Dynamic response of regenerator in cyclic flow system

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The dynamic response of isothermal and cyclic flow in a regenerator is investigated. A basic transfer-function model for the regenerator is derived and used to study the system dynamics of a regenerator connecting a reciprocating piston and a reservoir using two models, \( G_{nm}(s) \) and \( G_{mp}(s) \). The frequency response of the mass flow rate at the reservoir side of the regenerator induced by pressure or mass flow rate waves at the piston side is shown to approach zero at high frequency, but to approach a constant value of gain at low frequencies. The cut-off frequency \( f_{\text{cut}} \) and bandwidth depend on the regenerator design (wire mesh size, porosity, length, diameter, etc.). The criteria for the use of a quasi-steady approximation in regenerator modelling can then be determined by comparing the operating frequency with the regenerator bandwidth. It is also shown that attenuation of the mass flow rate fluctuation induced by the pressure waves is stronger for regenerators with a larger void volume and that the use of air or nitrogen as the working fluid will induce a stronger attenuation effect than helium.

Keywords: regenerators; thermodynamics; cyclic flow systems

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_{fr} )</td>
<td>Free-flow area of regenerator ((\text{m}^2))</td>
</tr>
<tr>
<td>( C_p )</td>
<td>Heat capacity at constant pressure ((\text{J kg}^{-1} \text{K}^{-1}))</td>
</tr>
<tr>
<td>( D )</td>
<td>Regenerator diameter ((\text{m}))</td>
</tr>
<tr>
<td>( d_h )</td>
<td>Hydraulic diameter of regenerator ((\text{m}))</td>
</tr>
<tr>
<td>( d_m )</td>
<td>Screen wire diameter ((\text{m}))</td>
</tr>
<tr>
<td>( f )</td>
<td>Cyclic frequency ((\text{Hz}))</td>
</tr>
<tr>
<td>( f_h )</td>
<td>Friction factor of regenerator</td>
</tr>
<tr>
<td>( L )</td>
<td>Regenerator length ((\text{m}))</td>
</tr>
<tr>
<td>( n )</td>
<td>Mass flow rate ((\text{kg s}^{-1}))</td>
</tr>
<tr>
<td>( N )</td>
<td>Rotation speed of oscillating piston ((\text{rev s}^{-1}))</td>
</tr>
<tr>
<td>( p )</td>
<td>Pressure ((\text{N m}^{-2}))</td>
</tr>
<tr>
<td>( R )</td>
<td>Universal gas constant ((\text{m}^2 \text{K}^{-1} \text{s}^{-2}))</td>
</tr>
<tr>
<td>( Re_h )</td>
<td>Reynolds number based on hydraulic diameter of regenerator (=d_h u_{\text{max}}/\nu)</td>
</tr>
<tr>
<td>( s )</td>
<td>Laplace transfer variable</td>
</tr>
<tr>
<td>( T_0 )</td>
<td>System mean temperature ((\text{K}))</td>
</tr>
<tr>
<td>( t )</td>
<td>Time ((\text{s}))</td>
</tr>
<tr>
<td>( u_{\text{max}} )</td>
<td>Peak velocity of gas inside regenerator ((\text{m s}^{-1}))</td>
</tr>
</tbody>
</table>

| \( V_p \) | Swept volume of piston \((\text{m}^3)\) |
| \( w \) | Velocity inside regenerator \((\text{m s}^{-1})\) |
| \( x \) | Position in regenerator \((\text{m})\) |

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>( \alpha )</td>
<td>First coefficient in momentum Equation (2)</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Second coefficient in momentum Equation (2)</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>Porosity of regenerator</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Gas density ((\text{kg m}^{-3}))</td>
</tr>
<tr>
<td>( \lambda_b )</td>
<td>Characteristic parameter, Equation (17)</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Kinematic viscosity of gas ((\text{m}^2 \text{s}^{-1}))</td>
</tr>
<tr>
<td>( \omega )</td>
<td>Angular frequency ((\text{rad s}^{-1}))</td>
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<th>Subscripts</th>
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<tr>
<td>1</td>
<td>Regenerator end at piston side</td>
</tr>
<tr>
<td>2</td>
<td>Regenerator end at reservoir side</td>
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</tbody>
</table>

