Dynamic response of regenerator in cyclic flow system

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The dynamic response of isothermal and cyclic flow in a regenerator is investigated. A basic transfer-function model for the regenerator is derived and used to study the system dynamics of a regenerator connecting a reciprocating piston and a reservoir using two models, $G_{mm}(s)$ and $G_{mp}(s)$. The frequency response of the mass flow rate at the reservoir side of the regenerator induced by pressure or mass flow rate waves at the piston side is shown to approach zero at high frequency, but to approach a constant value of gain at low frequencies. The cut-off frequency f_{cut} and bandwidth depend on the regenerator design (wire mesh size, porosity, length, diameter, etc.). The criteria for the use of a quasi-steady approximation in regenerator modelling can then be determined by comparing the operating frequency with the regenerator bandwidth. It is also shown that attenuation of the mass flow rate fluctuation induced by the pressure waves is stronger for regenerators with a larger void volume and that the use of air or nitrogen as the working fluid will induce a stronger attenuation effect than helium.

Keywords: regenerators; thermodynamics; cyclic flow systems

Nomenclature

A_{fr}	Free-flow area of regenerator (m^2)
C_p	Heat capacity at constant pressure ($J\ kg^{-1}\ K^{-1}$)
D	Regenerator diameter (m)
d_h	Hydraulic diameter of regenerator (m)
d_m	Screen wire diameter (m)
f	Cyclic frequency (Hz)
f_h	Friction factor of regenerator
L	Regenerator length (m)
$\dot{m}$	Mass flow rate ($kg\ s^{-1}$)
N	Rotation speed of oscillating piston ($rev\ s^{-1}$)
p	Pressure ($N\ m^{-2}$)
R	Universal gas constant ($m^2\ K^{-1}\ s^{-2}$)
Re_h	Reynolds number based on hydraulic diameter of regenerator ($= d_h u_{max}/\nu$)
s	Laplace transfer variable
T_0	System mean temperature (K)
t	Time (s)
u_{max}	Peak velocity of gas inside regenerator ($m\ s^{-1}$)

V_p	Swept volume of piston (m^3)
w	Velocity inside regenerator ($m\ s^{-1}$)
x	Position in regenerator (m)

Greek letters

α	First coefficient in momentum Equation (2)
β	Second coefficient in momentum Equation (2)
ε	Porosity of regenerator
ρ	Gas density ($kg\ m^{-3}$)
λ_b	Characteristic parameter, Equation (17)
ν	Kinematic viscosity of gas ($m^2\ s^{-1}$)
ω	Angular frequency ($rad\ s^{-1}$)

Subscripts

1	Regenerator end at piston side
2	Regenerator end at reservoir side

Superscripts

—	Mean value
~	Perturbation value

Regenerators used in cryocoolers are usually operated under transient or cyclic flow conditions. The regenerator connects the compression and expansion spaces such that the working fluid is heated or cooled by the regenerator matrix in the thermodynamic cycle. The reciprocating motion of the piston and displacer or the filling and exhausting processes of gas in cryocoolers such as Stir-

ling, Gifford-McMahon or pulse tube refrigerators will generate a cyclic pressure wave across the regenerator and induce cyclic flow through the regenerator. The thermal performance analysis of a cryocooler thus becomes extremely complicated since the transient solutions of unsteady state models will probably be involved. Quasi-steady approximations have, therefore,

been frequently used in regenerator modelling in order to simplify system analysis. However, no criterion has ever been derived to validate this approximation.

The phenomenon of a cyclic flow rate induced by a cyclic pressure wave across the regenerator is basically a system dynamics problem. The understanding of regenerator dynamics under cyclic flow conditions is thus quite important in the design of Stirling machines and cryo-coolers. Earlier research on regenerator performance with cyclic flow mainly focussed on modelling and predictions of transient temperature profiles (see, for example, references 1 and 2), regenerator inefficiency³⁻⁵ and heat transfer coefficient measurements^{6,7}. Very little attention has been paid to the dynamic behaviour of regenerators until recently.

