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Targeted origin placement for the autonomous gain–phase tailoring of piezoelectric sensors

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Abstract

Targeted origin placement can be shown to compliment mathematical tools such as the image method, window functions and two-sided Laplace transformation for designing a whole new class of piezoelectric sensors that can be used to tailor the gain and phase portions of the sensor transfer functions independently. It can be demonstrated that by properly applying the boundary conditions and the method of imaging, the targeted origin of this new class of sensor design can be traversed anywhere along the surface of the testing structure. In addition, the active sensor concept that integrates classical control theory with piezoelectric sensors can be adapted to further advance sensor technology. The design concept, theoretical derivations, numerical calculations and experimental verifications of implementing this class of new sensor design to influence flexible structure control and point sensor design methodologies are detailed.

1. Introduction

Metrology techniques that can be used to evaluate or improve the performance of flexible structures have been actively developed for the last two decades. Both piezoelectric point sensors and distributed sensors have been widely implemented to sense structure performance. The benefit of a point sensor is based on the fact that a user can take full advantage of its performance without knowing the full details of the sensor design, making a point sensor easy to use. As performance of point sensors is tightly related to sensor structure, the limitations of point sensors are typically influenced by the characteristics of sensor structures. Considering the influence of gain and phase, the usable bandwidth of traditional point sensors is typically no more than one-tenth to one-fifth of the first-mode resonant frequency if linearity and accuracy of better than 5–10% are desired. Taking into consideration that mode shapes or many structure vibration characteristics are defined once a structure is defined, distributed piezoelectric sensors and actuators have gone through significant technological advancement to optimize the overall performance over the last two decades [1–4]. Many innovative piezoelectric distributed sensors have been developed over the last two decades [5,6], which can tailor the

structure transfer function so as to influence distributed sensor transfer functions. As distributed sensors must be re-designed for each structure, applications of distributed sensors have not been as popular as point sensors.

In this paper, a new perspective that removes the apparent barrier between distributed and point piezoelectric sensors will be introduced. As autonomous gain–phase tailoring can be achieved by modifying the sensor transfer functions by adopting the various methodologies developed herein, this class of sensor was called APROPOS devices [7–9], where APROPOS is an acronym for autonomous phase–gain rotation/linear piezoelectric optimal sensing. How this new class of sensors can circumvent the apparent limitations imposed by the Bode gain–phase theorem [10, 11] will be examined briefly. It can be shown that an APROPOS device has a positive impact on both flexible structure control and on improving traditional point sensors. Methodologies that can be used to circumvent the limitations of the previously reported spatial filters [6], which were designed by considering propagation waves only and neglected the influence of evanescent waves such that spatial filters cannot be implemented in the case where testing structures cannot be considered as infinite when compared with the sensor size, are to be presented as well. For example, the method of

imaging was used first to expand the finite sensor structure to an infinite domain, which then permitted us to properly take the boundary conditions, the propagation waves and the evanescent waves into consideration. Secondly, the influence of the targeted origin, the effect of the various effective surface electrode shapes and the role of the interfacing circuits [5] was then examined in detail. On the other hand, it is known that piezoelectric material possesses a reciprocal relationship between mechanical deformation and the electric field; the above-mentioned sensor design concepts are equally suitable for actuator design. The classical active sensor design concepts that adopt control theory to tailor passive sensor transfer function are shown to be applicable to further advanced APROPOS device technology.

2. Theory of piezoelectric laminates

The piezoelectric effect, which was discovered by Pierre Curie and Jacques Curie [12], relates the conversion between electric and mechanical energies. Applying mechanical stress on these materials leads to a charge on the surface. In addition, strain can be generated by simply applying an external electric field and is characterized by a reciprocal relationship of piezoelectric materials.

2.1. Piezoelectric sensor equation

In 1969, Kawai [13] discovered that the piezoelectric polymer polyvinylidene fluoride (PVF₂) possesses approximately ten times more piezoelectric strength than more traditional piezoelectric polymers. PVF₂ has a characteristic of a *mm*2 symmetry and thus has only five different stress constants, d_{31} , d_{32} , d_{33} , d_{15} and d_{24} . Due to its flexibility, PVF₂ was widely adopted for studying flexible structure control. The constitutive equations of piezoelectric materials written in an IEEE compact matrix notation are [5, 14, 15]

$$T_p = c_{pq}^E S_q - e_{kp} E_k \quad D_i = e_{ip} S_q + \varepsilon_{ik}^S E_k, \quad (1)$$

and

$$S_p = s_{pq}^E T_q + d_{ip} E_i \quad D_i = d_{ip} T_q + \varepsilon_{ij}^T E_k, \quad (2)$$

where $i, k = 1-3$, $p, q = 1-6$, T_p , S_q are stresses and strains respectively, D_i is the electric displacement, E_k is the electric field, c_{pq} is the elastic stiffness constant, ε_{ij} is the permittivity of the piezoelectric material, d_{ip} is the piezoelectric strain/charge constant and e_{ip} is the piezoelectric stress/charge constant. As only thin piezoelectric laminae applied to plates are examined in this paper, only a charge generated along the direction of thickness D_3 is measurable externally. The constitutive equation (equation (2)) can be rewritten as [5, 15]

$$D_3 = \varepsilon_{33}^0 E_3 + d_{31}^0 P_0(x', y') T_1 + d_{32}^0 P_0(x', y') T_2 + d_{36}^0 P_0(x', y') T_6, \quad (3)$$

where the notation “ $'$ ” represents that the constant is defined with respect to the materials coordinate axes, and d_{36} is generated if the materials coordinate axes do not coincide with the attached structure principal axes. The relative polarization strength of the piezoelectric lamina $P_0(x', y')$ has values equal

Figure 1. Schematic diagram of a PoD sensor designed by using an APROPOS device.

to or less than one. The superscript “ 0 ” indicates the constant was obtained when $P_0(x', y') = 1$. From Gauss’s law, the closed-circuit charge signal $q_k(t)$ generated from the surface electrode of the k th lamina as strain appears can be expressed as [5, 15]

$$q_k(t) = -z_k^0 \int_S F P_0 \left[e_{31}^0 \frac{\partial^2 w}{\partial x^2} + e_{32}^0 \frac{\partial^2 w}{\partial y^2} + 2e_{36}^0 \frac{\partial^2 w}{\partial x \partial y} \right] dx dy, \quad (4)$$

where $k = 1$ or 3 in our experiments, z_k^0 is the distance between the neutral axis of the laminated plate and the middle of the k th lamina, S is the surface area covered by the piezoelectric material, $w = w(x, y)$ is the bending displacement of the laminated plate used and $F = F(x, y)$ is the effective surface electrode for the two-dimensional structure. Considering a one-dimensional cantilever plate [5, 15], the sensor equation in (4) can be reduced to

$$q_k(t) = -z_k^0 e_{31}^0 \int_S R(x) \frac{\partial^2 w}{\partial x^2} dx, \quad (5)$$

where $R(x)$ is the effective surface electrode for a one-dimensional structure and can be shown as

$$R(x) = \int_{-b/2}^{b/2} F(x, y) P_0(x, y) dy. \quad (6)$$

Selecting an appropriate effective surface electrode $R(x)$ forms the foundation of many innovative distributed sensors in the last two decades. For example, a modal sensor [5, 16] or a spatial filter [6] can be implemented by setting the effective surface electrode equal to some appropriate mathematical functions. It can be shown herein that the APROPOS device developed in this article is derived from similar mathematical foundations.