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<th>Description</th>
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<tr>
<td>~</td>
<td>Mean value</td>
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<tr>
<td>(-)</td>
<td>Perturbation value</td>
</tr>
</tbody>
</table>

Regenerators used in cryocoolers are usually operated under transient or cyclic flow conditions. The regenerator connects the compression and expansion spaces such that the working fluid is heated or cooled by the regenerator matrix in the thermodynamic cycle. The reciprocating motion of the piston and displacer or the filling and exhausting processes of gas in cryocoolers such as Stirling, Gifford-McMahon or pulse tube refrigerators will generate a cyclic pressure wave across the regenerator and induce cyclic flow through the regenerator. The thermal performance analysis of a cryocooler thus becomes extremely complicated since the transient solutions of unsteady state models will probably be involved. Quasi-steady approximations have, therefore,
been frequently used in regenerator modelling in order to simplify system analysis. However, no criterion has ever been derived to validate this approximation.

The phenomenon of a cyclic flow rate induced by a cyclic pressure wave across the regenerator is basically a system dynamics problem. The understanding of regenerator dynamics under cyclic flow conditions is thus quite important in the design of Stirling machines and cryocoolers. Earlier research on regenerator performance with cyclic flow mainly focussed on modelling and predictions of transient temperature profiles (see, for example, references 1 and 2), regenerator inefficiency 3-5 and heat transfer coefficient measurements 6,7. Very little attention has been paid to the dynamic behaviour of regenerators until recently.

Guo et al. 8 experimentally observed that there is a phase shift and amplitude attenuation for a cyclic pressure wave across a regenerator. Furthermore, they tried to qualitatively interpret the dynamic behaviour of a regenerator by using a four-end linear network model which was derived for cyclic flow in a regenerator using the revised Darcy’s law (momentum equation). However, such modelling leads to erroneous results due to ignorance of the second-order term in Darcy’s law. The regenerator model has not been used in the analysis of the dynamic response of the regenerator in a system.

The present study aims to derive a basic transfer-function model for a regenerator by using a small perturbation approximation and linear system theory. The regenerator model is further used to derive a system dynamics model to study the dynamic responses of a regenerator connecting a reciprocating piston and a reservoir (Figure 1). A criterion for the validation of the quasi-steady approximation in regenerator modelling will then be developed.

**Basic transfer-function model of regenerator**

A regenerator is essentially a porous medium made by stacking metallic wire screen meshes or by packing metal powders in a cylindrical vessel. In the derivation of the system dynamics model of regenerators, the following assumptions are made:

1. the flow in the regenerator is one-dimensional;
2. the working gas is an ideal gas;
3. gas viscosity $\nu$ is constant;
4. the porous medium is homogeneous and isotropic; and
5. there is no temperature difference across the regenerator.

![Figure 1 Cyclic flow system including regenerator](image)

**Governing equations**

The mass continuity equation is derived by applying conservation of mass to the control volume of a regenerator

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho \dot{m})}{\partial x} = 0 \quad (1)$$

The momentum equation of a regenerator is basically the revised Darcy’s law$$
\frac{\partial (\rho \dot{m})}{\partial t} + \rho \dot{V} \frac{\partial \rho}{\partial x} + \frac{\partial P}{\partial x} + \frac{\partial \rho (\dot{V}^2 \rho)}{\partial x} + \beta \rho (\dot{V}^2) = 0 \quad (2)
$$

For convenience, the velocity terms in Equations (1) and (2) are rewritten in terms of mass flow rate using the relation $\dot{m} = \rho A r \dot{V}$

$$\frac{\partial P}{\partial t} + \frac{\partial (\rho \dot{m})}{\partial x} + \frac{\partial P}{\partial x} + \frac{\partial \rho (\dot{V}^2 \rho)}{\partial x} + \frac{\partial \rho (\dot{V}^2 \rho)}{\partial x} + \beta \rho (\dot{V}^2) = 0 \quad (3)$$