Guo *et al.*⁸ experimentally observed that there is a phase shift and amplitude attenuation for a cyclic pressure wave across a regenerator. Furthermore, they tried to qualitatively interpret the dynamic behaviour of a regenerator by using a four-end linear network model which was derived for cyclic flow in a regenerator using the revised Darcy's law (momentum equation). However, such modelling leads to erroneous results due to ignorance of the second-order term in Darcy's law. The regenerator model has not been used in the analysis of the dynamic response of the regenerator in a system.

The present study aims to derive a basic transfer-function model for a regenerator by using a small perturbation approximation and linear system theory. The regenerator model is further used to derive a system dynamics model to study the dynamic responses of a regenerator connecting a reciprocating piston and a reservoir (Figure 1). A criterion for the validation of the quasi-steady approximation in regenerator modelling will then be developed.

Basic transfer-function model of regenerator

A regenerator is essentially a porous medium made by stacking metallic wire screen meshes or by packing metal powders in a cylindrical vessel. In the derivation of the system dynamics model of regenerators, the following assumptions are made:

- 1 the flow in the regenerator is one-dimensional;
- 2 the working gas is an ideal gas;
- 3 gas viscosity ν is constant;
- 4 the porous medium is homogeneous and isotropic; and
- 5 there is no temperature difference across the regenerator.

Figure 1 Cyclic flow system including regenerator

Governing equations

The mass continuity equation is derived by applying conservation of mass to the control volume of a regenerator

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho w)}{\partial x} = 0 \quad (1)$$

The momentum equation of a regenerator is basically the revised Darcy's law⁹

$$\frac{\partial(\rho w)}{\partial t} + \frac{\partial(\rho |w|w)}{\partial x} + \frac{\partial p}{\partial x} + \alpha \nu \rho (\varepsilon w) + \beta \rho (\varepsilon^2 |w|w) = 0 \quad (2)$$

For convenience, the velocity terms in Equations (1) and (2) are rewritten in terms of mass flow rate using the relation $\dot{m} = \rho A_{fr} w$

$$\frac{\partial p}{\partial t} + \frac{RT_0}{A_{fr}} \frac{\partial \dot{m}}{\partial x} = 0 \quad (3)$$

$$\frac{1}{A_{fr}} \frac{\partial \dot{m}}{\partial t} + \frac{1}{A_{fr}^2} \frac{\partial(\dot{m}|\dot{m}|/\rho)}{\partial x} + \frac{\partial p}{\partial x} + \frac{\alpha \varepsilon \nu}{A_{fr}} \dot{m} + \beta \rho \left(\frac{\varepsilon}{\rho A_{fr}} \right)^2 \dot{m}|\dot{m}| = 0 \quad (4)$$

Small perturbations and linearization

The momentum equation, Equation (4), should be modified in order to apply small perturbation for linearization. Since the gas flow inside the regenerator is not very high in practice, usually $Re < 500$, the inertia term (the second term) can be ignored. Furthermore, the last term can be omitted and the associated effect included in a modified parameter α' . Therefore, Equation (4) becomes

$$\frac{1}{A_{fr}} \frac{\partial \dot{m}}{\partial t} + \frac{\partial p}{\partial x} + \frac{\alpha' \varepsilon \nu}{A_{fr}} \dot{m} = 0 \quad (5)$$

It should be noted that α' will vary with mass flow rate, i.e. $\alpha' = f(\dot{m})$.

Under cyclically steady conditions, the mass flow rate and pressure can be written as the mean term and periodic perturbation term

$$\dot{m}(x, t) = \bar{\dot{m}}(x) + \tilde{\dot{m}}(x, t) \quad (6)$$

$$p(x, t) = \bar{p}(x) + \tilde{p}(x, t) \quad (7)$$

Hence, $\bar{\dot{m}}(x) = 0$, $\bar{p}(x) = p_0 = \text{constant}$. Substituting Equations (6) and (7) into the governing Equations (3) and (5), and assuming a small perturbation, i.e. $\tilde{p}(x, t) \ll \bar{p}(x)$, we obtain the following linear equations which are similar to an RLC electric circuit

