On the Autonomous Gain and Phase Tailoring Transfer Functions of Symmetrically Distributed Piezoelectric Sensors

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A new design concept for a distributed sensor, which was developed based on the principle that the strain distribution of an arbitrary finite plate structure can be expressed as the superposition of even and odd strain functions, is presented. The distributed sensors adopt a symmetric weighting electrode to match the symmetric distribution of the even parts of the strain in order to introduce a no-phase delay low-pass filter to tailor the sensor transfer function. Both the design concept and the experimental results are detailed herein. [DOI: 10.1115/1.1804994]

Introduction

The concept of using a piezoelectric distributed sensor to tailor sensor transfer functions in the field of flexible structure control [1,2] has been extensively investigated since the 1980s. Some of the most representative studies include modal sensors [3] and spatial filters [4–6]. These types of distributed sensors take advantage of the distributed nature of a piezoelectric lamina to offer a weighting function to the measured sensor signals induced by the structure motion. For example, the modal sensor uses a shaped electrode to match the distribution of a modal strain to pick up only prespecified modal information in order to overcome a spillover problem [6]. On the other hand, spatial filters offer a no-phase delay low-pass filter to the sensor transfer function by applying a spatially weighting function to the propagating waves. The difference between a modal sensor and a spatial filter is that a modal sensor matches a specific mode of a finite structure, while spatial filters, developed in the 1990s, offer the possibility to create a no-phase delay, gain tailored operation to sensors defined within an infinitely long beam structure. The limitation of modal sensors is that they need to be redesigned for different finite structures as the mode shapes are different for different boundary conditions and it is relatively difficult to implement for high-order modes because complex mode shapes and many alternating poling sections exist. In comparison, spatial filters must be placed on an approximately infinite, long beam structure and placed far from the boundaries in order to eliminate both the effects of boundaries and evanescent waves because spatial filters are unable to deal with such effects. Collins et al. noted that contributions of the evanescent waves, which emanate from the boundaries and other forced locations [4,5], can be eliminated by placing the spatial filters away from these locations by a distance of at least $\lambda/2\pi$, with λ as the wavelength. In this approach, the sensor equation related to the spatial filter only considers the propagating waves, which leads to a mathematical design tool based on the two-sided Laplace transform. In addition, no-phase delay filters have been proven possible in a spatial domain using this method [4,5] as long as the sensors are placed far from the boundaries and away from the forced conditions as mentioned above. Collins et al. also showed that spatial filters suffer additional phase information

from the propagating and evanescent waves generated by an actuator placed near it (not included as part of the original model and theory) [5].

The abovementioned limitations are the underlying reasons why distributed sensors have not been as popular as point sensors. Besides having to be redesigned for different structures, the realization of a modal sensor is also limited by the existence of nonlinear characteristics and a damping effect. Clark and Burke investigated the practical limitations toward achieving modal sensors with theoretical and experimental verifications [7]. Their results show that a modal sensor will experience some spillover in the sensor transfer function even for the case where there is the presence of only a slight mismatch to the desired modal strain. To overcome these shortcomings, Mukherjee and Joshi [8] developed a design procedure to identify the optimal spatial distribution of the distributed sensor to match the desired modal information with a finite element method [9] that details how to import structure characteristics into computer codes in the design process so as to minimize the abovementioned limitations of the modal sensors. Nonetheless, all of these methodologies face some difficulties for implementation to practical and commercial applications.

In comparison, adopting a spatial filter seems to be easier and the design thinking coincides with that of traditional controller implementations, which provides a smoother integration with the mainstream thoughts of today's control engineers. Nevertheless, the requirements of having to place spatial filters far from the boundaries of an approximated infinite beam structure impose serious limitations on its practical use. To circumvent the limitations of spatial filters, Hsu and Lee invented a series distributed sensors denoted as an Autonomous Phase-gain ROTation/linear Piezoelectric Optimal Sensing (*APROPOS*) device [10,11], which adopts mathematical tools, such as the *method of image* and *weighting function*, to expand structure wave modes into infinite domains and which offers zero weighting at the boundaries. This approach takes into account the boundary effects as well as the influence of evanescent waves, which were previously neglected in spatial filters, therefore, the *APROPOS* device has proven to have the ability to introduce a no-phase delay filtering effect for cases, such as small structures, long wavelength domains, and locations near or on the boundaries: Although the *APROPOS* device had made significant advancement toward the development of distributed sensors, the requirement of zero weighting on the boundaries hinders many applications. In addition to using shaped distributed sensors to perform signal processing in a spatial domain similar to that of the abovementioned sensors, shaped sensors were also used to

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Contributed by the Technical Committee on Vibration and Sound for publication in the JOURNAL OF VIBRATION AND ACOUSTICS. Manuscript received June 24, 2004; final revision June 24, 2004. Associate Editor: Chin An Tan.

minimize power consumption [12]. It should be noted that the spatial signal-processing methodology also holds for cases where inappropriate shape function is involved [13].

In this article, a design concept for distributed sensors that is based on matching the symmetric part of strain waves in a finite structure is detailed. This symmetrically distributed sensor was developed by adopting the simple fact that all mathematical functions can be divided into an even function and an odd function. By applying spatial integration to piezoelectric lamina, we can utilize the mathematical principle that integrates over a real domain for cases where the integrand is a product of two even functions or two odd functions and which always leads to a real-value function. On the other hand, integration over a real domain when the integrand is a product of an even function and an odd function leads to zero. Thus, it can be shown that symmetrically distributed sensors provide a symmetric weighting to the strain distribution that leads to a no-phase delay low-pass filter into the sensor transfer function, similar to the *APROPOS* device concept mentioned above. To denote the property that can introduce a no-phase delay low-pass filter to the sensor transfer function similar to that of the *APROPOS* device, this new class of sensor is denoted as a Symmetric *APROPOS* device. It can be shown that a Symmetric-*APROPOS*-device design concept provides us with a straightforward approach that allows it to be placed anywhere on a finite plate structure under all types of boundary conditions. The beauty of this device is that it need not operate like the *APROPOS* device reported by Hsu and Lee [10,11] earlier, which required incorporating mathematical concepts, such as *method of image* and *weighting function*, to deal with the boundary effects and the evanescent waves. In addition, it can also act as a point sensor that can measure the local information of a specific point on a structure with some predefined prefiltering actions. Due to the simplicity of its implementation, Symmetric *APROPOS* devices can be adopted for many practical cases. More specifically, one can be attached directly onto a finite structure without knowing the characteristics of the structure beforehand, which is a key feature of traditional point sensors. Both theoretical derivations and experimental results are detailed herein.