2.2. Wave modes in a one-dimensional plate

The governing equation of a one-dimensional cantilever plate formed by laminating a piece of 52 μm thick piezoelectric polymer lamina made of PVF₂ [13] on top of a 200 μm thick stainless steel shim (figure 1), assuming that the internal damping effect is negligible, can be expressed as [5, 15, 16]

$$D_{11} \frac{\partial^4 w(x, t)}{\partial x^4} + \rho A \frac{\partial^2 w(x, t)}{\partial t^2} = 0 \quad (7)$$

where D_{11} , ρ and A represent the flexural rigidity, density and cross-section of the plate, respectively. As the mechanical stiffness of the PVF₂ thin lamina is much smaller than that of a stainless steel shim plate, its mechanical stiffness can be neglected. Considering a cantilever plate with a fixed end at $x = -a$, a free end at $x = a$ and the external vibration applied from the fixed end, the boundary conditions of this plate can be expressed as

$$w(x = -a) = g e^{j\omega t}, \quad (8a)$$

$$\partial w(x = -a) / \partial x = 0, \quad (8b)$$

$$\partial^2 w(x = a) / \partial x^2 = 0, \quad (8c)$$

$$\partial^3 w(x = a) / \partial x^3 = 0. \quad (8d)$$

Considering only the harmonic structural response $w(x, t) = w(x) e^{j\omega t}$ of equation (7), this leads to

$$\frac{\partial^4 w}{\partial x^4} - k^4 w(x) = 0, \quad (9)$$

where

$$k^4 = \frac{\rho A}{D_{11}} \omega^2 \quad (10)$$

is the dispersion relationship, $j = \sqrt{-1}$, k is the wavenumber and ω is the frequency. The general solution of equations (7)–(9) can be expressed as [5]

$$w(x, t) = [w_{lp} e^{jkx} + w_{le} e^{kx} + w_{rp} e^{-jkx} + w_{re} e^{-kx}] e^{j\omega t} \quad (11)$$

where w_{lp} and w_{rp} are the wave mode amplitudes of the left and right propagating wave modes; w_{le} and w_{re} are the wave mode amplitudes of the left and right evanescent wave modes. These four constants are a function of the wavenumber k only, and will have different magnitudes and forms for different boundaries. The combined effects of these wave modes and the boundaries form the fundamental characteristics of the structure. That is, the information of the boundary and the local characteristics are merged in these wave mode amplitudes. These four wave modes also determine the propagation of the wave modes, which in turn depends on the wavenumber k . One thing that should be mentioned is that the amplitudes of these four wave modes are not only dependent on the boundary conditions but also dependent on the location of the targeted origin of the effective surface electrode $R(x)$ to be pasted on top of the sensor structure, which in this case is the cantilever plate. Taking a one-dimensional cantilever plate as an example and setting the targeted origin at the middle of the cantilever plate, the solutions of the four wave mode amplitudes can be solved by substituting equation (10) into equation (11) first and then further simplifying the results using the boundary condition equation (8) to arrive at

$$w_{lp} = \left[\frac{[(j+1)e^{4ak} + j-1]e^{jak} + 2je^{3jak+2ak}}{8je^{2jak+2ak} + (2je^{4ak} + 2j)e^{4jak} + 2je^{4ak} + 2j} \right] g, \quad (12a)$$

$$w_{le} = \left[\frac{(j-1)e^{4jak+ak} + 2je^{2jak+3ak} + (j+1)e^{ak}}{8je^{2jak+2ak} + (2je^{4ak} + 2j)e^{4jak} + 2je^{4ak} + 2j} \right] g, \quad (12b)$$

$$w_{rp} = \left[\frac{[(j-1)e^{4ak} + j+1]e^{3jak} + 2je^{jak+2ak}}{8je^{2jak+2ak} + (2je^{4ak} + 2j)e^{4jak} + 2je^{4ak} + 2j} \right] g, \quad (12c)$$

$$w_{re} = \left[\frac{(j+1)e^{4jak+3ak} + 2je^{2jak+ak} + (j-1)e^{3ak}}{8je^{2jak+2ak} + (2je^{4ak} + 2j)e^{4jak} + 2je^{4ak} + 2j} \right] g, \quad (12d)$$

where a and $-a$ represent a free end and a fixed end respectively and g is the acceleration of gravity. It should be noted that all the denominators in equation (12) are the same, which represent the full field characteristics of the structure. More specifically, solving the equations made by equating the denominators of equation (12) to zero yields the natural frequency of each structure natural mode. On the other hand, the numerators are different for different wave modes, which means different wave modes contribute different effects on the structure. That is, substituting equation (12) into equation (10) and then setting x to zero yields the local characteristics of the targeted origin $x = 0$, which is the superposition of the four wave mode amplitudes. With the targeted origin being placed at different locations, the four wave mode amplitudes lead to completely different forms; i.e., the numerators of these wave mode amplitudes represent the local information of the structure.

Substituting equation (11) into equation (5) forms the basic sensor equation of the autonomous gain-phase sensors, i.e. the APROPOS device. That is,

$$q(k) = -z_k^0 e_{31} k^2 \int_{\bar{s}} R(x) [-w_{lp} e^{jkx} + w_{le} e^{kx} - w_{rp} e^{-jkx} + w_{re} e^{-kx}] dx. \quad (13)$$

When a spatial filter was first developed in 1990 [6], it was 0.7 m in size, was placed on a beam of length 7.3 m and was located far from the boundaries in order to avoid the influence of the evanescent wave mode and boundary effects. That is, the sensor equation in that formulation included the effect of the propagating wave modes only. More specifically, previous spatial filters [6] were constructed by considering only the two propagating wave modes in equation (11) by using a two-sided Laplace transform

$$L\{R(x); s\} = \int_{-\infty}^{\infty} R(x) e^{-sx} dx \quad (14)$$

where $s = jk$ or $-jk$. It was known that under the above-mentioned special conditions, a no-phase-delay low-pass filter could be introduced by choosing an appropriate effective surface electrode $R(x)$. Unfortunately, these previous spatial filters [6] tended to fail when (1) the structure was finite, (2) the desired sensing point was close to the boundary or (3) the frequency of interest was low. All these three failure conditions are related to the case where the influence of the evanescent wave and the boundary effects should not have been neglected, which really represents most sensor structures. The method of imaging will be introduced herein to develop the desired APROPOS-device-based sensors for the cases where the above-mentioned effects must be taken into consideration.

Figure 2. Expanding wave modes to an infinite domain by using the method of imaging, where the solid line represents the original sensor structure and the broken line represents the infinite structure extended by using the method of imaging.

2.3. Method of imaging

As waves arrive at the boundary, they are reflected and then carry the influence to the boundary. The most familiar boundary conditions are a free end and fixed end, both of which are presented in a cantilever beam shown in figure 1. By using the method of imaging, which has been well developed for years [17], the waves present within a finite structure can be expanded into an infinite domain as shown in figure 2. More specifically, the bold line in figure 2 represents the original waves in the original finite structure, and the broken line represents the wave that incorporates the influence of the boundary condition and is propagated in an expanded infinite domain. As the wave propagates to the free end, it reflects with the very same wave shape without any phase shift. On the other hand, as the wave propagates to the fixed end, it reflects a 180° phase delay.

In other words, the wave is continuous at the free end and a null at the fixed end. That is, waves must become even functions at the free ends and odd functions at the fixed ends. These effects can be observed from figure 3, where the free end and the fixed end were chosen as the targeted origin respectively. It can be shown later that following this generic characteristic of wave modes, a low-pass filter without phase delay can be implemented at arbitrary locations on the structures, i.e. the targeted origin can be set anywhere on the structures, by properly taking the boundary conditions into consideration. Detailed implementation procedures related to the design of the APROPOS device by the method of imaging are discussed herein.