$$\frac{\partial \tilde{p}}{\partial t} + \frac{1}{C_F} \frac{\partial \tilde{m}}{\partial x} = 0 \quad (8)$$

$$L_F \frac{\partial \tilde{m}}{\partial t} + \frac{\partial \tilde{p}}{\partial x} + R_F \tilde{m} = 0 \quad (9)$$

where: R_F is the flow resistance per unit length of regenerator; C_F is the flow capacitance per unit length of regenerator; and L_F is the flow inductance per unit length of regenerator. According to analogy with an RLC

circuit, these terms are defined as

$$R_F \equiv \frac{\bar{\alpha} v_e}{A_{fr}} \quad (10)$$

$$C_F \equiv \frac{A_{fr}}{RT_0} \quad (11)$$

$$L_F \equiv \frac{1}{A_{fr}} \quad (12)$$

Here $\bar{\alpha}$ represents the parameter α' evaluated at the peak value of flow rate fluctuation $\dot{m}_{max}$. Equations (8) and (9) are the linearized equations of regenerators.

Solutions

Equations (8) and (9) can be solved by using the Laplace transform to yield (assuming zero initial conditions)

$$\begin{aligned} \tilde{m}(x, s) = & \frac{\tilde{m}_2(s)}{\sinh(\lambda_b L)} \sinh(\lambda_b x) \\ & + \frac{\tilde{m}_1(s)}{\sinh(\lambda_b L)} \sinh[\lambda_b(L - x)] \end{aligned} \quad (13)$$

$$\begin{aligned} \tilde{p}(x, s) = & \frac{\tilde{p}_2(s)}{\sinh(\lambda_b L)} \sinh(\lambda_b x) \\ & + \frac{\tilde{p}_1(s)}{\sinh(\lambda_b L)} \sinh[\lambda_b(L - x)] \end{aligned} \quad (14)$$

where

$$\tilde{m}(0, s) = \tilde{m}_1(s); \quad \tilde{m}(L, s) = \tilde{m}_2(s) \quad (15)$$

$$\tilde{p}(0, s) = \tilde{p}_1(s); \quad \tilde{p}(L, s) = \tilde{p}_2(s) \quad (16)$$

λ_b is the characteristic value defined as

$$\lambda_b^2 = s(R_F C_F + s L_F C_F) \quad (17)$$

Basic transfer-function model of regenerator

Equations (13) and (14) represent a distributed-parameter dynamics model accounting for the spatial variation of mass flow rate and pressure waves along the regenerator. It is preferable to write the dynamics model in terms of the boundary conditions of the regenerator and in a transfer-function form. Substituting Equation (13) into the Laplace form of Equation (8) and Equation (14) into the Laplace form of Equation (9) and letting $x = 0$, we obtain

$$\tilde{m}_2(s) = [\cosh(\lambda_b L)] \tilde{m}_1(s) - \left[s C_F \frac{\sinh(\lambda_b L)}{\lambda_b} \right] \tilde{p}_1(s) \quad (18)$$

$$\begin{aligned} \tilde{p}_2(s) = & - \left[(L_F s + R_F) \frac{\sinh(\lambda_b L)}{\lambda_b} \right] \tilde{m}_1(s) \\ & + [\cosh(\lambda_b L)] \tilde{p}_1(s) \end{aligned} \quad (19)$$

Hence, Equations (18) and (19) can be written in a matrix form

$$\begin{bmatrix} \tilde{m}_2(s) \\ \tilde{p}_2(s) \end{bmatrix} = \begin{bmatrix} \cosh(\lambda_b L) & -s C_F \frac{\sinh(\lambda_b L)}{\lambda_b} \\ -(L_F s + R_F) \frac{\sinh(\lambda_b L)}{\lambda_b} & \cosh(\lambda_b L) \end{bmatrix} \begin{bmatrix} \tilde{m}_1(s) \\ \tilde{p}_1(s) \end{bmatrix} \quad (20)$$

Equation (20) is the basic transfer-function model of a regenerator which accounts for the dynamic responses of the regenerator between the two boundaries.