Theory of Piezoelectric Laminates

The discovery of piezoelectric effect by Pierre and Jacques Curie in 1880 [14] triggered many important studies and applications. Piezoelectric materials possess a property of generating charges on the surface electrode when undergoing deformation. In addition, a mechanical strain is induced when an electrical field is applied to the piezoelectric material. These reciprocal relationships can be clearly identified by using the constitutive equations written in terms of the IEEE compact matrix notation [15]

$$S_p = s_{pq}^E T_q + d_{ip} E_i \quad (1a)$$

$$D_i = d_{ip} T_q + \varepsilon_{ij}^T E_j \quad (1b)$$

where $i, j = 1 \sim 3$ and $p, q = 1 \sim 6$. The first constitutive equation (1a) shows that both a mechanical stress T_q and an electrical field E_i can induce a mechanical strain S_p through the elastic compliance constant s_{pq}^E and the piezoelectric constant d_{ip} , respectively. Equation (1b) shows that both the mechanical stress T_q and electrical field E_j can induce an electrical displacement D_i through the piezoelectric constant d_{ip} and the permittivity constant ε_{ij}^T , respectively. Note that the superscripts E and T indicate that the material constants are obtained when the E or T fields are held constant. A polymeric piezoelectric material polyvinylidene fluoride (PVF₂), first discovered in 1969 by Kawai [16], was chosen as the piezoelectric thin sheet to verify the concepts to be presented in this paper. As PVF₂ possesses mm2 symmetry [16,17], its piezoelectric constant can be seen is

$$\begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{24} & 0 & 0 \\ d_{31} & d_{32} & d_{33} & 0 & 0 & 0 \end{bmatrix} \quad (2)$$

Since the distributed sensor in this article used a 52 μm thick PVF₂ thin sheet attached onto a 200 μm thick standard steel, which served as the flexible structure, a plane stress approximation was adopted [17,18]. The surface electrodes can thus be merely placed on the x_1 - x_2 plane and only the charges generated along the x_3 direction is measured. Considering the case where the principal axes of the piezoelectric lamina and that of the attached structure coincide, the constitutive equation (1b) can be reduced to

$$D_3 = \varepsilon_{33}^T E_3 + d_{31} P_0(x, y) T_1 + d_{32} P_0(x, y) T_2 \quad (3)$$

where $P_0(x, y)$ is the relative polarization strength of the piezoelectric lamina used with a value between 1 and -1 . The PVF₂ thin sheet used in this research was uniformly polarized throughout the thickness direction and where the relative polarization strength $P_0(x, y)$ was equal to one. Since the PVF₂ thin sheet used as the sensor can only detect charges generated from a thickness direction, Gauss' Law was adopted to obtain the closed-circuit charge signals detected by the surface electrodes [18]. The charge signals generated by the k th lamina of a three-layered PVF₂-shim metal-PVF₂ laminate with distance z_k to the midplane is [18]

$$q_k(t) = \int_S R(x, y) \left(e_{31} \frac{\partial u}{\partial x} + e_{32} \frac{\partial v}{\partial y} + \varepsilon_{33}^T E_3 \right) dx dy - z_k \int_S R(x, y) \left(e_{31} \frac{\partial^2 w}{\partial x^2} + e_{32} \frac{\partial^2 w}{\partial y^2} \right) dx dy \quad (4)$$

where $R(x, y)$ represents the effective surface electrode, u and v are the in-plane displacement fields in the x and y directions, and w is the out-of-plane displacement field. Note that the effective surface electrode $R(x, y)$ can offer additional design freedom. It is obvious from Eq. (4) that the effective surface electrode can offer a weighting function to structure strain for both in-plane and out-of-plane motions.

To verify the concept of a Symmetric *APROPOS* device, a one-dimensional plate was used where the principal axes of the attached piezoelectric lamina coincides with the plate. The sensor equation (4) can be further reduced to [3,18]

$$q_k(t) = -z_k e_{31} \int_{-a}^a R(x) \frac{\partial^2 w}{\partial x^2} dx \quad (5)$$

where $-a$ and a are the boundaries of the PVF₂ thin sheet placed on the one-dimensional plate, with length $2a$ and origin at the middle point of the PVF₂ thin sheet where it is placed on the one-dimensional plate. This is the basic sensor equation of both modal sensors and spatial filters. In other words, one of the main design concepts for all distributed sensors is to match the strain distribution of the structure. The modal sensor matches the modal strain of a specific modal shape of a finite structure [3]. The spatial filter offers a filter in the spatial domain based on the propagating waves in an approximately infinite, long flexible beam structure [4]. In the next section, it will be detailed how the Symmetric *APROPOS* device can offer a spatial-filtering effect without introducing any phase distortion in the sensor transfer function by using a symmetrically distributed piezoelectric sensor made of a piezoelectric thin film.

Concept of a Symmetric *APROPOS* Device

The concept of the Symmetric *APROPOS* device was developed based on the fact that any mathematical function can be expressed as the superposition of an even function and an odd function. In mechanical detection cases, the structure strain distribution of interest is separated into an even and an odd function.

That is, the strain distribution with respect to an arbitrarily defined location, which is set as the origin on a deformed structure, becomes [19]

$$\varepsilon(x) = \varepsilon_e(x) + \varepsilon_o(x) \quad (6)$$

where $\varepsilon_e(x)$ and $\varepsilon_o(x)$ are the corresponding even and odd functions of the strain $\varepsilon(x)$. Recalling that the sensor equations of distributed sensors are the integrals of the structure strain weighted by the effective surface electrode $R(x)$ [see Eq. (5)], the sensor equation can be expressed as

$$\int_{-a}^a R(x)\varepsilon(x)dx = \int_{-a}^a R(x)[\varepsilon_e(x) + \varepsilon_o(x)]dx \quad (7)$$

The integration range is from $-a$ to a , which is the area covered by the distributed sensor. For this type of integral, if the integrand is a product of two even functions or two odd functions, a real-value function will result. If the transfer function measured by using a specific sensor is a real value, only a 0° and/or 180° phase change will occur. If the integrand is the product of an even and an odd function, the integral from $-a$ to a as shown in Eq. (7) leads to zero. The properties of symmetric functions in integration had previously been adopted to spatial filters [4,5]. It had been mentioned by Collins et al. that the dynamic expression of a spatial filter can be simplified by using a symmetric pattern, where this symmetric pattern would make sure the transfer function of the two-sided Laplace transform was a real value and symmetrically distributed in a wave-number domain. Although the spatial filter design attempts to take advantage of the merits of symmetric functions, it fails to incorporate the contributions of boundaries and evanescent waves into the design based on a two-sided Laplace transform.

