bone (Fig. 2A). Consequently, the deflection of the bent wire exerts a reaction force directly on the distal radius fragment, thus providing anatomical reduction, increasing joint stability, and facilitating bony union. In the current study, the potential for fracture reduction and bony union was assumed to be closely related to the reaction force at the wire–bone interfaces. Accordingly, impingement and displacement of fragments, and loss of fixation can be well-controlled with the appropriate reaction force applied at the wire–bone interfaces. The hypothesis of this study was that the reaction force might be affected by some device-, bone-, and surgery-factors, such as bone quality, wire stiffness, and wire–bone contact length.

Fig. 2. (A) An intramedullary wire inserted into the radial medullary canal to treat the distal radius fracture (anterior-posterior view). (B) A mathematical model based on linearly elastic beam and foundation theory was developed to simulate the distal radius fracture treated with Kirschner wire. All symbols are defined in the text.

The bent shape of the intramedullary wire was determined by the shearing force P and the reaction force from the cancellous and cortical bone (Fig. 2B). The stainless steel wire was simulated as a cylindrical rod with a diameter of D_w , and assigned an elastic modulus E of 210 GPa. The interfacial behavior of wire–bone contact was simulated by a linearly elastic foundation spring-like element (Fig. 2B). Along the wire's axis, the mechanical effect of the cortical and cancellous bone on the bent wire was evaluated by two foundation moduli (K_a and K_b) and two contact lengths (L_a and L_b). The position of the cancellous foundation was assumed to be applied on the opposite side of the cortical foundation (Fig. 2B). The friction effect between the wire and the medullary bone was ignored. In addition, only one wire insertion and a two-dimensional reduction were considered. The symbol L_c was defined as the length of the distal radius fragment above the fracture site (Fig. 2A). The bone quality $R (=K_a/K_b)$ was defined as the ratio of the foundation moduli of the cortical to cancellous bone. The higher the R -value, the more it was representative of cancellous bone with a comparative strength of osteoporotic bone.

2.2. Model solutions and parametric study

For the biomechanical evaluation, a vertical shear load P was applied to the wire's tip at the entry point (Fig. 2B). The load–deflection equations of the bent wire were established on the basis of linearly elastic beam and foundation theory. Under the assumption of continuous wire–bone contact (Fig. 2B), the load–deflection relationships of the inserted wire at the proximal, middle and distal contact regions, respectively, were given as follows:

$$y_1'''(x) + \alpha y_1(x) = 0 \quad (0 \leq x \leq L_a) \quad (1)$$

$$y_2''''(x) + \frac{\alpha}{R} y_2(x) = 0 \quad (L_a \leq x \leq L_a + L_b) \quad (2)$$

$$y_3''(x_3) - \beta x_3 = 0 \quad (0 \leq x_3 \leq L_c) \quad (3)$$

$$\alpha = \frac{K_a}{EI}, \quad \beta = \frac{P}{EI}$$

where y_i ($i = 1-3$) was the deflection function of the inserted wire in the three contact regions, respectively, and both x and x_3 were the coordinate systems originating from the opposite wire tips (Fig. 2B). The symbols α and β were used for convenience, and the symbol I represents the area moment of inertia of the inserted wire. The general solutions to the load–deflection Eqs. (1–3) are given as follows:

$$y_1(x) = e^{\varepsilon x} [c_1 \cos(\rho x) + c_2 \sin(\rho x)] + e^{-\varepsilon x} [c_3 \cos(\rho x) + c_4 \sin(\rho x)]$$

$$y_2(x) = e^{\varepsilon/\sqrt[4]{R}x} [c_5 \cos(\rho x) + c_6 \sin(\rho x)] + e^{-\varepsilon/\sqrt[4]{R}x} [c_7 \cos(\rho x) + c_8 \sin(\rho x)]$$

$$y_3(x_3) = \frac{\beta x_3^3}{6} + c_9 x_3 + c_{10}$$

$$\varepsilon = |Re(-\sqrt[4]{\alpha}(-1)^{3/4})| \quad \rho = |Im(-\sqrt[4]{\alpha}(-1)^{3/4})|$$