Dynamic response of regenerator in cyclic flow system

System dynamics model of regenerator in cyclic flow system

The dynamic response of a regenerator in a cyclic flow system which consists of a reciprocating piston and a reservoir (Figure 1) will be studied here. For convenience, Equation (18) is rearranged to yield

$$\tilde{m}_1(s) = \left[\frac{s C_F \sinh(\lambda_b L)}{\lambda_b \cosh(\lambda_b L)} \right] \tilde{p}_1(s) + \frac{\tilde{m}_2(s)}{\cosh(\lambda_b L)} \quad (21)$$

Substituting Equation (14) into the Laplace form of Equation (9) and letting $x = L$ will yield

$$\tilde{p}_2(s) = \frac{\tilde{p}_1(s)}{\cosh(\lambda_b L)} - \left[\frac{L_F s + R_F}{\lambda_b} \frac{\sinh(\lambda_b L)}{\cosh(\lambda_b L)} \right] \tilde{m}_2(s) \quad (22)$$

We define location 1 of the regenerator at the piston side and the location 2 at the reservoir side. The dynamics of a regenerator connected in a cyclic flow system can be described by three transfer-function models, where $G_{pp}(s)$ represents the response of the pressure wave at the reservoir-side end induced by the pressure wave at the piston-side end and, since $\tilde{p}_2(s) = 0$ for a reservoir

$$G_{pp}(s) = \frac{\tilde{p}_2(s)}{\tilde{p}_1(s)} = 0 \quad (23)$$

On the other hand, from Equation (22), for $\tilde{p}_2(s) = 0$

$$\tilde{p}_1(s) = \left[\frac{L_F s + R_F}{\lambda_b} \sinh(\lambda_b L) \right] \tilde{m}_2(s) \quad (24)$$

therefore, combining Equations (24) and (21), we obtain

$$G_{mm}(s) \equiv \frac{\tilde{m}_2(s)}{\tilde{m}_1(s)} = \frac{1}{\cosh(\lambda_b L)} \quad (25)$$

where $G_{mm}(s)$ represents the response of the mass flow rate wave at the reservoir-side end induced by the piston-side mass flow rate wave.

The response of the mass flow rate wave at the reservoir-side end induced by the pressure wave at the piston-side end is represented by the transfer-function model $G_{mp}(s)$ which is derived from Equation (24)

$$G_{mp}(s) = \frac{\tilde{m}_2(s)}{\tilde{p}_1(s)} = \frac{1}{L_F s + R_F} \frac{\lambda_b}{\sinh(\lambda_b L)} \quad (26)$$

For convenience, Equation (26) can be written as [since $\tilde{p}_2(s) = 0$ for a regenerator connected to a reservoir]

$$G_{mp}(s) = \frac{\tilde{m}_2(s)}{\tilde{p}_1(s) - \tilde{p}_2(s)} = \frac{\tilde{m}_2(s)}{\Delta\tilde{p}(s)} = \frac{1}{L_F s + R_F \sinh(\lambda_b L)} \quad (27)$$

Equation (27) can be considered to be a dynamic form of Darcy's law which is used to describe the dynamic responses of a regenerator in the cyclic flow system.

Specifications of cyclic flow system

The regenerator used in the present analysis has a diameter $D = 40$ mm and a length $L = 30$ mm, which are the same as the values in the regenerator used by Tanaka *et al.*⁷ in an oscillating flow test. Four kinds of screen mesh wire are adopted in the analysis: 150, 200, 250 and 400 mesh. The physical properties of the regenerators are listed in Table 1, where the porosities are calculated using Chang's formula¹⁰ and the parameters α and β in the momentum Equation (2) are converted from the friction correlation for oscillatory flow tests by Tanaka *et al.*⁷

$$f_h = \frac{A}{Re_h} + B \quad (28)$$

where $A = 175$ and $B = 1.60$, according to Tanaka *et al.*⁷, and $Re_h (= u_{\max} d_h / \nu)$ is the Reynolds number based on a regenerator hydraulic diameter d_h and a peak velocity of gas inside the regenerator void volume $u_{\max}$

$$d_h = \frac{4\epsilon d_m}{\phi(1 - \epsilon)} \quad (29)$$

$\phi = 4$ for screen wire meshes and f_h is the friction factor defined as

$$f_h = \frac{\Delta p d_h}{\rho u_{\max}^2 L / 2} \quad (30)$$

where $\Delta p (= p_1 - p_2)$ and $u_{\max}$ are, respectively, the pressure drop across the regenerator and the peak local velocity.