Taking into account all of the above arguments on integrals of odd and even functions and the fact that the effective surface electrode $R(x)$ can also be split into an even function $R_e(x)$ and an odd function $R_o(x)$ [Eq. (7)], we find that

$$\int_{-a}^a R(x)\varepsilon(x)dx = \int_{-a}^a R_e(x)\varepsilon_e(x)dx + \int_{-a}^a R_o(x)\varepsilon_o(x)dx \quad (8)$$

It is clear from Eq. (8) that the even and odd parts of the effective surface electrode $R(x)$ match the even and odd functions of the strain $\varepsilon(x)$, respectively. More specifically, if the effective surface electrode $R(x)$ is an even function $R_e(x)$ [i.e., $R_e(x) = R_e(-x)$], only the symmetric strain $\varepsilon_e(x)$ will be measured and the resultant of the integral will be a real-value number. Similarly, an odd-function effective surface electrode $R_o(x)$ [i.e., $R_o(x) = -R_o(-x)$] will merely measure the anti-symmetric $\varepsilon_o(x)$ and the integral resultant will also be a real-value number [19]. This concept is very straightforward and does not require taking into consideration conditions, such as zero weighting at the boundaries as those of the *APROPOS* device [10,11]. More specifically, the design of a Symmetric *APROPOS* device is based on matching even and odd parts of the strain function with a symmetric or anti-symmetric effective surface electrode so as to pick up the matching information of the apparent strain distribution. More specifically, by choosing the shape of the weighting function $R(x)$ as either an even or an odd function, only one of the two terms on the right-hand side of Eq. (8) will remain. That is, the evaluation of the right-hand side of Eq. (8) will be a single real value (i.e., the phase changes can either be 0° or 180° , but not both). When we look at Eqs. (7) and (8), it appears that this symmetrically distributed sensor does not resolve the influence of a phase change as a real-value number changes from 0° to 180° . This undesirable phase uncertainty arises from the fact that the shape functions of both symmetrically and anti-symmetrically distributed sensors can contain either positive or negative parts as determined by its positive or negative poling direction. More specifically, a symmetrically or anti-symmetrically distributed sensor can then disturb the

phase distribution of the testing structure in a certain frequency range (i.e., several 180° phase jumps will occur). To eliminate this phase uncertainty, a Symmetric *APROPOS* device must be designed without having positive and negative parts (i.e., all poling directions of the piezoelectric sensor under the Symmetric *APROPOS* device need to remain identical). Similarly, an anti-Symmetric *APROPOS* device will have an identical poling direction within the whole positive segment. It is to be noted that the poling direction of the negative segments, which are on opposite sides of the targeted origin, is opposite to that of the positive segments. Adopting this design criterion, the phase of the Symmetric *APROPOS* device or anti-Symmetric device is determined solely by the monitored strain distribution and, thus, will not be influenced by the design of the effective surface electrode. That is, no 180° -phase shift or jump will appear in the symmetrically distributed sensor. The abovementioned no-phase delay associated with the gain-tailoring behavior achieved by using a Symmetric *APROPOS* device of the same polarization profile is called an autonomous gain-phase behavior.

In the following sections, the Symmetric *APROPOS* device applied to a fourth-order structure system was studied theoretically and experimentally to demonstrate this concept.

Sensor Equation of a Symmetric *APROPOS* Device

The governing equation of a one-dimensional plate can be rewritten as [20]

$$D_{11}\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A \frac{\partial^2 w(x,t)}{\partial t^2} = 0 \quad (9)$$

where D_{11} is the flexural rigidity of the composite plate made by a three-layered PVF₂ shim metal-PVF₂ laminate and ρA is the product of density ρ and the cross-sectional area of plate A . There are two ways to solve the general solution. The first one is by using the characteristic polynomials to consider the time-harmonic excitation cases [i.e., $w(x,t) = w(x)e^{j\omega t}$] [4]. The characteristic polynomials can be viewed as the superposition of the fundamental waves (i.e., propagating and evanescent waves). More specifically, Eq. (9) can be rewritten as

$$\frac{\partial^4 w}{\partial x^4} - k^4 w(x) = 0 \quad (10)$$

where

$$k^4 = \frac{\rho A}{D_{11}} \omega^2 \quad (11)$$

is the dispersion relationship between the wave number k and angular frequency ω . The general solution $w(x,t)$ for Eqs. (9)–(11) can be expressed as [10,20]

$$w(x,t) = [w_{lp}e^{ikx} + w_{le}e^{kx} + w_{rp}e^{-ikx} + w_{re}e^{-kx}]e^{j\omega t} \quad (12)$$

where w_{lp} and w_{rp} are the wave-mode amplitudes of the left- and right-propagating wave modes, w_{le} and w_{re} are the wave-mode amplitudes of the evanescent wave modes, and $j = (-1)^{1/2}$. The second approach to solving the differential equation is to take the normal mode expansion, which considers the displacement field as [20,21]

$$w(x,t) = \sum_{i=1}^{\infty} \phi_i(x)A_i(t) \quad (13)$$

where $\phi_i(x)$ and $A_i(t)$ are the mode shape and modal coordinate of mode i . The general solution can be expressed as

$$w(x,t) = \sum_{i=0}^{\infty} \{ [A_i \cosh(\bar{k}_i x) + B_i \sinh(\bar{k}_i x) + C_i \cos(\bar{k}_i x) + D_i \sin(\bar{k}_i x)] A_i(t) \} \quad (14)$$

where A_i , B_i , C_i , and D_i are the amplitudes for i th mode determined by the boundary conditions. Note that the wave mode amplitudes of the evanescent and propagating waves shown in Eq. (12) are functions of wave number k_i only and the structure characteristics are embedded in the amplitudes [10]. Mathematically, the characteristic polynomial uses the four complex wave-mode amplitudes, which are functions of wave number k , to shift the phase of these propagating and evanescent waves to match the boundary conditions. That is, the characteristic polynomial represents the structure characteristics at the wave-number domain, and thus, it can be converted back to the frequency domain by using the dispersion relationship shown in Eq. (11). On the other hand, the amplitudes for normal mode expansion are real-value numbers (i.e., the imaginary part is zero for each mode). The mathematical meaning of a normal mode expansion is to multiply the proper constants to a series of real-value functions to match the boundary conditions. The characteristic of the structure is thus embedded in the modal coordinates $A_i(t)$ for mode i . Since only this series of modal shapes can exist in this finite structure, its motion must be the superposition of these mode shapes (i.e., the physical meaning of a normal mode expansion). One thing that should be noted is that the hyperbolic sine and hyperbolic cosine functions in Eq. (14) are contributed by the evanescent waves, while the sine and cosine functions are from the propagating waves.