3. Design concepts of APROPOS device by method of imaging

It is clear from figure 3 that we can view an APROPOS device as a kind of weighting function acting on the wave modes. That is, an APROPOS device exerts weighted influence on the waves propagating between the boundaries. Following these concepts, if the weighting exerted by the APROPOS device on a discontinuous boundary is zero [7,8], an APROPOS device will see a continuous wave in an infinite domain. In this case, the two-sided Laplace transform indicated in equation (14) can be re-written directly. More specifically, once the method of imaging (figures 2 and 3) is implemented, the targeted origin can be placed anywhere on the sensor structure. Furthermore, the left and right wave modes propagating to and away from the targeted origin will be identical. With this symmetry condition in place, the sensor equation equation (12) can be further reduced to

$$q(k) = -z_k^0 e_{31} k^2 \left[-(w_{lp} + w_{rp}) \int_{-\infty}^{\infty} R(x) e^{-jkx} dx + (w_{le} + w_{re}) \int_{-\infty}^{\infty} R(x) e^{-kx} dx \right]. \quad (15)$$

Figure 3. Schematic diagram of (a) an even-function and (b) an odd-function APROPOS device, both of which have targeted origins set at a free end and a fixed end, respectively.

In fact, the above-mentioned symmetry of the wave propagation is the reason why a low-pass filter without phase lag can be constructed as the information on the waves that represent future and past vibrations existing simultaneously with respect to the origin.

One thing that should be noted is that the no-phase-delay low-pass filter is introduced by considering the propagating and evanescent wave modes in an infinite domain. That is, the autonomous gain–phase filter developed by using an APROPOS device was really developed within the infinite domain. This understanding illustrates that the autonomous gain–phase filter behaviour is generic with respect to the sensor structure and the effective surface electrode chosen, which is independent from the fundamental characteristics of the sensor structure as well as the local information of the targeted origin. In other words, the APROPOS-device-based sensor output is the superposition of the characteristics of the structure, the local characteristics of the targeted origin and the effective surface electrode chosen. As such, APROPOS devices can be viewed as point sensors for measuring the local information of the structure at the targeted origin, which possesses the full characteristics of the flexible structure on which the APROPOS device is mounted. Some of the design concepts that expand the performance of point sensors by introducing an APROPOS device are detailed.

3.1. Developing a point-distributed sensor

The performance of traditional piezoelectric point sensors is strongly constrained by the sensor structure characteristics. Once the sensor structure is determined, the fundamental sensor performance is basically decided. On the other hand, a distributed sensor can tailor its performance with the design of an effective surface electrode. Despite this advantage, distributed sensors are still not as popular as point sensors since they must be re-designed for each structure under testing. To arrive at the best of both worlds, a concept of a point-distributed sensor (PoD sensor) [7–9] is thus being developed to enhance the performance of a point sensor by integrating the distributed piezoelectric sensor technology. More specifically, the concept and know-how of distributed sensors can be integrated into the design of point sensors to create PoD sensors. Implementing PoD sensors requires design of the distributed sensor for the specific point sensor structure only once. Once the PoD sensor

Figure 4. Comparison of time domain signals generated from a point sensor, a photonic sensor and an accelerometer under 1 Hz (a), 4 Hz (b) and 10 Hz (c) external vibrations.

is created, it can be used the same way as any other traditional point sensor. The configuration of a PoD sensor is shown in figure 1. Under this condition, the influences of the boundary effects and the evanescent waves are not negligible. It can be shown herein that a two-sided Laplace transform can be directly adopted to circumvent the limitations of the previously reported spatial filter concept [6] by considering the influence of the boundary effects and evanescent waves.

Since an APROPOS device can introduce a no-phase-delay low-pass filter in the structure system transfer function, the original structure response will not be influenced and the gain will be dropped for frequencies higher than the corner frequency of the added low-pass filter. Adopting this effect into a control loop can enhance the system stability as the gain margin of the overall system is raised. Comparing the concept of adopting an APROPOS device with a flexible structure area, the meaning of a no-phase-delay low-pass filter for a

Figure 5. The experimental set-up for comparing the properties of an APROPOS device and a commercial accelerometer with a photonic sensor serving as the reference signal.

PoD sensor can be understood from a different perspective. The performance of a traditional piezoelectric point sensor is basically set once the sensor structure is decided and thus not much can be done to tailor the sensor system response afterwards except by means of electronic filtering. That is, the characteristics of a sensor structure clearly determine the performance of the sensor itself. It was previously mentioned that with the strong influence of a structure natural resonance, the usable bandwidth of a point sensor is typically limited to the range of one-tenth to one-fifth of the first mode. The reason why a point sensor bandwidth is limited can be further illustrated by the normal mode expansion. The bending displacement of the structure expressed by the normal expansion is

$$w(x, t) = \sum_{i=1}^{\infty} A_i(t) \Phi_i(x), \quad (16)$$

where $A_i(t)$ is the modal coordinates of the i th mode, and $\Phi_i(x)$ represents the i th mode shape. It is clear from equation (16) that the vibration of the structure can be viewed as the combination of all modes; i.e., the effect of all modes exists at any vibration frequency. The influence of each mode is represented by the magnitude of the modal coordinate. Applying a swept sine vibration to a structure, when the vibration frequency moves closer to the natural frequency of a mode, the vibrating shape of the structure will become closer to this specific mode. This is one of the fundamental reasons why the usable bandwidth of a sensor is limited: due to the resonance behaviour of the sensor structure, which undoubtedly affects the performance of the sensors. The effects are clearly demonstrated by the experimental results as shown in figures 4 and 5.

The specimen (figure 5) that serves as the sensor structure for the APROPOS device shown in figure 5 has its first-mode resonant frequency at 9 Hz. The photonic sensor [18], which uses the reflective light intensity to measure the relative spacing between the specimen and the tip of the sensor $h(t)$, was chosen

to measure the displacement of the shaker exerted on the specimen in order to generate the reference signal needed as its output will not be influenced by the sensor structure at all. The influence of the first mode is apparent from the data shown in figure 4, which shows the various sensor signal time traces recorded by using a digital oscilloscope. It is clear that when the vibration experienced by the specimen is near the resonance frequency of the first mode (figure 4(b)), the sensor signal will be strongly influenced by the first mode. It should be noted that the time trace output from the accelerometer is free from the contamination of the first mode even when the shaker vibrates at 10 Hz, which is very close to the resonant frequency of the first mode. The only problem lies in the considerable noise associated with the output signal. This effect can be easily understood by considering that the resonance frequency of the accelerometer used has its first resonant frequency at 60 kHz, which is typical for a commercial accelerometer so as to increase its bandwidth. It is also due to this high first resonant frequency that the sensitivity of these accelerometers will be low at a low frequency. Adopting the flexible sensor structure as shown in figure 5 to make a point sensor, called a PoD sensor, we offer a way to improve the low-frequency sensor response. However, the flexibility of the sensor structure also lowers its first resonance frequency significantly when compared with traditional accelerometers, which in turn shrinks its usable bandwidth significantly. This trade-off is the main problem faced by all sensor designers, as the bandwidth and the sensitivity cannot be optimized simultaneously. However, applying the concept of an APROPOS device to the flexible sensor structure, a built-in low-pass filter can be introduced to significantly reduce or even eliminate the noise generated by the first mode. It should be noted that the above-mentioned built-in low-pass filter is very different from the traditional low-pass filter applied by electronic means, as an APROPOS device generates no phase delay in the transfer function of the original system. It can be shown that the usable bandwidth can be enhanced up to almost 90% of the first-mode frequency when an APROPOS device is implemented.