where 10 integration constants c_i ($i = 1-10$) were solved by 10 boundary conditions. The symbols Re and Im , respectively, denote the real and imaginary parts of the complex number in the square bracket. In this study, a 10-mm cross-sectional size at the radius fracture site was modeled; thus, the lateral deflection at the wire's entry point ($x = L$) was set at 10 mm, i.e., $y_3(0) = 10$. The deflection and slope at the wire tips ($x = 0$) were assumed to be zero, i.e., $y_1(0) = 0$ and $y_1'(0) = 0$. The deflection, slope, bending moment and shear force were continuous at the adjoining points between the three regions. These continuous conditions were used to calculate the integration constants and wire deflection at the three contact regions. For a given set of wire–bone contact lengths (L_a , L_b , and L_c), the bent shape of the inserted wire with a structural stiffness ($E \times I$) was determined by the shearing force (P) and the foundation moduli (K_a and K_b) of the cancellous bone and cortex. That is, the proximal and distal ends of the inserted wire were deflected 10 mm, $y_3(0) = 10$, (Fig. 2A). Hence, the inserted wire was assumed to be deflected in accordance with the actual surgical situation where the curved wire and the longitudinal axis of the radius form an angle.

For Ulson's procedure, straight Kirschner wires are inserted from the cortex of the radial styloid, and they deflect while being drilled along the inner wall of the opposite cortex without penetrating into the underlying cortex. In the model, the Kirschner wire was regarded as a beam with a circular cross-section. The interaction between the wire and the underlying bone was simulated as an elastic foundation (Fig. 2B). According to Winkler's assumption [13], the reaction force of the foundation at every point is proportional to the deflection of the wire at that point. The foundation modulus of bone was determined by a simplified beam model. A bone tube with a constant cross-sectional area (A) was used to simulate a bony structure ($A = \pi \times \phi^2/4$) and was subjected to a shearing force (S) at the free end, while being clamped at the other end. The lateral deflection δ of the bone tube at the free end can be calculated as $\delta = (S \times l)/(A \times Y)$ [14]. The symbols Y and l are the elastic modulus and the length of the bone tube, respectively. The modulus K (N/m²) of continuous spring-like elements at the free end was estimated from the following equation: $\phi \times K \times \delta = S$ [13]. Consequently, the foundation modulus was defined and expressed as $F/(A \times \delta)$. In this study, $k = (A \times Y)/(\phi \times l)$ and is proportional to the elastic modulus of the bone. If the values of the material (Y) and geometry parameters (A , ϕ , and l) are assigned, the foundation moduli (K_a and K_b) of cortical and cancellous bone can be estimated. In this study, the cortical modulus was 35 MPa and the cancellous modulus was determined by the bone quality $R (=K_a/K_b)$.

Parametric analyses were performed to evaluate the effects of bone quality (R : 10, 50 and 100), wire–cortex contact

Fig. 3. The deflection of the inserted wire along the wire's length based on the coordinate of $y_1 - x$ with $D_w = 2.0$ mm for three R -values of bone quality.

(L_a : 20, 30 and 40 mm), and wire diameter (D_w : 1.8, 2.0 and 2.5 mm) on the stability of the wire–bone construct. The construct stability was described by the reaction force at the wire–cortex interface. The value of the wire length $L (=L_a + L_b + L_c)$ was assumed to be 130 mm in the parametric study (Fig. 2B). The wire–bone cortex reaction force from $x=0$ to $x=L_a + L_b$ was calculated based on the lever-fulcrum principle. The fulcrum of the wire–bone construct was assumed to lie in the wire–cancellous bone region at the location $x = \xi = \int_{L_a}^{L_a+L_b} (Kby_2x) dx / \int_{L_a}^{L_a+L_b} (Kby_2) dx$. The distributed reaction force F at the wire–cortex interface was assumed to be concentrated at the location $x=L_a/2$. The reaction force F to restore the distal bone fragment was determined by the lever-fulcrum principle: $F = P \times (L - \xi) / (\xi - L_a/2)$. All integration constants, wire deflection and reaction force were simultaneously solved using Mathematica software, Edition 5 (Wolfram Research Inc., Champaign, IL, USA).