Substituting Eq. (14) into the sensor equation of the distributed sensor on a one-dimensional plate [Eq. (5)] leads to

$$q_k(t) = -z_k e_{31} A(t) \int_{-a}^a R(x) \sum_{i=1}^{\infty} (\phi_{ie}'' + \phi_{ip}'') dx \quad (15)$$

where $\phi_{ip} = C_i \cos(\bar{k}_i x) + D_i \sin(\bar{k}_i x)$ and $\phi_{ie} = A_i \cosh(\bar{k}_i x) + B_i \sinh(\bar{k}_i x)$ represent the displacement fields contributing from the evanescent and propagating waves, respectively. The sensor equations from an evenly distributed effective surface electrode $R_e(x)$ or an oddly distributed function $R_o(x)$ are

$$q_e(t) = -z_k e_{31} A(t) \int_{-a}^a R_e(x) \sum_{i=1}^{\infty} (\phi_{ie}'' + \phi_{ip}'')_e dx \quad (16a)$$

$$q_o(t) = -z_k e_{31} A(t) \int_{-a}^a R_o(x) \sum_{i=1}^{\infty} (\phi_{ie}'' + \phi_{ip}'')_o dx \quad (16b)$$

where the subscript e and o represent whether the effective surface electrode is either even or odd.

For a symmetric structure, it is well known that the modal shape is alternatively symmetric and anti-symmetric (i.e., if the odd modes are symmetric with respect to the middle point, then the even modes will be anti-symmetric). For example, a free-free beam and simply supported beam both satisfy these conditions [22]. Symmetric *APROPOS* devices are generally applicable to these types of structures. The sensor equation of a Symmetric *APROPOS* device on a cantilever plate, which is not symmetric with its boundary conditions, is chosen as an example to demonstrate that the Symmetric *APROPOS* device concept can be applied to a case more general than symmetric structures. The details of this example can be seen in the following section.

Sensor Equation of a Symmetric *APROPOS* Device on a Cantilever Plate

The general solution of a one-dimensional plate in terms of a normal mode expansion is [22]

$$w(x, t) = \sum_{i=0}^{\infty} \{ [\cosh(\bar{k}_i x) - \sigma_i \sinh(\bar{k}_i x) - \cos(\bar{k}_i x) + \sigma_i \sin(\bar{k}_i x)] A_i(t) \} \quad (17)$$

where σ_i is the resultant wave mode amplitude after being normalized. Figure 1 is a normalized schematic of a modal strain

Fig. 1 Schematic of a modal strain distribution for a one-dimensional plate in terms of (a) $\phi_i''(x)$, (b) $\phi_{ie}''(x)$, and (c) $\phi_{ip}''(x)$

expressed in terms of (a) $\phi_i''(x)$, (b) $\phi_{ie}''(x)$, and (c) $\phi_{ip}''(x)$ for a one-dimensional cantilever plate with the origin placed at the middle of the plate, where $\phi_i''(x) = \phi_{ie}''(x) + \phi_{ip}''(x)$ is the modal strain of the i th mode as indicated in Eqs. (13) and (14). Note that the strain $\phi_{ie}''(x)$ is alternatively symmetric and anti-symmetric, with respect to odd and even modes in Fig. 1(b). On the contrary, strain $\phi_{ip}''(x)$ is alternatively anti-symmetric and symmetric with respect to odd and even modes in Fig. 1(c). This alternating behavior shows that symmetric and anti-symmetric parts of the modal strains expressed with respect to middle point of a cantilever plate are actually $\phi_{ie}''(x)$ and $\phi_{ip}''(x)$. For odd modes, the strains $\phi_{ie}''(x)$ and $\phi_{ip}''(x)$ are the even function and the odd function, respectively. On the other hand, the strains $\phi_{ie}''(x)$ and $\phi_{ip}''(x)$ are the odd function and the even function for even modes.

Considering Fig. 1(a), it is clear that modal shapes are very complicated at higher order modes. This is the reason why modal sensors [3] have not been used at higher-order modes. On the other hand, evanescent waves cannot be neglected for cases where either the strain of interest is located near the boundaries or where there are low wave numbers [see Fig. 1(b)]. This limitation is the

Fig. 2 Schematic of a Symmetric APROPOS device with respect to the first mode

reason why spatial filters must be placed far from the boundaries or are used only with high wave numbers [4,5]. It should be mentioned that when we discuss low wave numbers, we are comparing it characteristically to wave numbers of lower modes that can be transferred to a frequency domain by using a dispersion relationship as shown in Eq. (11). That is, the finite structure can only be approximated as an infinite structure at higher-order modes in a traditional spatial-filter design. This is the reason why a traditional spatial-filter design can only be adopted in cases where the sensor of interest is much smaller than the structure and is placed at locations far from the boundaries. In comparison, a Symmetric APROPOS device concept can be adopted to finite structures.

It was mentioned above that symmetric and anti-symmetric parts of strain distributions with respect to middle points of a cantilever plate are strain induced by $\phi''_{ie}(x)$ and $\phi''_{ip}(x)$. A Symmetric APROPOS device that matches the symmetric strain at this symmetric point is detailed herein to clearly show how it works, although there is seldom a need to have the origin of the Symmetric APROPOS device at the middle of the cantilever plate, in practical cases. The strain distribution of this middle point can certainly be separated into a combination of an even function and an odd function, which reveals the design advantage of a Symmetric APROPOS device and where the user need not be concerned about the placement of the sensor and can apply it directly without suffering from any additional phase information.