The relation between parameters α and β and the modified parameter α' is derived as

$$\alpha' = \alpha + \frac{\beta}{\nu} u_{\max} = \alpha + \frac{\beta}{d_h} Re_h \quad (31)$$

where $\alpha = A/2d_h^2$ and $\beta = B/2d_h$.

The peak velocity $u_{\max}$ is related to the driving mechanism and rotation speed of the piston. For the crank case used by Tanaka *et al.*⁷, $u_{\max}$ follows the relation

$$u_{\max} = \frac{1.042\pi V_p N}{60 A_r \epsilon} = \frac{1.042\pi V_p f}{A_r \epsilon} \quad (32)$$

where V_p is the swept volume of piston. The parameter α' modified for a cyclic mass flow rate thus becomes

$$\bar{\alpha} = \alpha + \frac{\beta}{\nu} \times \frac{1.042\pi V_p f}{A_r \epsilon} \quad (33)$$

It is noted that the modified parameter $\bar{\alpha}$ is a function of the frequency of cyclic flow rate or piston motion.

Frequency responses of regenerator in cyclic flow system

From Equation (33) $\bar{\alpha}$ is seen to be a function of α , β , ν , f , ϵ , etc.; while α and β are a function of d_h alone, according to Equation (31). Since d_h is a function of screen wire diameter d_m and porosity ϵ of the regenerator, which are determined by the standard screen wire mesh size, $\bar{\alpha}$ is seen to be a function of standard screen wire mesh size, gas viscosity ν and operating frequency.

Figure 2 shows the values of $\bar{\alpha}$ determined at different Reynolds numbers Re_h and oscillation frequencies of the piston f for helium gas in four generators packed with different screen wire meshes. $\bar{\alpha}$ values for air, nitrogen and helium gases with a 200 mesh screen are shown in Figure 3. The use of air or nitrogen as the working fluid in a regenerator is seen to introduce a much greater pressure loss, about 5 times more than that using helium gas.

Frequency response of $G_{mm}(s)$

The frequency response of flow rate fluctuation at the reservoir side $\tilde{m}_2$ can be seen from the Bode plots of $G_{mm}(j\omega)$ as shown in Figure 4. It is seen that both the gain and the phase tend to decrease more rapidly with increasing frequency for the regenerator with a dense wire screen mesh. This means that the attenuation of flow rate fluctuation at the reservoir-side end will be stronger for regenerators with larger wire screen mesh (dense

Figure 2 $\bar{\alpha}$ value for various regenerators

Table 1 Regenerator properties

	150 mesh	200 mesh	250 mesh	400 mesh
Wire diameter d_m (mm)	0.06604	0.05334	0.04064	0.02540
Porosity ϵ	0.732	0.707	0.735	0.686
Hydraulic diameter d_h (mm)	0.1803	0.1287	0.1127	0.0554
α (mm ⁻²)	2691.6	5281.8	6886.6	28417.0
β (mm ⁻¹)	4.4370	6.2155	7.0972	14.417

Figure 3 α value for various working fluids

Figure 4 Frequency responses of $G_{mm}(s)$ for various regenerators

screen). The magnitude of unity (i.e. 0 db) represents the situation where there is no attenuation of the flow rate fluctuation at the reservoir-side end.

It is more convenient to define a bandwidth at -3 db gain which specifies the cut-off frequency f_{cut} in order to understand the frequency lower limit under which there is very low attenuation of the dynamic fluctuations.