3. Results

The deflection pattern and reaction force were substantially affected by the extent of the wire–bone contact, the wire stiffness and the ratio of foundation moduli of cortex to cancellous bone. Fig. 3 shows how the bone quality significantly affects the deflection pattern of the inserted wire, particularly in the middle and distal regions ($L_a \leq x \leq L$). The deflection along the wire's axis increased substantially as the bone quality tended towards osteoporosis with a larger R -value (Fig. 3). This implied a greater deflection of the inserted wire between the cortex and the entry point for the osteoporotic bone, thus indicating easier insertion into the

Fig. 4. The deflection of the inserted wire along the wire's length based on the coordinate of $y_1 - x$ with $R = 10$ and $D_w = 2.0$ mm for three L_a -values of contact length.

canal. Fig. 4 shows the deflection pattern of the inserted wire for three L_a -values of contact length. Within the wire–cortex contact region ($0 \leq x \leq L_a$), the wire was forced to make full contact with the intramedullary wall of the cortex. The shorter contact length with the cortex makes the wire deflect much more in the middle and distal regions (Fig. 4). The deflection curves of the inserted wire for three various D_w values of wire diameter are shown in Fig. 5. For the wire with 2.5-mm diameter, the wire at the tip ($x \cong 0$) approximated

Fig. 5. The deflection of the inserted wire along the wire's length based on the coordinate of $y_1 - x$ with $R = 10$ for three different D_w values of wire diameter.

Fig. 6. The reaction force at the interfaces between the wire and the bone in the parametric study on bone quality, wire diameter, and contact length.

a point-contact mode in comparison with the other two wire diameters (Fig. 5).

Apart from wire deflection (Figs. 3, 4, and 5), the bony reaction force was chosen as the index of stability and reduction for the wire–bone construct. In the parametric analyses of bone quality, wire diameter, and contact length, the reaction forces induced by the deflected wire from $x=0$ to $x=L_a+L_b$ are shown in Fig. 6. The reaction force F was 17.5, 15.7, and 12.6 N for bone quality R ranging from 10, 50, to 100, respectively. Increasing the contact length L_a from 20 to 30 to 40 mm, the reaction force rose from 14.2 to 19.3 to 29.4 N, respectively. Full contact of the wire with the cortex resulted in a greater reaction force in that region. For the 1.8-, 2.0-, and 2.5-mm wire diameter, the reaction force was 13.2, 21.4, and 40.2 N, respectively. By increasing the wire diameter, the elastic deformation of the surrounding cortical and cancellous bone rapidly increased as a result of the higher structural stiffness of the wire, thus resulting in a stronger reaction force. Even for a 10-fold decrease in bone quality, the reaction force was reduced by only 28.0% (Fig. 6). By comparison, the reaction force was markedly increased by 107% and 205% for the stable wire–cortex contact ($L_a=20, 30$, to 40 mm) and the larger wire diameter ($D_w=1.8, 2.0$, and 2.5 mm), respectively. This revealed that surgical planning for the entry point, insertion angle, and structural stiffness of the wire was more efficient than bone quality at stabilizing the whole wire–bone construct.

4. Discussion

In a distal radius fracture, the induced loads caused by the contractions of the surrounding soft tissues at the fracture zones commonly lead to fragment dislocation [6]. Sufficient support of the bone fragments at the fracture site is an indicator of healing of the radius. In order to achieve suitable reduction and safe retention, the Kapandji technique with several Kirschner wires was developed to treat this type of fracture. Historically, Ulson's procedure has been shown to

restore sufficient radial height and volar tilting, with a better surgical outcome. In addition, this method provides a dynamic compressive force on the fracture segment, instead of only static intra-fragmental fixation as with the Kapandji technique [6,7,15]. The purpose of the current study was to investigate the factors influencing the stability and the reduction of the wire-wrist construct within the post-operative period.

The ratio of the foundation moduli of the cortical to cancellous bone was assumed to affect stability and reduction of a distal radius fracture. For a constant cortex modulus (K_a), the cancellous bone was nearly osteoporotic for larger R -values. Increasing the R -value from 10 to 100, the percentage of wire deflection in the middle region ($L_a \leq x \leq L_a+L_b$) and the bony reaction were increased (Figs. 3 and 6), indicating that the strength of the cancellous bone affects the wire deflection and the bony reaction. For denser cancellous bone, the insertion of the wire becomes comparatively more difficult and deflection in the middle region ($L_a \leq x \leq L_a+L_b$) was decreased (Fig. 3). These results correlated well with the clinical observation that the wire was difficult to insert into the canal and to bend following the cortical wall for the denser cancellous bone. This finding suggests that the mechanical support of the denser cancellous bone provides a more stable stiffness of the fractured wrist (Fig. 6). Various clinical reports have also shown that the comminuted distal radius fracture with osteoporotic bone has been a difficult management problem in terms of restoring wrist anatomy and function [1,2].