Small perturbations and linearization

The momentum equation, Equation (4), should be modified in order to apply small perturbation for linearization. Since the gas flow inside the regenerator is not very high in practice, usually $Re < 500$, the inertia term (the second term) can be ignored. Furthermore, the last term can be omitted and the associated effect included in a modified parameter $\lambda$. Therefore, Equation (4) becomes

$$\frac{1}{A_r} \frac{\partial \dot{m}}{\partial t} + \frac{1}{A_r} \frac{\partial (\dot{m} \dot{V} \rho)}{\partial x} + \frac{\partial P}{\partial x} + \alpha \dot{V} \dot{m} + \beta \rho (\dot{V}^2) \dot{m} = 0 \quad (5)$$

It should be noted that $\lambda$ will vary with mass flow rate, i.e. $\lambda = f(\dot{m})$.

Under cyclically steady conditions, the mass flow rate and pressure can be written as the mean term and periodic perturbation term

$$\dot{m}(x, t) = \bar{m}(x) + \tilde{m}(x, t) \quad (6)$$

$$P(x, t) = \bar{P}(x) + \tilde{P}(x, t) \quad (7)$$

Hence, $\bar{m}(x) = 0$, $\bar{P}(x) = P_0 = \text{constant}$. Substituting Equations (6) and (7) into the governing Equations (3) and (5), and assuming a small perturbation, i.e. $\tilde{P}(x, t) \ll \bar{P}(x)$, we obtain the following linear equations which are similar to an RLC electric circuit

$$\frac{\partial \tilde{P}}{\partial t} + \frac{1}{C_F} \frac{\partial \tilde{m}}{\partial x} = 0 \quad (8)$$

$$L_F \frac{\partial \tilde{m}}{\partial t} + \frac{\partial \tilde{P}}{\partial x} + R_F \tilde{m} = 0 \quad (9)$$

where: $R_F$ is the flow resistance per unit length of regenerator; $C_F$ is the flow capacitance per unit length of regenerator; and $L_F$ is the flow inductance per unit length of regenerator. According to analogy with an RLC
Dynamic response of regenerator: B.J. Huang and C.W. Lu

Here \( \dot{\alpha} \) represents the parameter \( \alpha' \) evaluated at the peak value of flow rate fluctuation \( \dot{\alpha}_{\text{max}} \). Equations (8) and (9) are the linearized equations of regenerators.

**Solutions**

Equations (8) and (9) can be solved by using the Laplace transform to yield (assuming zero initial conditions)

\[
\tilde{m}(x, s) = \frac{\tilde{m}_2(s)}{\sinh(\lambda_b x)} \sinh(\lambda_b x) + \frac{\tilde{m}_1(s)}{\sinh(\lambda_b L)} \sinh(\lambda_b(L - x))
\]

Equation (20) is the basic transfer-function model of a regenerator which accounts for the dynamic responses of the regenerator between the two boundaries.

**Basic transfer-function model of regenerator**

Equations (13) and (14) represent a distributed-parameter dynamics model accounting for the spatial variation of mass flow rate and pressure waves along the regenerator. It is preferable to write the dynamics model in terms of the boundary conditions of the regenerator and in a transfer-function form. Substituting Equation (13) into the Laplace form of Equation (8) and Equation (14) into the Laplace form of Equation (9) and letting \( x = 0 \), we obtain

\[
\tilde{m}_2(s) = [\cosh(\lambda_b L)] \tilde{m}_1(s) - \left[ \frac{SC_F \sinh(\lambda_b L)}{\lambda_b} \right] \tilde{p}_1(s)
\]

\[
\tilde{p}_2(s) = -\left[ \frac{(L_F s + R_F)}{\lambda_b} \right] \tilde{m}_1(s) + [\cosh(\lambda_b L)] \tilde{p}_1(s)
\]

Hence, Equations (18) and (19) can be written in a matrix form

\[
\begin{bmatrix}
\hat{m}_2(s) \\
\hat{p}_2(s)
\end{bmatrix} =
\begin{bmatrix}
\cosh(\lambda_b L) & -sC_F \sinh(\lambda_b L) \\
\frac{sC_F \sinh(\lambda_b L)}{\lambda_b} & \cosh(\lambda_b L)
\end{bmatrix}
\begin{bmatrix}
\hat{m}_1(s) \\
\hat{p}_1(s)
\end{bmatrix}
\]

Equation (20) is the basic transfer-function model of a regenerator in a cyclic flow system.