Figure 2 is a schematic of a Symmetric APROPOS device that matches the symmetric distribution of the strain $\phi''_{ie}(x)$ and rejects the anti-symmetric distribution of the strain $\phi''_{ip}(x)$ of the first mode (i.e., ϕ''_{ip}). On the contrary, an anti-symmetric sensor will match the anti-symmetric distribution of the strain $\phi''_{ip}(x)$ and thereby reject the symmetric distribution of the strain $\phi''_{ie}(x)$ of the first mode (i.e., ϕ''_{ie}). An anti-symmetric APROPOS device can be easily implemented by reverse poling or by flipping over the piezoelectric thin film. In this approach, the Symmetric APROPOS device can offer a weighting function to the magnitude of the modal strain without introducing any phase delay.

Figure 3 shows one of the possibilities of using a Symmetric APROPOS device to offer a no-phase delay low-pass filter to the sensor transfer function in a spatial domain. Recalling that the sensor equation of piezoelectric sensors [Eq. (5)] is a surface integral with respect to strain distribution in a spatial domain; the physical effects of Fig. 3 clearly indicate that spatially distributed strain information becomes insignificant at higher-order modes due to cancellation with respect to the tenth propagating strain wave as shown in Fig. 3. In comparison, the variation of the strain waves is slow at the lower-order modes and, therefore, leads to fewer cancellations (see the first propagating strain wave in Fig. 3). This effect implies that distributed sensors have the ability to offer some sort of low-pass filter in a spatial domain. Nevertheless, this simple explanation does not lead to a no-phase lead

Fig. 3 Schematic of a Symmetric APROPOS device with respect to the first mode (thick line) and tenth mode (thin line)

or a no-lag behavior. It is only with the introduction of a Symmetric APROPOS device and the integral property of distributed sensors that a no-phase delay low-pass filter becomes possible. The distributed sensor shown in shaded area in Fig. 3 is one such example.

Figure 4 is a schematic of the concept of the Symmetric APROPOS device, where $G_0(s)$ is the original transfer function of the cantilever plate (thick gray line), $G_s(s)$ is the sensor transfer function that has been tailored by the Symmetric APROPOS device (thin dark line), and $F_l(s)$ is the transfer function of a low-pass filter (dashed-dot line). In the case where the low-pass filter is made by using an electrical filter, an additional phase delay will inevitably be introduced to the sensor transfer function $G_E(s)$ (dashed line). Similar to what was reported about spatial filters [4,5], modal sensors [3], APROPOS devices [10,11], such as the Symmetric APROPOS device presented in this article, provide us with an easy alternative toward implementing a "signal-processing filter" that does not suffer from phase delay when gain tailoring is implemented. More specifically, if a no-phase delay low-pass filter can be developed by using any of the distributed sensor approaches mentioned above, the phase distribution of the

Fig. 4 Effect of a symmetric distributed sensor using a Bode Plot, where $G_0(s)$ is the original transfer function of the cantilever plate (thick gray line), $G_s(s)$ is the sensor transfer function that has been tailored by the Symmetric APROPOS device (thin dark line), $F_l(s)$ is the transfer function of a low-pass filter (dashed-dot line), and $G_E(s)$ is the phase distribution of the sensor transfer function filtered by an electrical filter (dashed line)

Fig. 5 (a) Schematic of a symmetric piezoelectric distributed sensor, (b) Schematic of the effect of a Symmetric APROPOS device

sensor transfer function will remain the same (thin dark line). Examining this issue from this signal-processing perspective, provides us with a powerful design freedom, achievable by using the abovementioned spatial domain approaches to enhance phase margin, usable bandwidth, sensitivity, and frequency selectivity of a piezoelectric system. In other words, this line of thought provides us with a way to circumvent the limitation imposed by the Bode-gain-phase theorem, which clearly dictates the gain and phase relationship for all noncausal systems. Therefore, we know that by using a Symmetric APROPOS device to control a flexible structure, the phase margin can be easily enhanced. In other words, a no-phase delay compensator can be induced into the spatial domain before the control loop. In summary, in the case where a Symmetric APROPOS device matches the symmetric property of strain waves, a no-phase delay low-pass filter can be introduced. This symmetrically distributed sensor can be used with finite structures and for low wave-number regimes. It should be mentioned that the Symmetric APROPOS device can be placed at any location on a finite one-dimensional plate structure directly. In next section, a Symmetric APROPOS device made by an exponential function will be detailed.

Implementation of a Symmetrically Distributed Piezoelectric Sensor

Consider the case of a Symmetric APROPOS device developed by using an evenly distributed effective surface electrode with respect to the origin located at the middle point of the cantilever plate [see Fig. 5(a)]. It should be noted that the Symmetric APROPOS device was also implemented symmetrically with respect to the y-axis to eliminate the rotation behavior of the cantilever plate [3]. Since the symmetric exponential function $e^{-\alpha|x|}$ has been proven to offer a second-order pole in a wave-number domain and in one order pole frequency domain by Hsu and Lee

in 2002 [10,11], this exponential function was used as the basic spatial function of the Symmetric APROPOS device, i.e.,

$$R_e(x) = e^{-\alpha|x|} [H(x+a) - H(x-a)] \quad (18)$$

where the Heaviside step function is used to illustrate the Symmetric APROPOS device is a finite distributed sensor within distance $2a$. This sensor is placed between $-a$ to a , with length $2a$, and where α is the prespecified corner wave number of the no-phase delay low-pass filter. Substituting this Symmetric APROPOS device and the displacement field of the one-dimensional cantilever plate [Eq. (17)] into the sensor equation (15), the sensor transfer function becomes

$$q(t) = -z_k e_{31} \sum_{i=1}^{\infty} A_i(t) \left\{ \frac{\alpha \bar{\phi}_{ip}}{\alpha^2 + k_i^2} + \frac{\alpha \bar{\phi}_{ie}}{\alpha^2 - k_i^2} \right\} \quad (19)$$

where

$$\begin{aligned} \bar{\phi}_{ie} &= 2\phi''_{ie}(0) - \alpha e^{-\alpha a} [\phi'_{ie}(a) + \phi'_{ie}(-a)] \\ &\quad - k_i^2 e^{-\alpha a} [\phi'_{ie}(a) - \phi'_{ie}(-a)] \end{aligned} \quad (20a)$$

$$\begin{aligned} \bar{\phi}_{ip} &= 2\phi''_{ip}(0) - \alpha e^{-\alpha a} [\phi''_{ip}(a) + \phi''_{ip}(-a)] \\ &\quad + k_i^2 e^{-\alpha a} [\phi'_{ip}(a) - \phi'_{ip}(-a)] \end{aligned} \quad (20b)$$