It can be seen from Table 2 that f_{cut} decreases with increasing wire mesh number. However, it can be easily shown that the phase angle at f_{cut} remains constant at -57.66° . This indicates that only a phase lag for the reservoir-side flow rate induced by the piston-side flow rate fluctuation has to be considered for a cyclic frequency lower than f_{cut} . For instance, for the regenerator length and diameter used in the present study, f_{cut} for the 400 mesh screen design is 90 Hz. Therefore, only a phase lag of $<57.66^\circ$ needs to be considered for a cyclic frequency <90 Hz.

Table 2 Cut-off frequency of $G_{mm}(j\omega)$ for helium gas

Wire screen mesh number	f_{cut} (Hz)	Phase angle at f_{cut} ($^\circ$)
150	304	-57.66
200	234	-57.66
250	205	-57.66
400	90	-57.66

The effect of the working fluid on $G_{mm}(s)$ is seen to be quite strong from Figure 5. For air and nitrogen gases, the magnitude and phase of $G_{mm}(j\omega)$ drop more rapidly than with helium gas. The cut-off frequency f_{cut} or bandwidth is reduced two-fold for air and nitrogen gas. However, the phase lag at f_{cut} still remains constant (-57.66°).

The attenuation and phase shift of flow rates at the two ends of the regenerator can be clearly seen from Figure 6, which is drawn for 400 mesh regenerator operating at 100 Hz with helium gas. $\dot{m}_2$ is seen to lag behind $\dot{m}_1$ by 57.66° .

Table 3 shows that the bandwidth of the regenerator for air or nitrogen in the 200 mesh regenerator is much narrower than for helium gas. However, the corresponding phase shifts at f_{cut} are the same, at -57.66° . This indicates that helium gas is a superior working fluid to air or nitrogen gas.

Figure 5 Frequency responses of $G_{mm}(s)$ for various working fluids

Figure 6 Time responses of $\dot{m}_2$ induced by $\dot{m}_1$ for 400 mesh regenerator

Table 3 Cut-off frequency of $G_{mm}(j\omega)$ for 200 mesh regenerator

Working fluid	f_{cut} (Hz)	Phase angle at f_{cut} ($^\circ$)
Air	103	-57.66
Nitrogen	104	-57.66
Helium	234	-57.66

Frequency response of $G_{mp}(s)$

The frequency response of flow rate fluctuation at the reservoir-side end $\dot{m}_2$ induced by a pressure wave across the regenerator $\Delta\bar{p}$ can be seen from the Bode plots of $G_{mp}(j\omega)$ for helium gas, as shown in Figure 7. Both the gain and the phase angle are seen to decrease more rapidly with increasing frequency for a loose wire screen mesh. The bandwidth f_{cut} and the corresponding phase angle ϕ_{cut} are shown in Table 4. It is seen that the bandwidth increases with increasing mesh number. This is due to the fact that for a loose regenerator, the flow capacitance effect is stronger and causes a greater attenuation effect (with narrower bandwidth) on the flow rate fluctuation in response to the pressure waves across the regenerator.

An example of the time response of the reservoir-side mass flow rate $\dot{m}_2$ induced by a pressure drop

$\Delta p (= p_1 - p_2)$ for a 400 mesh regenerator at 100 Hz is shown in Figure 9.

It is quite interesting to see that Equation (27) will be reduced to a constant gain of $A_{fr}/(\bar{\alpha}v_eL)$ and zero phase shift at very low frequencies ($\omega \rightarrow 0$), that is

$$\lim_{\omega \rightarrow 0} |G_{mp}(j\omega)| = \frac{1}{R_t} \quad (34)$$

where R_t is the total flow resistance of the regenerator, defined as

$$R_t \equiv \frac{\bar{\alpha}v_eL}{A_{fr}} = R_F L \quad (35)$$

This shows that the regenerator will obey Darcy's law at low frequencies. That is, the quasi-steady assumption of Darcy's law holds for the reservoir-side flow rate attenuation induced by the pressure drop across the regenerator, to within a bandwidth approximately. However, a small phase shift in the reservoir-side flow rate response exists, as shown in Table 4, and has to be corrected if the quasi-steady assumption of Darcy's law is used.