The performance enhancement of a PoD sensor designed by using an APROPOS device can be further illustrated by looking at figure 6. In figure 6, $G_0(s)$ is the original transfer function of the sensor structure, $G_s(s)$ is the sensor transfer function that has been tailored by the APROPOS device, $F_l(s)$ is the no-phase-delay low-pass filter introduced by the APROPOS device, BW_0 is the original bandwidth of the sensor structure and BW_s is the enhanced bandwidth of the sensor with the APROPOS device. Thus, the PoD sensor designed with an APROPOS device can be used as a point sensor that simultaneously has the benefit of having good low-frequency sensitivity and a wide bandwidth. It will be shown that using a sensor and actuator pair to form an active PoD sensor can further improve sensor performance.

3.2. The sensor equation of an APROPOS device

It should be noted again from figure 3 that an APROPOS device has two basic sensor configurations. One configuration is an even function created by making the polarization profiles on both sides of the targeted origin identical and placing the targeted origin at the free end. The other configuration is an odd

Figure 6. Effect of an APROPOS device on a point sensor shown by using a Bode plot, where ‘au’ stands for arbitrary unit.

function created by reversing the effective surface electrode and placing the targeted origin at the fixed end. Designing an even-function APROPOS device by using equation (15), where the weighting at the non-continuous boundary is zero, leads to

$$q(k) = -z_k^0 e_{31} k^2 \left[-(w_{lp} + w_{rp}) \int_{-\infty}^{\infty} R_e(x) e^{-jkx} dx + (w_{le} + w_{re}) \int_{-\infty}^{\infty} R_e(x) e^{-kx} dx \right]. \quad (17)$$

Similarly, the sensor equation for an odd-function APROPOS device is

$$q(k) = -z_k^0 e_{31} k^2 \left[-(w_{lp} + w_{rp}) \int_{-\infty}^{\infty} R_o(x) e^{-jkx} dx + (w_{le} + w_{re}) \int_{-\infty}^{\infty} R_o(x) e^{-kx} dx \right], \quad (18)$$

where R_e is an even function as $R_e(x) = R_e(-x)$, and R_o is an odd function as $R_o(x) = -R_o(-x)$. The transform variables s of the Laplace transform are jk , $-jk$, k , and $-k$. In addition, it is clear that both equations (17) and (18) are actually two-sided Laplace transforms [19, 20]. The influence of the odd and even functions mentioned above can also be examined by drawing comparisons from the Fourier transform, as a Fourier transform can be viewed as equivalent to a two-sided Laplace transform if and only if the transform variable is limited to jk and $-jk$. It is known that in a Fourier transform, if $R_e(x)$ is an even function, the transform will be real, i.e. no imaginary part exists. On the other hand, if $R_o(x)$ is odd, the transformation will lead to a function of a pure imaginary part, i.e. with a zero real part. These odd- and even-function effects can also be considered as originating from the time reversal property, which is independent of the transform variable adopted. On the other hand, the transform variable s of the two-sided Laplace transform can be viewed as representing the spatial reversal by substituting the even and odd functions into equations (17) and (18) before reversing the spatial domain.

In summary, the transformed function will be even or odd solely depending on whether the function to be transformed is even or odd according to equations (17) and (18). The corresponding properties in the Fourier transform lie in the fact that the real and imaginary part of a complex function is an even and odd function, respectively. In addition, it is also easy to find that the transfer function of an odd function has an extra transform variable $j\omega$ in the numerator compared with the case where there is an even function in the Fourier transform. This property also holds for a two-sided Laplace transform, where the domain reversal is independent of the transform variable s . With the above symmetry argument [21], the sensor equation can be further simplified from equations (17) and (18) to

$$q(k) = -z_k^0 e_{31} k^2 [-(w_{lp} + w_{rp})G_p(s) + (w_{le} + w_{re})G_e(s)] \quad (19)$$

and

$$\begin{aligned} q(k) &= -z_k^0 e_{31} k^2 [-(w_{lp} + w_{rp})[(jk)G_p(s)] \\ &\quad + (w_{le} + w_{re})[(k)G_e(s)]] \\ &= -z_k^0 e_{31} k^2 \Delta(k) [-(w_{lp} + w_{rp})[jG_p(s)] \\ &\quad + (w_{le} + w_{re})[G_e(s)]] \end{aligned} \quad (20)$$

where equation (19) is the transfer function of an even-function APROPOS device and equation (20) is the transfer function of an odd-function APROPOS device such that $G_p(s)$ and $G_e(s)$ represent the transfer functions of the propagating wave modes and evanescent wave modes respectively. The term $\Delta(k)$ represents the additional zero which is introduced by the odd-function APROPOS device. It is worth noting that the transfer variables are different for propagating wave modes and evanescent wave modes. It should also be noted that the sensor equation of the APROPOS device developed above is applied merely at an infinite domain. In the next section, it will be shown that the method of imaging can be used to expand the application areas of APROPOS devices by mapping finite-domain sensor structures to the infinite domain.

3.3. APROPOS device by method of imaging

It was shown above that the newly developed APROPOS device has the ability to introduce a no-phase-delay low-pass filter to the transfer function of a sensor structure by simply choosing a symmetrically distributed effective surface electrode. That is, once a symmetrically distributed APROPOS device with respect to the targeted origin in the infinite domain is constructed, a low-pass filter can be introduced that will not introduce any phase lag.

By the method of imaging, it is easy to consider the characteristics of waves with respect to the boundary. It is clear from figure 3 that waves remain continuous when reflecting back from a free end. If the targeted origin of the APROPOS device is placed closer to the free end of a cantilever plate, the symmetric effective surface electrode cannot be finished; i.e., translating the surface electrode will have electrode that is not zero at the free end. Taking the concept of the method of imaging where the waves are continuous when reflecting back from the free end, the unfinished APROPOS device can be completed by just adding an additional effective surface electrode back from the free end as shown in figure 7(a) [17,21]. Similarly, as waves disappear at the fixed end, the unfinished

Figure 7. Design of an APROPOS device with respect to a free end (a) and a fixed end (b) and their corresponding representation in an infinite domain (c), where the symbols '+' and '-' represent the polarization direction of the piezoelectric lamina covered by the effective surface electrode.

APROPOS device can be eliminated at the fixed end by merely adding an additional electrode back to the structure with an opposite polarization. That is, the 180° phase shift can be taken care of by adopting an opposite-polarization electrode (figure 7(b)) [17,21]. Extending an APROPOS device into an infinite domain with respect to the fixed end can be achieved by using the approach shown in figure 7(c).

In summary, the APROPOS device shown in figure 7 can be reconstructed by merely using the method of imaging and the polarization profile of the APROPOS device to extend it to the infinite domain. It should be noted that an additional electrode can be implemented by adding or subtracting the effective surface electrode equation $R(x)$ in equation (14).

The above discussions clearly demonstrate that it is possible to retrieve information from any location including the boundary of the structure. Extending this line of thought, APROPOS devices can be used to enhance the performance of flexible structures. More specifically, APROPOS devices can be viewed not only as distributed sensors but also as point sensors in the flexible structure control field. By using the concept of the method of imaging, an APROPOS device breaks free from the limitations relating to the boundary effect and the size of the testing structure as encountered by previous spatial filters [6].