It is shown in Fig. 1(b) that the strain $\phi''_{ie}(x)$ is symmetric and anti-symmetric for odd and even modes with respect to the middle point (i.e., the origin). Since both the integral and the derivative of an even or an odd function become odd or even, respectively [23], and the fields that represent the evanescent bending angle deformation are the integral of the corresponding evanescent strain fields ϕ''_{ie} , they will be anti-symmetric and symmetric for odd and even modes with respect to the middle point. On the other hand, the propagating bending angle fields ϕ'_{ip} are symmetric and anti-symmetric for odd and even modes with respect to the middle point. It is obvious from Eqs. (19) and (20) that this Symmetric APROPOS device will reject the strain ϕ''_{ip} and the displacement ϕ'_{ip} in an odd mode, and in the case of an even mode, ϕ''_{ie} and ϕ'_{ie} will be rejected by this Symmetric APROPOS device. More specifically, the first and second terms of Eq. (20a), i.e., $\phi''_{ie}(0)$ and $[\phi'_{ie}(a) + \phi'_{ie}(-a)]$, will be zero because the evanescent strain ϕ''_{ie} is an odd function, and the third term $[\phi'_{ie}(a) - \phi'_{ie}(-a)]$ is also zero as the evanescent bending angle is an even function. On the contrary, the contributions of the odd mode in Eq. (20b) are zeros due to the distribution of the anti-symmetric propagation strain ϕ''_{ip} and symmetric propagation bending angle ϕ'_{ip} . The sensor equations of this Symmetric APROPOS device for odd and even modes can then be written as

$$q_{\text{odd mode}}(t) = -z_k e_{31} \sum_{i=1}^{\infty} A_i(t) \frac{\alpha \bar{\phi}_{ie}}{\alpha^2 - k_i^2} \quad (21a)$$

$$q_{\text{even mode}}(t) = -z_k e_{31} \sum_{i=1}^{\infty} A_i(t) \frac{\alpha \bar{\phi}_{ip}}{\alpha^2 + k_i^2} \quad (21b)$$

Substituting the dispersion relationship of a one-dimensional plate [Eq. (11)], Eq. (19) can be further transferred to the frequency domain

$$q(t) = z_k e_{31} \left(\frac{D_{11}}{\rho A} \right)^{1/2} \sum_{i=1}^{\infty} A_i(t) \frac{\alpha}{\bar{\alpha} + \omega} \{ \bar{\phi}_{ip} + \bar{\phi}_{ie} \} \quad (22)$$

where $\bar{\alpha}$ is the transferred corner frequency. This is a -20 dB/decade no-phase delay low-pass filter without any phase delay in the frequency domain. Note that the strain of the origin $\phi''_i(0)$ will

be measured in the sensor transfer function [see Eq. (20)]. Thus the Symmetric *APROPOS* device is a distributed sensor that can monitor the local characteristics of a specific point on the attached structure, which is identical to the targeted origin used in *APROPOS* devices [10]. In the next section, it will be shown that the Symmetric *APROPOS* device is the only point sensor that can really measure the strain information of a specific point on a one-dimensional plate structure. Consider again Eq. (20), where the second and the third terms are the local information at the boundaries of the Symmetric *APROPOS* device, we see that the Symmetric *APROPOS* device measures the local characteristics of the origin coupled with the no-phase delay filtering effects induced by the sensor and the boundary effects. Figure 5(b) shows the characteristics of this Symmetric *APROPOS* device expressed in Eq. (22), where the symmetrically shaped decaying electrode $e^{-\alpha|x|}$ offers a no-phase filtering effect $G(\omega) = \alpha/(\alpha + \omega)$ in a frequency domain. On the other hand, the boundaries of this distributed sensors that measures the structure characteristics of these two local locations (ϕ''_{ie} , ϕ''_{ip} , ϕ'_{ie} , and ϕ'_{ip} at $x=a$ and $-a$) and the middle of the distributed sensor [$\phi''_{ie}(0) + \phi''_{ip}(0)$], which is the targeted origin and retrieves the strain information of this specific point, are the two primary contributing factors for this Symmetric *APROPOS* device. The general solution related to the charge induced by the Symmetric *APROPOS* device $q_s(t)$ can thus be expressed from Eq. (16a) as

$$q_s(t) = z_k e_{31} \left(\frac{D_{11}}{\rho A} \right)^{1/2} \sum_{i=1}^{\infty} A_i(t) \{ \phi''_i(0) + [B_e(a, t) + B_e(-a, t)] G(\omega) \} \quad (23a)$$

where $G(\omega)$ is the induced no-phase delay low-pass filter, and $B_e(a, t)$ and $B_e(-a, t)$ are the measured symmetric part of the strain on the boundary of the sensor. Similarly, the general solution related to the charge induced by the anti-symmetric *APROPOS* device can be expressed from (16b) as

$$q_a(t) = z_k e_{31} \left(\frac{D_{11}}{\rho A} \right)^{1/2} \sum_{i=1}^{\infty} A_i(t) \{ \phi''_i(0) + [B_o(a, t) + B_o(-a, t)] G(\omega) \}, \quad (23b)$$

where $B_o(a, t)$ and $B_o(-a, t)$ are the measured anti-symmetric part of the strain on the boundary of the sensor. Note that both the symmetric and anti-symmetric *APROPOS* device can monitor the local strain of the targeted origin $\phi''_i(0)$. This is due to the fact that both the symmetric and anti-symmetric parts of the strain are defined with respect to the origin. More specifically, only the origin is the location that cannot be separated into an even function and/or an odd function.

It should be mentioned that the even and odd functions of the strain with respect to an arbitrary location on a structure cannot be identical to the one at the middle point of the cantilever plate because the normal modes are only alternatively even and odd at the middle point. Nevertheless, this alternating even-and-odd-function phenomenon also holds for nodal and anti-nodal points of modal strain distribution. For example, one quarter of the full length of a cantilever plate is a nodal point for the third mode and can also satisfy the abovementioned conditions.