**System dynamics model of regenerator in cyclic flow system**

The dynamic response of a regenerator in a cyclic flow system which consists of a reciprocating piston and a reservoir (Figure 1) will be studied here. For convenience, Equation (18) is rearranged to yield

\[
\tilde{m}_1(s) = \frac{sC_F \sinh(\lambda_b L)}{\lambda_b \cosh(\lambda_b L)} \tilde{p}_1(s) + \frac{\tilde{m}_2(s)}{\cosh(\lambda_b L)}
\]

Substituting Equation (14) into the Laplace form of Equation (9) and letting \( x = L \) will yield

\[
\tilde{p}_2(s) = -\frac{\tilde{p}_1(s)}{\cosh(\lambda_b L)} - \left[ \frac{L_F s + R_F \sinh(\lambda_b L)}{\lambda_b \cosh(\lambda_b L)} \right] \tilde{m}_2(s)
\]

We define location 1 of the regenerator at the piston side and the location 2 at the reservoir side. The dynamics of a regenerator connected in a cyclic flow system can be described by three transfer-function models, where \( G_{pp}(s) \) represents the response of the pressure wave at the reservoir-side end induced by the pressure wave at the piston-side end and, since \( \tilde{p}_2(s) = 0 \) for a reservoir

\[
G_{pp}(s) = \frac{\tilde{p}_2(s)}{\tilde{p}_1(s)} = 0
\]

On the other hand, from Equation (22), for \( \tilde{p}_2(s) = 0 \)

\[
\tilde{p}_1(s) = \left[ \frac{L_F s + R_F \sinh(\lambda_b L)}{\lambda_b} \right] \tilde{m}_2(s)
\]

therefore, combining Equations (24) and (21), we obtain

\[
G_{mn}(s) = \frac{\tilde{m}_2(s)}{\tilde{m}_1(s)} = \frac{1}{\cosh(\lambda_b L)}
\]

where \( G_{mn}(s) \) represents the response of the mass flow rate wave at the reservoir-side end induced by the piston-side mass flow rate wave.

The response of the mass flow rate wave at the reservoir-side end induced by the pressure wave at the piston-side end is represented by the transfer-function model \( G_{mp}(s) \) which is derived from Equation (24)

\[
G_{mp}(s) = \frac{\tilde{m}_2(s)}{\tilde{p}_1(s)} = \frac{1}{L_F s + R_F \sinh(\lambda_b L)}
\]
For convenience, Equation (26) can be written as [since \( \bar{p}_2(s) = 0 \) for a regenerator connected to a reservoir]

\[
G_{mp}(s) = \frac{\bar{m}_1(s)}{\bar{p}_1(s) - \bar{p}_2(s)} \Delta \bar{p}(s) = \frac{1}{L_2 s + R_2 \sinh(\lambda_2 L)} \cdot \bar{m}_1(s)
\]  

(27)

Equation (27) can be considered to be a dynamic form of Darcy's law which is used to describe the dynamic responses of a regenerator in the cyclic flow system.

**Specifications of cyclic flow system**

The regenerator used in the present analysis has a diameter \( D = 40 \text{ mm} \) and a length \( L = 30 \text{ mm} \), which are the same as the values in the regenerator used by Tanaka et al.\(^7\) in an oscillating flow test. Four kinds of screen mesh wire are adopted in the analysis: 150, 200, 250, and 400 mesh. The physical properties of the regenerators are listed in Table 1, where the porosities are calculated using Chang's formula\(^10\) and the parameters \( \alpha \) and \( \beta \) in the momentum Equation (2) are converted from the friction correlation for oscillatory flow tests by Tanaka et al.\(^7\)

\[
f_h = \frac{A}{Re_h} + B
\]  

(28)

where \( A = 175 \) and \( B = 1.60 \), according to Tanaka et al.\(^7\), and \( Re_h(= \frac{u_{\text{max}} d_h}{v}) \) is the Reynolds number based on a regenerator hydraulic diameter \( d_h \) and a peak velocity of gas inside the regenerator void volume \( u_{\text{max}} \)

\[
d_h = \frac{4ed_m}{\phi(1 - \epsilon)}
\]  

(29)

\( \phi = 4 \) for screen wire meshes and \( f_h \) is the friction factor defined as

\[
f_h = \frac{\Delta \rho d_h}{\rho u_{\text{max}}^2 L/2}
\]  

(30)

where \( \Delta \rho (= p_1 - p_2) \) and \( u_{\text{max}} \) are, respectively, the pressure drop across the regenerator and the peak local velocity.