It is interesting to note that the phase lag increases abruptly from 250 to 400 mesh. The 400 mesh regenerator has a wider bandwidth but a larger phase lag. On the other hand, the 150 mesh regenerator has a narrower bandwidth and almost no phase lag, so that the quasi-steady assumption can hold approximately for both the magnitude and phase shift.

Table 5 shows that for the 200 mesh regenerator, the bandwidth of air or nitrogen for $G_{mp}(j\omega)$ is much narrower than for helium gas. However, the corresponding phase lag at f_{cut} is smaller for air or nitrogen. This verifies that the use of helium gas as a working fluid is superior to air or nitrogen gas since the bandwidth is much wider and the phase lag is smaller.

Figure 7 Frequency responses of $G_{mp}(s)$ for various regenerators

Table 4 Cut-off frequency of $G_{mp}(j\omega)$ for helium gas

Wire screen mesh number	f_{cut} (Hz)	Phase angle at f_{cut} (°)
150	48	-1.47
200	65	-3.70
250	74	-5.60
400	115	-30.0

Figure 8 Frequency responses of $G_{mp}(s)$ for various working fluids

Figure 9 Times responses of $\dot{m}_2$ induced by Δp for 400 mesh regenerator

Table 5 Cut-off frequency of $G_{mp}(j\omega)$ for 200 mesh regenerator

Working fluid	f_{cut} (Hz)	Phase angle at f_{cut} (°)
Air	9	-0.5
Nitrogen	10	-0.6
Helium	65	-3.7

Discussion and conclusions

A basic transfer-function model for isothermal and cyclic flow in a regenerator is derived and used to study the system dynamics of a regenerator connecting a reciprocating piston and a reservoir via Bode plots.

The frequency response of the mass flow rate at the reservoir side of the regenerator induced by the pressure and mass flow rate waves at the piston side are shown to approach zero at high frequencies, but approach a constant value in gain at low frequencies so that a quasi-steady approximation can hold. The cut-off frequency f_{cut} which defines the bandwidth depends on the regenerator design (wire mesh size, porosity, length, diameter, material, etc.). f_{cut} for the flow rate to flow rate response $G_{\text{mm}}(j\omega)$ decreases with increasing mesh number of the regenerator. This indicates that the attenuation of mass flow rate fluctuation is stronger for regenerators with a lower void volume (i.e. more dense screen). The use of air or nitrogen gas will introduce stronger attenuation than helium. However, the phase angle at f_{cut} remains constant irrespective of screen mesh numbers and working fluids.

For the reservoir-side flow rate response induced by the pressure waves $G_{\text{mp}}(j\omega)$ the cut-off frequency or bandwidth f_{cut} increases with increasing regenerator mesh number. This indicates that the attenuation of mass flow rate fluctuation is stronger for regenerators with a larger void volume (i.e. loose screen). The use of air or nitrogen gas will introduce a stronger attenuation than helium. However, a small phase lag exists for the reservoir-side flow rate response and needs to be corrected if the quasi-steady assumption of Darcy's law is used.

Regenerators used in Stirling machines or cryocoolers are usually operated under transient or cyclic flow conditions. The thermal performance analysis is thus quite complicated since transient solutions of unsteady state models are involved. The present analysis can be used to simplify the modelling of the regenerator by using a quasi-steady approximation, according to the operating frequency of the machine and the bandwidth of the regenerator. A system dynamics model of the regenerator similar to Equation (27) can be easily derived according to the present analysis for analysis of a real Stirling

cryocooler. However, the pressure fluctuation at the reservoir side will be non-zero [i.e. $\tilde{p}_2(s) \neq 0$] due to the introduction of another reciprocating displacer and a cold head in a Stirling cryocooler.

The basic transfer-function model of a regenerator [Equation (20)] derived in the present study can also be used to develop linear network models of Stirling or pulse tube cryocoolers by connecting similar models one by one for each component of the machine, in a similar way to Guo¹¹. The thermal performance prediction of cryocoolers can then be simplified a great deal.

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