In summary, in the case where the Symmetric *APROPOS* device matches the symmetric part of the modal strain, a no-phase delay low-pass filter can be introduced. When operating in a finite domain, a Symmetric *APROPOS* device does not offer zero weighting at the boundaries of the finite structure. In the next section, the concept of expanding a symmetric *APROPOS* device concept to a new and innovative point sensor will be detailed.

Symmetric *APROPOS* Device as a Point Sensor

Considering Eq. (20a), which indicates that a Symmetric *APROPOS* device can monitor the local strain at the origin, we see

that the term $2\phi''_{ie}(0)$ is much larger than the sum of the second and third terms $\alpha e^{-\alpha a} [\phi''_{ie}(a) + \phi''_{ie}(-a)] - k_i^2 e^{-\alpha a} [\phi'_{ie}(a) - \phi'_{ie}(-a)]$, which is a type of point sensor. More specifically, if a is small when compared to the wavelength of interest [see Fig. 5(b)], then $\phi''_{ie}(0)$, $\phi''_{ie}(a)$, and $\phi''_{ie}(-a)$ are of the same order per Fig. 3. That is, $2\phi''_{ie}(0) \gg \alpha e^{-\alpha a} [\phi''_{ie}(a) + \phi''_{ie}(-a)] - k_i^2 e^{-\alpha a} [\phi'_{ie}(a) - \phi'_{ie}(-a)]$ will hold if $e^{-\alpha a} \ll 2$ or $\alpha \gg -\ln(2)/a$. Similarly, this condition holds when considering the propagating wave case shown in Eq. (20b). The sensor equation of the Symmetric *APROPOS* device can then be simplified to

$$q(t) = z_k e_{31} \left(\frac{D_{11}}{\rho A} \right)^{1/2} \sum_{i=1}^{\infty} A_i(t) \phi''_i(0) \frac{\alpha}{\alpha + \omega} \quad (24)$$

which is a distributed sensor that can measure the local strain of the origin coupled with a no-phase delay low-pass filter. This is identical to the concept of the targeted origin in an *APROPOS* device. The only limitation is that the Symmetric *APROPOS* device cannot measure the local characteristic at the boundaries of a finite structure.

In comparison with an *APROPOS* device [10], a Symmetric *APROPOS* device does not require adopting the concept of *method of image* or the superposition of a distributed sensor to offer zero weighting at the boundaries of the effective surface electrode as well as mapping the domain to an infinite domain. The Symmetric *APROPOS* device is more straightforward, as only a symmetric effective surface electrode is needed. An *APROPOS* device (which is different from a Symmetric *APROPOS* device), does not require that $\alpha \gg -\ln(2)/a$ because in an *APROPOS* device, the spatial weighting is zero at the boundaries. Physically, the requirement of the Symmetric *APROPOS* device (i.e., $e^{-\alpha a} \ll 2$), is reduced to the spatial weighting at the boundaries of the effective surface electrode. Thus, the strain information of the targeted origin can be emphasized against that of the boundaries due to the induced spatial weighting factor. It should be noted that this concept also holds for anti-symmetric *APROPOS* devices because they also measure the local characteristics of the targeted origin and monitor the anti-symmetric part of the structure information at their boundaries [Eq. (23b)]. Consider the case where the anti-symmetric *APROPOS* device monitors the local strain. This is quite similar to that of Symmetric *APROPOS* devices because the condition $e^{-\alpha a} \ll 2$ holds. Physically, this condition corresponds to the case where the spatial weighting at the boundaries is small when compared to the targeted origin, and the sensor is small in size when compared to the wavelength. In this situation, the sensor equation shows that an anti-symmetric *APROPOS* device, acting like a point sensor, is identical to Eq. (24), which is also identical to a symmetric *APROPOS* device.

Traditionally, a small uniform sensor is treated as a point sensor that measures the local information of the location placement [24]. The various commercialized strain gages are the most representative products [25]. However, these types of sensors are limited to the assumption that the strain covered by this type of point sensor is uniform. This rule will not apply if the strain distribution is not uniform. This effect can be easily shown in the following example.

Consider a uniform piezoelectric sensor with its effective surface electrode as

$$R(x) = H(x+a) - H(x-a) \quad (25)$$

where the origin of the uniform sensor is placed at the middle point of a cantilever plate. Substituting Eq. (25) into Eq. (15), we obtain

$$q(t) = -z_k e_{31} \sum_{i=1}^{\infty} A_i(t) \{ [\phi'_{ie}(b) - \phi'_{ie}(-b)] + [\phi'_{ip}(b) - \phi'_{ip}(-b)] \} \quad (26)$$

This equation shows that this uniform sensor measures the angle difference between the boundaries. That is, the uniform sensor matches the anti-symmetric mode in terms of angle and can be perceived as a clear representation of the concept of a Symmetric *APROPOS* device. Note that the uniform sensor does not really measure the actual local strain information of a single point. It measures the difference of the bending angles of its boundaries, which equals the strain only in the case where uniform strain is present. More specifically, a traditional strain gage is not really a sensor that measures local strain in a rigorous mechanical sense. It is a sensor that measures the difference of the bending angles in the cantilever case as mentioned here. It should be noted that this uniform sensor does not introduce any additional filtering effect to the transfer function of the structure. It is clear from the comparison of Eqs. (23) and (26) that the Symmetric *APROPOS* device mentioned above matches the strain information of the structure and the uniform sensor measures the difference between bending angles at the boundaries. In order to make a fair comparison, a constant $20 \times \log_{10}(k)$ must be added to the uniform sensor transfer function as expressed in a wave-number domain. From the dispersion relationship [i.e., Eq. (11)], the constant becomes $10 \times \log_{10}(\omega)$ if the transfer function expressed in the frequency domain is to be compared as a result of the dispersion relationship. With the introduction of the abovementioned constants, the theoretical prediction related to the no-phase delay low-pass filter introduced between the Symmetric *APROPOS* device and the uniform sensor can then be experimentally verified herein.