Figure 8. Schematic diagram of expansion of a finite cosine function to an infinite domain by the method of imaging. (This figure is in colour only in the electronic version)

Table 1. A right-sided and left-sided Laplace-transformed pair, where $F_r(s) = \int_0^\infty f(x)e^{-sx} dx$, $F_l(s) = \int_{-\infty}^0 f(x)e^{-sx} dx$.

$F(x)$	$F_r(s) = \mathcal{L}\{f(x)\}$	$F_l(s) = \mathcal{L}\{f(x)\}$
$e^{-\alpha x }$	$\frac{1}{\alpha + s}$	$\frac{1}{\alpha - s}$
$\sin(\beta x)$	$\frac{\alpha}{s^2 + \alpha^2}$	$-\frac{\alpha}{s^2 + \alpha^2}$
$\cos(\beta x)$	$\frac{s}{s^2 + \beta^2}$	$-\frac{s}{s^2 + \beta^2}$

3.4. Placing the targeted origin at the boundary

Before introducing the concept of placing the targeted origin of the APROPOS device at the boundaries, the interaction between the sine or cosine functions and the two-sided Laplace transform by the method of imaging will be discussed first. Since the two-sided Laplace transform [19, 20] of a cosine function or a sine function is a delta function in the s domain, it is possible to subtract either a cosine or a sine function from the originally desired effective surface electrode in order to make sure the final effective surface electrode has a magnitude of zero at the boundary, which is a condition that must be met at the fixed end. In addition, the final performance of the APROPOS device generated by the resultant effective surface electrode will be identical to that of the originally desired function in the infinite domain. In other words, the subtraction of a cosine function is a way of placing the targeted origin at the boundaries when the sensor structure is finite. More specifically, once the cosine function subtraction is performed, the sensor equation can be expanded directly to the infinite domain and the powerful two-sided Laplace transform approach is then immediately available to us. In summary, integrating the method of imaging and a cosine function approach can lead to an unambiguous representation of the boundary effect (figures 7 and 8); i.e., the effective surface electrode will be an odd function at a fixed end and an even function at a free end.

With the concept introduced above, the finite cosine function can be transformed completely to the infinite domain by taking into consideration the interactions between the free and the fixed ends as shown in figures 7 and 8. Considering a two-sided Laplace transform table shown in table 1 [19, 20, 22, 23], the two-sided Laplace transform of a cosine function is obviously zero. The fact that a cosine function was transformed to zero by the two-sided Laplace transform indicates that it will not have any influence on the sensor transfer function. In physical implementations, the introduction of the cosine function does influence the construction procedures needed for the sensor designed. Considering the cosine function formed

Figure 9. Schematic diagram of an APROPOS device with a fixed end as its targeted origin.

in figures 7 and 8, the wavenumber of the cosine function was and should be chosen to be lower than all the natural mode wavenumbers in order to make the APROPOS device easy to implement. On the other hand, the delta function induced by the cosine function is conscientiously placed far from the resonant wavenumber of the structure mode in the s domain; i.e., the cosine function does not pick out any specific information on the structure characteristics. With this choice, all of the vibration modes of the structure of interest will have a higher frequency than the cosine function introduced. It should be noted that by subtracting either a sine or a cosine function to make the effective surface electrode null at the boundaries provides us with an opportunity to use the method of imaging to expand the sensor design from a finite sensor structure to an infinite domain even when the targeted origin is placed at or very near the boundaries. This domain expansion behaviour can also be achieved by using a window function [24]. Even though a detailed description of this approach is the primary focus of a separate paper [21, 25], it should be evident that the fundamental concept of mapping a finite sensor length to an infinite domain enables the introduction of a two-sided Laplace transform tool.

It should also be noted that the two-sided Laplace transform automatically introduces a sign change on opposite sides as shown in table 1. In fact, it can be easily shown that this phenomenon is the underlying reason why an APROPOS device can tailor gain without introducing phase lag, as this autonomous gain–phase behaviour seems to violate the Bode gain–phase theorem. That is, once a symmetric APROPOS device is implemented, and the mathematical mode of a two-sided Laplace transform is constructed, a no-phase-delay low-pass filter is automatically introduced. For example, choosing the effective surface electrode of $e^{-\alpha|x|}$ with α as the corner wavenumber can yield the desired autonomous gain–phase low-pass filter due to its symmetry characteristics with respect to the origin $x = 0$, which happens to be the targeted origin. One thing that should be noted is that the design performed in the wavenumber domain can be easily transformed to the frequency domain by using a dispersion relationship shown in equation (9).

Two specific APROPOS devices with their targeted origin set at the free end and at the fixed end were implemented by using the cosine function and the translation concept presented in this article. To set the targeted origin at the fixed end, the APROPOS device was expanded to the infinite domain by subtracting a sine function (see figure 9) in order to make sure

the effective surface electrode was null at the boundaries. More specifically, the function of the effective surface electrode was set to $R(x) = e^{-\alpha|x|} - c_1 \sin(\beta x)$, where the constant c_1 was the constant chosen to make sure the weighting introduced by the effective surface electrode at the free end was zero. This approach makes an APROPOS device easy to implement at a fixed end. The transfer function of this sensor can then be rewritten as

$$q(k) = -z_k^0 e_{31} k^2 \left[-(w_{lp} + w_{rp}) \frac{2s}{\alpha^2 - s^2} + (w_{le} + w_{re}) \frac{2s}{\alpha^2 - s^2} \right], \quad (21)$$

where the transform variables s of the propagating wave modes and evanescent wave modes are jk and k respectively. It should be noted that equation (21) represents the transfer function of an odd-function APROPOS device. An additional transform variable s was introduced into the numerator of the transfer function of the autonomous gain-phase low-pass filter when compared with that of an even-function APROPOS device. Extra zeros are thus being induced in the structure system when an odd function is introduced.

Similarly, this concept can be adopted to set the targeted origin at the free end. In this case, the effective surface electrode becomes $R(x) = e^{-\alpha|x|} - c_2 \cos(\beta x)$, where the constant c_2 is chosen to make sure the weighting induced by the effective surface electrode is null at the fixed end. It is clear from figure 7 that the cosine function is zero at the free end; it is not possible to expand an APROPOS device to an infinite domain by merely subtracting a cosine function. This hindrance can easily be solved by incorporating the concept of the method of imaging again from figure 7. That is, an APROPOS device can be implemented by choosing a larger constant β and finishing the cosine function with an additional electrode as shown in figure 10. In this case, the transfer function can be rewritten as

$$q(k) = -z_k^0 e_{31} k^2 \left[-(w_{lp} + w_{rp}) \frac{2\alpha}{\alpha^2 - s^2} + (w_{le} + w_{re}) \frac{2\alpha}{\alpha^2 - s^2} \right], \quad (22)$$

where the transform variables s of the propagating waves and evanescent waves are jk and k respectively. It is clear from equations (21) and (22) that the autonomous gain-phase low-pass filter introduced by the APROPOS device will not cause any phase delay. It is also clear from equations (21) and (22) that an even-function APROPOS device creates a -40 dB/decade autonomous gain-phase low-pass filter in a wavenumber domain, and the odd-function APROPOS device forms a -20 dB/decade autonomous gain-phase low-pass filter in a wavenumber domain.