Similar to the bone quality, the extent of the wire–bone contact plays a significant role in stability and reduction of a distal radius fracture (Figs. 4 and 5). The contact length between the wire and bone depends on the entry point, insertion angle, structural stiffness of the inserted wire, and the supporting strength of the cortical and cancellous bone (Fig. 2A). Consequently, the terms α and α/R were denoted as the flexural rigidity of the wire and the elasticity of the surrounding media, respectively. From a biomechanical point of view, they are the important factors influencing the shape of the wire deflection and the reaction force in the contact region ($0 \leq x \leq L_a+L_b$) [16]. Inserting the wire from the radial styloid resulted in a greater contact length ($=L_a+L_b$). Conversely, the contact length was decreased if the wire was inserted into the radius at a point closest to the ulna. By increasing the contact length from 20 to 40 mm, the reaction force increased by 107% because of the more stable support of the surrounding cortex (Fig. 6). However, the reaction force of the elastic wire was drastically reduced if the tip of the wire was pre-bent for easy insertion into the intramedullary canal of the radius because the local yielding of the pre-bent wire caused the majority of the elastic restoring ability for maintaining the stability and reduction of the wire–bone construct to be lost. Conversely, the wire with higher stiffness provided a greater reaction force at the wire–bone interfaces (Fig. 6).

In the parametric study (Figs. 3, 4, and 5), the deflection curves of the wire within the canal were analogous to

a parabola with somewhat different curvature in the middle and distal ranges. However, in terms of the biomechanics, the wire deflection cannot be seen as a precise index for understanding the structural stability and anatomical reduction of the whole wire–bone construct. The wire–bone reaction was indeed the overriding force for stabilizing the dislocation of the opposite fractured fragments pulled by muscular contractions. Fig. 6 shows that the denser cancellous bone provides a more stable wire–bone construct, thus facilitating fragment reduction and bone union. Comparatively, the contact length and wire diameter had much more significant effects on the wire–bone reaction force than the bone quality did. For example, a 10-fold change in R -value of the bone quality has only a mild influence on the contact reaction. However, doubling the contact length or increasing the wire diameter from 1.8 to 2.5 mm significantly augmented the contact reaction more than twofold (Fig. 6). In a clinical application, selecting the appropriate contact length and wire diameters is very important for creating a stable reduction of the fracture, and for avoiding insufficiency or over-correction. In fact, there is an interesting relationship between bone quality and wire diameter which may explain why the 1.8 mm K-wires are appropriate for Colles's fracture fixation in a clinical setting. In osteoporotic bone quality ($R=100$) the contact reaction (capacity to withstand strain) is around 12 N, and the 1.8 mm K-wire provided a similar contact reaction to bone; thus this diameter wire is more suitable for Ulson's procedure in osteoporotic bone than say the 2.0 or 2.5 mm K-wire for the same contact length (Fig. 6). Above all, the higher wire stiffness is more efficient for stabilizing the wire–bone construct. This suggests that surgical planning in terms of entry point, insertion angle, and structural stiffness of the wire is essential in order to optimize the stabilization of the whole wire–bone construct.

This study was aimed at formulating the parameters of bone quality, wire diameter, and contact length in order to investigate their effects on the reaction force at the wire–bone interfaces. However, there were several limitations inherent in this mathematical model. For Ulson's procedure, only one wire and a two-dimensional reduction were considered; the influences of splints or extra-skeletal fixators were ignored (Fig. 2A and B). Consequently, the predicted results could not provide information on the axial rotation and the flexion/extension reduction for the fractured fragment. In addition, the interaction between muscular contractions and wire–bone reactions were neglected; thus, the predicted results hold only for the wrist reduction without muscular contractions. Also, a continuous wire–bone contact throughout the wire length ($0 \leq x \leq L$) was assumed. This might overestimate the influence of the osteoporotic cancellous bone and unstable fractured fragments. Finally, the foundation modulus coupled with the viscoelastic properties of the bone and soft tissues was also not considered. Hence,

finite-element analysis and experimental evaluation should be conducted in future studies in order to validate the results of this study and to investigate the wire–bone interaction mechanism and three-dimensional reduction in more detail.

Conflict of interest statement

There was no financial support about this numerical research. No benefit of any kind will be received either directly or indirectly by the authors.

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