Experiment Setup

To experimentally verify the concept of a Symmetric *APROPOS* device, a 140 mm (L) $\times$ 22 mm (W) $\times$ 0.02 mm thick stainless steel shim in a one-dimensional cantilever plate configuration was chosen as the test bed for the finite structure configuration. Two layers of 52 μ m thick PVF₂ laminae with their electrical ground connections facing outside [3] were attached to each side of the cantilever plate. Coaxial cables were used to eliminate the influence of EMI (electromagnetic interference) in order to obtain experimental data of a good signal-to-noise ratio. As the normal mode expansions of a one-dimensional cantilever plate can be directly separated into even and odd functions with respect to the middle point, the axes and origin of choice are shown in Fig. 5(a). A Symmetric *APROPOS* device, designed by considering the structural base, was attached to one side of the structure by

$$R(x) = e^{-4|x|} [H(x+0.05) - H(x-0.05)] \quad (27)$$

which serves as the effective surface electrode. The corner wave number was chosen to be 4, which is lower than the characteristic wave number of the first mode. The length of the Symmetric *APROPOS* device was 100 mm long. The other side was a uniform sensor with

$$R(x) = H(x+0.07) - H(x-0.07) \quad (28)$$

which, as the effective surface electrode, is attached to the other side of the stainless steel shim to serve as the comparative sensor. The experiment setup and sensor placement are shown in Figs. 5(a) and 6. The photonic sensor [26] and an accelerometer were used to measure the excitation of the shaker at a low- and high-frequency regime, respectively. The sensor transfer function of the uniform sensor was measured by using the accelerometer to measure the excitation of the shaker as the reference signal. The transfer function obtained is shown as the gray line in Fig. 7. We took the resonance frequency of the uniform sensor as the reference to retrieve all related material properties (Fig. 7). For example, the material property $D_{11}/\rho A$ was found to be 0.101846 and the corner frequency of the low-pass filter was then calculated to be 5

Fig. 6 Experiment setup of a symmetric piezoelectric sensor

Hz. Looking at Eq. (22), the transfer function of the symmetric *APROPOS* device expressed in a frequency domain becomes

$$q(t) = -0.319133 z_k e_{31} \sum_{i=1}^{\infty} A_i(t) \frac{4}{5+\omega} \{ \bar{\phi}_{ie} + \bar{\phi}_{ip} \} \quad (29)$$

Figure 7 shows the transfer function of the Symmetric *APROPOS* device (black line) and the uniform sensor (gray line), both of which are interfaced with a charge amplifier as the interface circuit and where the accelerometer output is used as the reference signal. Note that $10 \times \log_{10}(\omega)$ was added to the transfer function of the uniform sensor to make a fair comparison due to the difference in physical quantities measured. It is obvious from Fig. 7 that a no-phase delay low-pass filter can be successfully implemented into a sensor transfer function.

To further experimentally verify a no-phase delay low-pass filter as induced by the design of a Symmetric *APROPOS* device, the transfer function was measured with respect to the uniform sensor directly. Figure 8 shows the ratio between the transfer function of the symmetric *APROPOS* device and that of the uniform sensor, which cancel out the structure transfer function. This clearly demonstrates the filtering effect of the Symmetric *APROPOS* device. The interface circuits of the uniform sensor and Symmetric *APROPOS* device were chosen to be a charge amplifier and current amplifier due to the availability of the in-

Fig. 7 Transfer function of a symmetric distributed sensor (dark line) versus an uniform sensor (gray line)

Fig. 8 Transfer function of a no-phase delay low-pass filter introduced by a symmetric piezoelectric distributed sensor

struments at hand. The experimental results obtained are shown in Fig. 8. Since the corner frequency was 5 Hz, the transfer function was measured by using two frequency bands to clearly demonstrate the data. From 1 to 10 Hz, a 100 points measurement with sensitivity 2 nA/V was used as the current amplifier. From 10 to 100 Hz, a 500 points measurement with sensitivity 100 nA/V was used as the current amplifier. A 33.9794 dB was added from 10 to 100 Hz to eliminate the difference between the sensitivities of the interface circuit. As the current amplifier induces a one-order high pass filter to the transfer function when compared to that of the charge amplifier, a $20 \times \log_{10}(\omega)$ and 90° must be added by multiplying the transfer function. In addition, $10 \times \log_{10}(\omega)$ was added to the numerator of the transfer function to transfer the uniform sensor output so as to have a compatible physical quantity when compared to that of a symmetric *APROPOS* device. Thus, a $-10 \times \log_{10}(\omega)$ and -90° must be added to the gain and the phase of the transfer function of the uniform sensor to compensate for the additional gain and phase induced by the current amplifier. Note that the obtained data clearly demonstrates that there is a no phase delay introduced by this low-pass filter (i.e., the symmetric *APROPOS* device can successfully be implemented to finite structures of low wave numbers).

Conclusions

Both theoretical and experimental results were obtained to verify the concept of Symmetric *APROPOS* devices. A Symmetric *APROPOS* device circumvents the limitations faced by modal sensors, spatial filters, and *APROPOS* devices. Combining the understanding that any strain function can be separated into even and odd functions and that corresponding mathematical theorems relate to integrals of even and odd functions across a symmetric region with respect to the origin, Symmetric *APROPOS* devices are shown to be applicable to any finite one-dimensional plate structure, even for cases with low frequency ranges. By matching symmetric and anti-symmetric properties of a strain distribution, a no-phase delay low-pass filter can be easily implemented to the sensor transfer function. This newly developed concept is applicable directly to different one-dimensional plate structures and can certainly be extended to other finite structures without neglecting the contributions of evanescent waves. Applying this newly developed concept to the field of flexible structure control, Symmetric *APROPOS* devices can increase the gain margin of the control loop. The stability of the system can then be easily enhanced. On the other hand, Symmetric *APROPOS* devices can serve as point sensors that measure the local structure information

of the structure. It has been proven that symmetric piezoelectric distributed sensors can distinguish evanescent and propagating waves by matching symmetric and anti-symmetric properties of these strain waves. In summary, for a finite one-dimensional plate structure, Symmetric *APROPOS* devices can introduce a no-phase delay low-pass filter to the sensor transfer function by matching symmetric properties of the strain waves at any location on the structure of interest except at its boundaries.

Acknowledgments

The authors would like to thank the continuous and generous financial support of the National Science Council of Taiwan, R.O.C., through Grants No. NSC 85-2622-E-002-017R, No. NSC 86-2622-E-002-023R, No. NSC 88-2218-E-002-005, and No. NSC 88-2622-E-002-001. The authors would also like to express their appreciation to Measurement Specialties, Sensor Products Division (MSI) for unconditionally providing all the PVF₂ films used in fabricating the piezoelectric devices discussed in this research work.

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