This effect indicates that none of the propagating wave modes introduces a phase delay to the APROPOS device output. The evanescent wave modes have a similar effect as well. This characteristic can be further explained by substituting jk , $-jk$, k and $-k$ for s . The overall influence induced by using an APROPOS-device-created autonomous gain-phase filter is a real function for each wave mode. From the displacement field of a one-dimensional plate equation (11), it is easy to verify that the strain of the targeted

Figure 10. Schematic diagram of an APROPOS device with a free end as the targeted origin.

origin is $w''(x) = -(w_{lp} + w_{rp}) + (w_{le} + w_{re})$, which represents the local characteristic of the targeted origin at this point. Comparing APROPOS devices to a targeted origin placed at a free and a fixed end (equations (21) and (22)), it is obvious that the filtering effect is introduced with respect to each wave mode of the structure. As mentioned before, the wave mode amplitudes represent the characteristics of the full structure and the local information of the targeted origin. The physical characteristics of the APROPOS device are a no-phase-delay low-pass filter that is introduced for every wave mode amplitude of the structure, and the characteristic of each of them is not altered. Additional zeros will be introduced to regimes between propagation and evanescent waves when the targeted origin is set at the fixed end. The following discusses the case where the targeted origin has been set at the free end. Substituting the transform variable, jk for propagating waves and k for evanescent waves, equation (22) can be rewritten as

$$q(k) = -z_k^0 e_{31} k^2 2\alpha \times \left[\frac{-(w_{lp} + w_{rp})(\alpha^2 - k^2) + (w_{le} + w_{re})(\alpha^2 + k^2)}{(\alpha^2 + k^2)(\alpha^2 - k^2)} \right] \\ = -z_k^0 e_{31} k^2 2\alpha \{ \alpha^2 [-(w_{lp} + w_{rp}) + (w_{le} + w_{re})] + k^2 [(w_{lp} + w_{rp}) + (w_{le} + w_{re})] \} / \{ (\alpha^2 + k^2)(\alpha^2 - k^2) \}^{-1}. \quad (23)$$

Since these four-wave mode amplitudes represent the characteristics of the targeted origin that was placed at the free end, it can be easy to verify from the boundary condition of the one-dimensional plate. It is known that at the free end, there is no bending moment. That is, $-(w_{lp} + w_{rp}) + (w_{le} + w_{re}) = 0$, and equation (23) can be further reduced to

$$q(k) = -z_k^0 e_{31} k^2 2\alpha \frac{k^2 [(w_{lp} + w_{rp}) + (w_{le} + w_{re})]}{(\alpha^2 + k^2)(\alpha^2 - k^2)}. \quad (24)$$

From the dispersion relationship equation (10), it can be seen that an APROPOS device introduces a -20 dB/decade low-pass filter in a frequency domain. Similarly, a -10 dB/decade low-pass filter in the frequency domain will be introduced by the APROPOS device with a targeted origin set at the fixed end. Thus the combination of the various wave modes, weighting functions, method of imaging, polarization profile and two-sided Laplace transform can create a no-phase-delay low-pass filter. It should be noted that by using the above-mentioned concept, an APROPOS device can operate at a low frequency and can take the influence of the boundary effect and evanescent waves into consideration, all of which were not achievable by previously reported spatial filters [6].

Figure 11. Schematic diagram of an APROPOS device by feedback control.

4. Active APROPOS device

The reciprocal relationship between piezoelectric sensors and actuators makes them natural tools for flexible structure control. Adopting feedback control techniques for piezoelectric sensors and actuator pairs can change the closed-loop pole location and improve system performance and/or lower the influence of the system parameter changes, i.e. reduce system sensitivity [26].

Integrating sensors and actuator pairs by using a control loop is known to be able to further tailor the sensor response. Taking an active accelerometer as an example, the sensor reacts to vibration-induced sensor structure deformation with an output of some electric signal. The output signal is then fed back to the actuator to null the sensor structure deformation. Measuring the feedback signal leads to a quantity that is proportional to the external vibration, which is the quantity of interest. As the sensor deformation is null, it will remain in the linear region for all measurement ranges. This basic notion is the reason why active sensors have better performance than passive sensor systems and is also why an active APROPOS device was developed. This concept can also be used to improve the system performance of flexible structures or to develop an active APROPOS device sensor as shown in figure 11.

When an APROPOS device is adopted to influence the sensor performance for feedback control loop, the overall loop can be made stable even before the loop is completed. That is, the system can be made stable as a low-pass filter is introduced by an APROPOS device to tailor the passive sensor gain performance without sacrificing the phase, all of which can be used to enhance the total system gain margin. Due to the reciprocal relationship between piezoelectric sensors and actuators, once an APROPOS-device-based sensor is used, a corresponding APROPOS-device-based actuator with identical characteristics can also be created. With the many discussions on creating APROPOS-device-based sensors, a similar no-phase-delay, low-pass filter that can act as a compensator can then be introduced by designing an APROPOS-device-based actuator. In other words, a complete feedback loop can be formed successfully by merely using an active APROPOS device actuator. It is certainly also true that all of the traditional electronically based compensators can be integrated into the feedback loop to enhance the overall performance again. With the above-mentioned approach, the freedom available to pursue feedback control for flexible structure control and for point sensor design is greatly enhanced.

APROPOS devices were designed by using wave modes; i.e., the negative feedback loop exerted by an actuator will

Figure 12. Schematic diagram of a sensor and actuator of an active APROPOS device, where the sensor output was amplified A -fold before being fed back to the actuator.

introduce negative waves back to the structure. More specifically, an APROPOS-device-based feedback control is different from the traditional feedback control as it really is a feedback of the wave modes. The design thinking of an active APROPOS device must be examined from the foundation of wave modes. Considering the configuration shown in figure 12, the targeted origins of the sensor were chosen at the origin of the coordinate system and the targeted origin of the actuator was set at x_0 , which was intentionally chosen to be different from the targeted origin of the sensor in order to demonstrate the effect of the targeted origin shift. During operation, the sensor signal was amplified A -fold before being sent on to the actuator. Following the aforementioned concepts, an APROPOS device can be introduced into an infinite domain without the influence of a structure boundary. Thus the overall sensor output of an active APROPOS device with different targeted origin placement can be derived as the measured wave modes minus the amplified feedback waves as follows:

$$q(k) = -z_k^0 e_{31} k^2 \left\{ - \left[(w_{lp} + w_{rp}) \int_{-\infty}^{\infty} f_s(x) e^{-jkx} dx - (w_{lp} + w_{rp})_{x_0} \int_{-\infty}^{\infty} A f_a(x)_{x_0} e^{-jkx} dx \right] + \left[(w_{le} + w_{re}) \int_{-\infty}^{\infty} f_s(x) e^{-kx} dx - (w_{le} + w_{re})_{x_0} \int_{-\infty}^{\infty} A f_a(x)_{x_0} e^{-kx} dx \right] \right\} \quad (25)$$

where the $f_s(x)$ and $f_a(x)$ represent the effective surface electrode of the sensor and the actuator, and the constant A is the feedback gain. It should be noted that x_0 represents the distance between the targeted origins of the actuator and the sensor, $(w_{lp} + w_{rp})_{x_0}$ and $(w_{le} + w_{re})_{x_0}$ represent the wave mode

amplitude as calculated by using x_0 as the targeted origin and $f_a(x)_{x_0}$ represents the surface electrode as written with respect to the targeted origin x_0 . It is obvious that the wave mode amplitude depends on the targeted origin chosen. The form of equation (25) strongly suggests that the filtering effect induced by the selection of the effective surface electrode of the active APROPOS device is independent of the boundaries and the structure characteristics, i.e. performed as if it is in the infinite domain. As integrating the wave information in the infinite domain forms the filtering effect, it is easy to shift the origin of the actuator to the targeted origin of the sensor with the same integration. Equation (25) is thus reduced to

$$q(k) = -z_k^0 e_{31} k^2 \times \left\{ -(w_{lp} + w_{rp}) \int_{-\infty}^{\infty} [f_s(x) - Af_a(x)] e^{-jkx} dx \right. \\ \left. \times (w_{lp} + w_{rp}) \int_{-\infty}^{\infty} [f_s(x) - Af_a(x)] e^{-kx} dx \right\}. \quad (26)$$

It is clear from equation (26) that the overall performance of the active APROPOS device can be regarded as having chosen an effective surface electrode $R(x)$ to be

$$R(x) = f_s(x) - Af_a(x). \quad (27)$$

The effects of a passive and active APROPOS device can be verified by using the following experiments.