The relation between parameters \( \alpha \) and \( \beta \) and the modified parameter \( \alpha' \) is derived as

\[
\alpha' = \alpha + \frac{\beta}{v} u_{\text{max}} = \alpha + \frac{\beta}{d_h} Re_h
\]  

(31)

where \( \alpha = A/2d_h^2 \) and \( \beta = B/2d_h \).

The peak velocity \( u_{\text{max}} \) is related to the driving mechanism and rotation speed of the piston. For the crank case used by Tanaka et al.\(^7\), \( u_{\text{max}} \) follows the relation

\[
u_{\text{max}} = \frac{1.042 \pi V_p N}{60A \epsilon} = \frac{1.042 \pi V_p f}{A \epsilon}
\]  

(32)

Table 1: Regenerator properties

<table>
<thead>
<tr>
<th>Wire diameter ( d_m ) (mm)</th>
<th>Porosity ( \epsilon )</th>
<th>Hydraulic diameter ( d_h ) (mm)</th>
<th>( \alpha ) (mm(^{-2}))</th>
<th>( \beta ) (mm(^{-1}))</th>
</tr>
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<tbody>
<tr>
<td>0.06604</td>
<td>0.05334</td>
<td>0.04064</td>
<td>0.02540</td>
<td>0.0554</td>
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where \( V_p \) is the swept volume of piston. The parameter \( \alpha' \) modified for a cyclic mass flow rate thus becomes

\[
\tilde{\alpha} = \alpha + \frac{\beta}{v} \frac{1.042 \pi V_p f}{A \epsilon}
\]  

(33)

It is noted that the modified parameter \( \tilde{\alpha} \) is a function of the frequency of cyclic flow rate or piston motion.

**Frequency responses of regenerator in cyclic flow system**

From Equation (33) \( \tilde{\alpha} \) is seen to be a function of \( \alpha, \beta, v, f, \epsilon \), etc.; while \( \alpha \) and \( \beta \) are a function of \( d_h \) alone, according to Equation (31). Since \( d_h \) is a function of screen wire diameter \( d_m \) and porosity \( \epsilon \) of the regenerator, which are determined by the standard screen wire mesh size, \( \tilde{\alpha} \) is seen to be a function of standard screen wire mesh size, gas viscosity \( v \) and operating frequency.

**Figure 2** shows the values of \( \tilde{\alpha} \) determined at different Reynolds numbers \( Re_h \) and oscillation frequencies of the piston \( f \) for helium gas in four generators packed with different screen wire meshes. \( \tilde{\alpha} \) values for air, nitrogen and helium gases with a 200 mesh screen are shown in **Figure 3**.

**The use of air or nitrogen as the working fluid in a regenerator is seen to introduce a much greater pressure loss, about 5 times more than that using helium gas.**

**Frequency response of \( G_{mp}(s) \)**

The frequency response of flow rate fluctuation at the reservoir side \( \bar{m}_2 \) can be seen from the Bode plots of \( G_{mp}(j\omega) \) as shown in **Figure 4**. It is seen that both the gain and the phase tend to decrease more rapidly with increasing frequency for the regenerator with a dense wire screen mesh. This means that the attenuation of flow rate fluctuation at the reservoir-side end will be stronger for regenerators with larger wire screen mesh (dense

**Table 1** Regenerator properties

<table>
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<tr>
<th>Wire diameter ( d_m ) (mm)</th>
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<th>Hydraulic diameter ( d_h ) (mm)</th>
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</tbody>
</table>

Cryogenics 1993 Vol 33, No 11 1049
The effect of the working fluid on $G_{mm}(s)$ is seen to be quite strong from Figure 5. For air and nitrogen gases, the magnitude and phase of $G_{mm}(j\omega)$ drop more rapidly than with helium gas. The cut-off frequency $f_{cut}$ or bandwidth is reduced two-fold for air and nitrogen gas. However, the phase lag at $f_{cut}$ still remains constant ($-57.66^\circ$).