5. Experimental set-up

To verify the effect of APROPOS devices, a 140 mm long, 22 mm wide and 0.02 mm thick standard steel shim constructed to become a one-dimensional cantilever plate was chosen as the sensor structure. The configuration shown in figures 9 and 10 was designed by making sure the effective surface electrode chosen can verify both the concepts of the targeted origin translation and the effects of the even and odd functions as well. The wave mode amplitudes of the propagating and evanescent waves for the cantilever plate can be simplified from equation (12) by summing the corresponding numerators and considering the case where $|e^{jak}| = 1$ and e^{ak} dominate when the wavenumbers are high. The magnitude of the propagating wave modes and the evanescent wave modes becomes

$$w_{lp} + w_{rp} \approx e^{jak} (e^{2ak} + 1) [2(e^{2ak} + 1) + j(e^{2ak} - 1)] \quad (28a)$$

$$w_{le} + w_{re} \approx e^{ak} (e^{2jak} + 1) [2(e^{2ak} + 1) + j\delta] \quad (28b)$$

where δ approaches zero. Further simplifying equations (26a) and (26b) yields the wave mode amplitudes of the propagating and evanescent waves of the cantilever plate at high frequency or high wavenumber as

$$|w_{lp} + w_{rp}| \approx \sqrt{5} e^{4ak}, \quad (29a)$$

and

$$|w_{le} + w_{re}| \approx 2e^{3ak}. \quad (29b)$$

Substituting the parameters for the experimental configuration used such that $D_{11}/\rho A = 0.101846$ in the dispersion relationship equation (10) first and then into equations (29a) and (29b), it can be seen that the wave mode amplitude of the evanescent waves will be 10% less than the propagating wave

Figure 13. The experimental set-up of an APROPOS device.

at 256 Hz or when the wavenumber is beyond 28.322. In other words, the effect of the evanescent waves will not be able to be neglected for these kinds of short structure. It should also be noted that co-axial cables were used to transmit the signal lines and that two PVF₂ laminae with their electrical ground connections facing outside [7, 8] were deployed to completely eliminate the influence of EMI (electromagnetic interference) and to increase the signal-to-noise ratio. The corner frequency of the no-phase-delay low-pass filter created by the APROPOS device was set at 5 Hz.

The experimental set-up is shown in figure 13, where a photonic sensor was used to obtain the relative displacement between the photonic sensor tip and the shaker head. This displacement was used as the reference signal and the signal swept range was from near DC to lower than 10 Hz. At a higher-frequency swept range, the reference signal was taken from the accelerometer mounted on top of the shaker head. The transfer function of a uniformly distributed sensor was used as the basic distributed sensor signal in order to examine the effect of the APROPOS device. The physical meaning of the uniform sensor output is the bending angle at the free end of the one-dimensional plate as predicted by the sensor equation (equation (5)). The uniform sensor output represents the full characteristics of the sensor structure resonance and can thus be used as a reference signal for verifying APROPOS device effects. As APROPOS-device-based sensors measure the local strain of the targeted origin, which can be further filtered by using the electrode designed to introduce the autonomous gain-phase tailoring, the transfer function of uniform sensor must add an jk term to form compatible data for a valid comparison. By using the dispersion relationship of the one-dimensional plate equation (10), the transfer function in the frequency domain must be plus $10 \times \log 10(w)$ to the measured data of the uniform sensor.

The effective surface electrode function of the free-end APROPOS device was chosen to be $R(x) = e^{-4|x|} - 3.16497655 \cos(0.00915117x)$. On the other hand, $R(x) = e^{-4|x|} - 0.5712 \sin(0.01122x)$ was chosen as the effective surface electrode for the fixed-end APROPOS device. By using the dispersion relationship of equation (9) and the experimental data of the uniformly distributed sensor, the corner frequency of this APROPOS device was calculated to be 5 Hz. From equations (21) and (22), the designed transfer functions of the APROPOS device shown in figures 9 and 10

Figure 14. The log–log plot of both the transfer function of a sensor structure under the influence of an APROPOS device, where the targeted origin was set at (1) a free end (thick curve) and (2) an uniform sensor (thin curve) and (3) a theoretical value from the uniform sensor (grey curve).

become

$$q(k) = -z_k^0 e_{31} k^2 \left[-(w_{lp} + w_{rp}) \frac{2jk}{4^2 + k^2} + (w_{le} + w_{re}) \frac{2k}{4^2 - k^2} \right] \quad (30a)$$

$$q(k) = -z_k^0 e_{31} k^2 \left[-(w_{lp} + w_{rp}) \frac{8}{4^2 - k^2} + (w_{le} + w_{re}) \frac{8}{4^2 - k^2} \right]. \quad (30b)$$

The frequency responses of the APROPOS device for the free end and for the fixed end are shown in figures 14 and 15. In figure 14, the free-end APROPOS device evidently produces a -20 dB/decade low-pass filter in the transfer function of the sensor structure output. In figure 15, the fixed-end APROPOS device produces a -10 dB/decade low-pass filter in the transfer function of the sensor structure output. Comparing figures 14 and 15, the zeros introduced by the odd function can be shown clearly. Note that the first extra zero introduced occurs after the second mode. One of the main benefits of being able to introduce gain tailoring capability without altering the phase distribution is the capability to increase the system stability.

It should be noted that the targeted origin of the designed APROPOS device was placed at the boundary of the one-dimensional plate, which made it difficult to measure the local information of the targeted origin to show the transfer function of the APROPOS device. To clearly demonstrate the effect of the APROPOS device, a 400-point swept sine measurement was performed to measure the frequency response of the APROPOS sensor from near DC to near 10 Hz as shown in figures 16(a) and (b). It is obvious from figure 10 that at the designed corner frequency of the APROPOS devices (around 5 Hz), no phase lag was introduced. This gain tailoring capability, which does not sacrifice the phase distribution

Figure 15. The log–log plot of both a transfer function of a sensor structure under the influence of an APROPOS device where the targeted origin is set at (1) a fixed end (thick curve) and (2) an uniform sensor (thin curve) and (3) a theoretical value from the uniform sensor (grey curve).

significantly, improves our capability of improving the system stability.

Using an APROPOS device as shown in figure 10 as the sensor and an APROPOS device as shown in figure 9 as the actuator, these two APROPOS devices formed an active APROPOS device. The experimental set-up is shown in figure 17. The transfer functions of the active APROPOS device (thin curve) and the free-end APROPOS-device-based sensor (thick curve) are shown in figure 18 to demonstrate the influence of the feedback effect on the performance of the APROPOS devices. It is also clear that an active APROPOS device successfully reduces the gains of resonant modes. The interaction of a feedback control with an APROPOS device is clearly shown in figure 18 even though only a simple negative feedback control was considered. It is obvious from figure 18 that the additional no-phase-delay, low-pass filter introduced by the fixed-end actuator further increases the gain difference between mode 1 and mode 2 from 12.66 to 23.13 dB, which demonstrates the feedback concept of an active APROPOS device.

6. Conclusions

Both theoretical and experimental results were shown to verify the successful implementation of both a passive and an active APROPOS device. The concept of a targeted origin translation was also verified by the method of imaging. It was shown clearly that an APROPOS device could detect testing structure properties at any location, even at the boundaries. Combining the powerful design thinking offered by the method of imaging and weighting functions, an APROPOS device can be used to construct PoD sensors, which integrate the advantages of both point and distributed sensors. A PoD sensor has the advantage

Figure 16. The log–log plot of a transfer function of the sensor structure under the influence of an APROPOS device where the targeted origin is set at a free end (a) and a fixed end (b).

of ease of use like that of a point sensor and has the design freedom offered by an APROPOS device. It was shown that both free and fixed ends of a cantilever structure play important roles in the design of APROPOS devices once the concept of wave modes is introduced. In addition, it was shown that different boundary conditions could only lead to different w_{rp} , w_{lp} , w_{re} and w_{le} , which in turn will influence the overall sensor transfer functions. By using a symmetrically distributed effective surface electrode, an autonomous gain–phase low-pass filter without phase shift can be introduced to the transfer function of the sensor output. Furthermore, the performances of odd- and even-function APROPOS devices are also shown, which indicate that an extra transform variable s will appear in the numerator of the odd function when compared with that of an even function. With the many different wave modes, the corresponding effects are all different.

In addition, the active sensor concept developed based on wave modes can also be verified. A no-phase-delay feedback control loop was found to be different from traditional control theory. Also, the integration of a classical control theory with piezoelectric sensors and actuators was also adapted to further advance flexible structure control and PoD sensor technology. More specifically, an APROPOS device provides a designer with an opportunity to adjust the system gain without affecting the system phase. That is, the classic dilemma of increasing a system gain which automatically leads to a loss of the phase margin can be avoided by using the methodology shown in this article. It was also shown that all of these added advantages could lead to a much wider sensor bandwidth

Figure 17. Experimental set-up of an active APROPOS device.

and a much higher gain for the desired frequency range. In summary, both odd-function and even-function APROPOS devices can produce a no-phase-delay low-pass filter to reduce the influence of the natural resonant modes on the desired transfer functions of the sensors. With the concept shown in this article, the technique of evaluating the flexible structure performance is greatly advanced. Applying an APROPOS device to create a PoD sensor, the usable bandwidth can be increased up to 80–90% of the first-mode frequency. The gain performance of a point sensor can thus be tailored without introducing any phase delay by using an APROPOS device

Figure 18. The closed-loop transfer function of a free-end sensor and a fixed-end actuator (thick curve), and the transfer function of an APROPOS device with a targeted origin placed at a free end (thin curve).

as presented in this paper. On the other hand, integrating an APROPOS device with feedback control theory has been shown to provide us with many more degrees of freedom for transfer function tailoring that includes sensors, actuators and overall control systems.

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References

- [1] Prak A, Elwenspoek M and Fluitman J H J 1992 Selective mode excitation and detection of micromachined resonators *J. Microelectromech. Syst.* **1** 179–86
- [2] Tzou H S and Hollkamp J J 1994 Collocated independent modal control with self-sensing orthogonal piezoelectric actuators (theory and experiment) *Smart Mater. Struct.* **3** 277–84
- [3] Tzou H S, Zhong J P and Natori M 1993 Sensor mechanics of distributed shell convolving sensors applied to flexible rings *Trans. ASME* **115** 40–6
- [4] Preumont A and Francois A 1999 Piezoelectric array sensing for real-time, broad-band sound radiation measurement *ICAST '99: Proc. Int. Conf. on Adaptive Structure and Technologies (Paris, 1999)* (Lancaster, PA: Technomic) pp 37–44
- [5] Lee C K 1992 Piezoelectric laminates: theory and experiments for distributed sensors and actuators *Intelligent Structural Systems* ed H S Tzou and G L Anderson (Dordrecht: Kluwer) pp 75–167
- [6] Miller D W, Collins S A and Peltzman S P 1990 Development of spatially convolving sensors for structural control applications *Proc. AIAA/ASME/ASCE/AHS: Proc. Conf. on Structures, Structural Dynamics and Materials (Long Beach, CA)* pp 2283–97
- [7] Lee C K, Hsu Y H, Lin C T, Hsiao W H, Shih H C, Hsu S H and Hu H S 1999 Implementing APROPOS device as point sensors *ICAST'99: Proc. 10th Int. Conf. on Adaptive Structures and Technologies* (Lancaster, PA: Technomic) pp 53–60
- [8] Hsu Y H, Lee C K, Hsiao W H, Lin C T, Shih H C, Hsu S H and Hu H S 1999 APROPOS device for control-structure interactions *ICAST'99: Proc. 10th Int. Conf. on Adaptive Structures and Technologies* (Lancaster, PA: Technomic) pp 45–52
- [9] Lee C K, Lin C T, Hsiao W H and Shih H C 2000 Device and method for measuring vibration *US Patent Pending*
- [10] Bode H W 1945 *Network Analysis and Feedback Amplifier Design* (New York: Van Nostrand-Reinhold)
- [11] Rohrs C E, Melsa J L and Donald G S 1993 Bode phase plot—straight line approximation *Linear Control System* int. edn (Singapore: McGraw-Hill) pp 244–62 section 5.4
- [12] Cady W G 1946 *Piezoelectricity* vol 1 (New York: McGraw-Hill) pp 1–8
- [13] Kawai H 1969 The piezoelectricity of poly(vinylidene fluoride) *Japan. J. Appl. Phys.* **8** 975–6
- [14] 1987 Piezoelectricity *ANSI/IEEE Standard 176* (Piscataway, NJ: IEEE)

- [15] Lee C K 1990 Theory of laminated piezoelectric plates for the design of distributed sensors/actuators: part 1. Governing equations and reciprocal relationships *J. Acoust. Soc. Am.* **87** 1144–58
- [16] Lee C K and Moon F C 1990 Modal sensors/actuators *ASME J. Appl. Mech.* **57** 434–41
- [17] Graff K F 1975 *Wave Motion in Elastic Solids* (New York: Dover)
- [18] MTI2000 1991 *Fotonic Sensor Instruction Manual*
- [19] Bracewell R N 1978 *The Fourier Transform and its Application* 2nd edn (New York: McGraw-Hill)
- [20] van der Pol B and Bremmer H 1959 *Operational Calculus: Based on the Two-Sided Laplace Integral* 2nd edn (Paris) (Cambridge: Cambridge University Press)
- [21] Lee C K and Hsu Y H 2000 Piezoelectric transducer apparatus with independent gain and phase characteristics control based on the fourth-order model *US Patent Pending*
- [22] Grossman D 1988 *Advanced Engineering Mathematics* (New York: Harper and Row)
- [23] O'Neil P V 1991 *Advanced Engineering Mathematics* 3rd edn (Boston, MA: PWS)
- [24] Harris F J 1978 On the use of windows for harmonic analysis with the discrete Fourier transform *Proc. IEEE* **66** 51–83
- [25] Hsu Y H and Lee C K 2001 Extending point-sensor performance by incorporating distributed-sensor and window functions concept *AIAA J.* submitted
- [26] Dorf B 1998 *Modern Control System* 8th edn (New York: Addison-Wesley)