The attenuation and phase shift of flow rates at the two ends of the regenerator can be clearly seen from Figure 6, which is drawn for 400 mesh regenerator operating at 100 Hz with helium gas. $m_2$ is seen to lag behind $m_1$ by $57.66^\circ$. Table 3 shows that the bandwidth of the regenerator for air or nitrogen in the 200 mesh regenerator is much narrower than for helium gas. However, the corresponding phase shifts at $f_{cut}$ are the same, at $-57.66^\circ$. This indicates that helium gas is a superior working fluid to air or nitrogen gas.

Table 2 Cut-off frequency of $G_{mm}(j\omega)$ for helium gas

<table>
<thead>
<tr>
<th>Wire screen mesh number</th>
<th>$f_{cut}$ (Hz)</th>
<th>Phase angle at $f_{cut}$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>304</td>
<td>$-57.66$</td>
</tr>
<tr>
<td>200</td>
<td>234</td>
<td>$-57.66$</td>
</tr>
<tr>
<td>250</td>
<td>205</td>
<td>$-57.66$</td>
</tr>
<tr>
<td>400</td>
<td>90</td>
<td>$-57.66$</td>
</tr>
</tbody>
</table>

Table 3 Cut-off frequency of $G_{mm}(j\omega)$ for 200 mesh regenerator

<table>
<thead>
<tr>
<th>Working fluid</th>
<th>$f_{cut}$ (Hz)</th>
<th>Phase angle at $f_{cut}$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>103</td>
<td>$-57.66$</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>104</td>
<td>$-57.66$</td>
</tr>
<tr>
<td>Helium</td>
<td>234</td>
<td>$-57.66$</td>
</tr>
</tbody>
</table>
Dynamic response of regenerator: B.J. Huang and C.W. Lu

Frequency response of \( G_{mp}(s) \)

The frequency response of flow rate fluctuation at the reservoir-side end \( \dot{m}_2 \) induced by a pressure wave across the regenerator \( \Delta p \) can be seen from the Bode plots of \( G_{mp}(j\omega) \) for helium gas, as shown in Figure 7. Both the gain and the phase angle are seen to decrease more rapidly with increasing frequency for a loose wire screen mesh. The bandwidth \( f_{cut} \) and the corresponding phase angle \( \phi_{cut} \) are shown in Table 4. It is seen that the bandwidth increases with increasing mesh number. This is due to the fact that for a loose regenerator, the flow capacitance effect is stronger and causes a greater attenuation effect (with narrower bandwidth) on the flow rate fluctuation in response to the pressure waves across the regenerator.

An example of the time response of the reservoir-side mass flow rate \( \dot{m}_2 \) induced by a pressure drop \( \Delta p = (p_1 - p_2) \) for a 400 mesh regenerator at 100 Hz is shown in Figure 9.

It is quite interesting to see that Equation (27) will be reduced to a constant gain of \( \Delta p/(\dot{m} El) \) and zero phase shift at very low frequencies \((\omega \to 0)\), that is

\[
\lim_{\omega \to 0} |G_{mp}(j\omega)| = \frac{1}{R_t}
\]

where \( R_t \) is the total flow resistance of the regenerator, defined as

\[
R_t = \frac{\dot{m} E_l}{\Delta p} = R_p L
\]

This shows that the regenerator will obey Darcy's law at low frequencies. That is, the quasi-steady assumption of Darcy's law holds for the reservoir-side flow rate attenuation induced by the pressure drop across the regenerator, to within a bandwidth approximately. However, a small phase shift in the reservoir-side flow rate response exists, as shown in Table 4, and has to be corrected if the quasi-steady assumption of Darcy's law is used.

It is interesting to note that the phase lag increases abruptly from 250 to 400 mesh. The 400 mesh regenerator has a wider bandwidth but a larger phase lag. On the other hand, the 150 mesh regenerator has a narrower bandwidth and almost no phase lag, so that the quasi-steady assumption can hold approximately for both the magnitude and phase shift.

Table 5 shows that for the 200 mesh regenerator, the bandwidth of air or nitrogen for \( G_{mp}(j\omega) \) is much narrower than for helium gas. However, the corresponding phase lag at \( f_{cut} \) is smaller for air or nitrogen. This verifies that the use of helium gas as a working fluid is superior to air or nitrogen gas since the bandwidth is much wider and the phase lag is smaller.

---

**Table 4** Cut-off frequency of \( G_{mp}(j\omega) \) for helium gas

<table>
<thead>
<tr>
<th>Wire screen mesh number</th>
<th>( f_{cut} ) (Hz)</th>
<th>Phase angle at ( f_{cut} ) (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>48</td>
<td>-1.47</td>
</tr>
<tr>
<td>200</td>
<td>65</td>
<td>-3.70</td>
</tr>
<tr>
<td>250</td>
<td>74</td>
<td>-5.60</td>
</tr>
<tr>
<td>400</td>
<td>115</td>
<td>-30.0</td>
</tr>
</tbody>
</table>

**Table 5** Cut-off frequency of \( G_{mp}(j\omega) \) for 200 mesh regenerator

<table>
<thead>
<tr>
<th>Working fluid</th>
<th>( f_{cut} ) (Hz)</th>
<th>Phase angle at ( f_{cut} ) (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>9</td>
<td>-0.5</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>10</td>
<td>-0.6</td>
</tr>
<tr>
<td>Helium</td>
<td>65</td>
<td>-3.7</td>
</tr>
</tbody>
</table>
Discussion and conclusions

A basic transfer-function model for isothermal and cyclic flow in a regenerator is derived and used to study the system dynamics of a regenerator connecting a reciprocating piston and a reservoir via Bode plots.

The frequency response of the mass flow rate at the reservoir side of the regenerator induced by the pressure and mass flow rate waves at the piston side are shown to approach zero at high frequencies, but approach a constant value in gain at low frequencies so that a quasi-steady approximation can hold. The cut-off frequency \( f_{cut} \) which defines the bandwidth depends on the regenerator design (wire mesh size, porosity, length, diameter, material, etc.). \( f_{cut} \) for the flow rate to flow rate response \( G_{mm}(j\omega) \) decreases with increasing mesh number of the regenerator. This indicates that the attenuation of mass flow rate fluctuation is stronger for regenerators with a lower void volume (i.e. more dense screen). The use of air or nitrogen gas will introduce stronger attenuation than helium. However, the phase angle at \( f_{cut} \) remains constant irrespective of screen mesh numbers and working fluids.

For the reservoir-side flow rate response induced by the pressure waves \( G_{mp}(j\omega) \) the cut-off frequency or bandwidth \( f_{cut} \) increases with increasing regenerator mesh number. This indicates that the attenuation of mass flow rate fluctuation is stronger for regenerators with a larger void volume (i.e. loose screen). The use of air or nitrogen gas will introduce a stronger attenuation than helium. However, a small phase lag exists for the reservoir-side flow rate response and needs to be corrected if the quasi-steady assumption of Darcy’s law is used.

Regenerators used in Stirling machines or cryocoolers are usually operated under transient or cyclic flow conditions. The thermal performance analysis is thus quite complicated since transient solutions of unsteady state models are involved. The present analysis can be used to simplify the modelling of the regenerator by using a quasi-steady approximation, according to the operating frequency of the machine and the bandwidth of the regenerator. A system dynamics model of the regenerator similar to Equation (27) can be easily derived according to the present analysis for analysis of a real Stirling cryocooler. However, the pressure fluctuation at the reservoir side will be non-zero [i.e. \( p_2(s) \neq 0 \)] due to the introduction of another reciprocating displacer and a cold head in a Stirling cryocooler.

The basic transfer-function model of a regenerator [Equation (20)] derived in the present study can also be used to develop linear network models of Stirling or pulse tube cryocoolers by connecting similar models one by one for each component of the machine, in a similar way to Guo\(^{11}\). The thermal performance prediction of cryocoolers can then be simplified a great deal.

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