LINEAR NETWORK ANALYSIS OF SPLIT-TYPE STIRLING REFRIGERATOR

B.J. Huang and C.W. Lu
Department of Mechanical Engineering, National Taiwan University, Taipei, Taiwan

A linear network model is developed for split-type Stirling refrigerators. We obtain an equivalent network and two transfer-function functions from which the system performance can be evaluated. Implementation of the linear network model in the system design analysis of split-type Stirling refrigerator is shown satisfactory.

INTRODUCTION

Stirling refrigerator operates at a cyclic state and the physical process is thus transient. The system design using conventional heat transfer analysis requires a sophisticated computing equipment and suffers from numerical instability and high computing cost. A linear network model is thus developed in the present study for the system design analysis of split-type Stirling refrigerator (Figure 1).

EQUIVALENT NETWORK OF STIRLING REFRIGERATOR

The linearly-perturbed models of the components can be derived from the governing equations and linearization method. Connecting the equivalent circuits of the components according to the process of Stirling refrigerator, we obtain the equivalent network as shown in Figure 2. Only block diagram can be drawn in Figure 2 for the connecting tube and regenerator since the distributed models are derived for them.

TRANSFER FUNCTIONS OF STIRLING REFRIGERATOR

Solving the linear dynamics equations, we obtain two transfer functions for Stirling refrigerator:

\[
G_{dp}(s) = \frac{\tilde{X}_d(s)}{X_p(s)} = \left[ \frac{G_3(s)G_5(s)/G_7(s) + G_2(s)}{1 - G_5(s)} \right] \times \left[ \frac{1 - G_5(s)Z(s)/Z_0(s)}{1 + G_5(s)Z(s)/Z_0(s)} \right] \times \frac{sp_A \tilde{A}}{RT_e} \tag{1}
\]

\[
G_{ep}(s) = \frac{\tilde{P}_e(s)}{X_p(s)} = \left[ \frac{G_3(s)G_5(s)/G_7(s)G_8(s)G_9(s)}{1 - G_5(s)} \right] \times \left[ \frac{1 - G_5(s)Z(s)/Z_0(s)}{1 + G_5(s)Z(s)/Z_0(s)} \right] \times \frac{sp_A \tilde{A}}{RT_e} \tag{2}
\]

where

\[
Z_1(s) = \frac{G_8(s) + G_7(s)Z_0(s)\tanh[\Gamma_1(s)L_1]}{G_7(s) + [G_6(s)/Z_0(s)]\tanh[\Gamma_1(s)L_1]}.
\]

\[
G_1(s) = D_{dw}(s) + D_{dr}(s) \cdot R_{wp}(s); G_2(s) = D_{dw}(s) \cdot R_{wm}(s); G_3(s) = G_1(s) + G_2(s) \cdot W_{mp}(s); G_4(s) = R_{zp}(s) + R_{mp}(s) \cdot W_{mp}(s); G_5(s) = G_2(s) \cdot W_{mp}(s); G_6(s) = R_{mp}(s) \cdot W_{mp}(s); G_7(s) = \frac{G_2(s)G_8(s)}{1 - G_5(s)}; G_8(s) = \frac{G_2(s)G_9(s)}{1 - G_5(s)}; G_9(s) = \frac{G_2(s)G_8(s)}{1 - G_5(s)}; G_{10}(s) = \frac{G_2(s)G_8(s)}{1 - G_5(s)};
\]

\[
G_{11}(s) = R_{mm}(s) \left[ 1 + \frac{W_{mm}(s)G_2(s)}{1 - G_5(s)} \right] - E_{mm}(s) \left[ \frac{R_{mp}(s) + G_2(s)G_8(s)}{1 - G_5(s)} \right] - \frac{E_{mm}(s)}{1 - G_5(s)} ;
\]

\[
\tilde{n}_{hi}(s) = \frac{sp_A \tilde{A}}{1 + Z_1(s)/Z_0(s)} \tilde{X}_p(s).
\]
It is found that the piston displacement $X_p$ is the system input of a split-type Stirling refrigerator. The system outputs are the displacer displacement $X_d$ and the expansion space pressure $p_e$. The Stirling refrigerator thus belongs to a single-input-multiple-output system (SIMO) in linear systems.

**STIRLING REFRIGERATOR PERFORMANCE CALCULATION**

**Maximum Available Cooling Capacity**

The maximum available cooling capacity can be evaluated by integrating the pressure $p_e(t)$ and volume $V_e(t)$ of the expansion space, where $V_e(t)$ is related to $X_d(t)$. Since the piston motion as well as the associated pressure waves approaches sinusoidal, $X_d(t)$ and $p_e(t)$ can be computed simply from the gain and phase of the frequency response functions, $G_d(j\omega)$ and $G_p(j\omega)$. Assuming that $a_{dp}$ is the gain of $G_d(\omega)$ with phase $\phi_d$ leading the piston; $a_{ep}$ is the gain of $G_p(\omega)$ with phase $\phi_e$ leading the piston. Then, the maximum available cooling capacity is

$$Q_{max} = f \int_0^{2\pi} p_e dV_e = f \pi a_{ep} a_{dp} X_p^2 A_d \sin(\phi_d - \phi_e)$$

where $X_p$ is the amplitude of the piston; $f$ is the operating frequency; $\theta$ is the piston angle; $p_{co}$ and $X_{do}$ are the amplitudes of $p_e(t)$ and $X_d(t)$, respectively.

**Net Cooling Capacity**

The net cooling capacity $Q_{net}$ can be evaluated by subtracting the heat losses from the maximum available cooling capacity $Q_{max}$. There are four types of heat losses: namely, heat conduction loss of regenerator $Q_{cond}$, enthalpy flow loss of regenerator $Q_{enthalpy}$, shuttle heat loss of displacer $Q_{shuttle}$, and hysteresis loss of spring $W_{frf}$ [2]. The heat loss due to the gas leakage through the clearance between the displacer and the cylinder wall is related to the seal design, manufacturing process, and material used. It is ignored in the present analysis. The effect of gas leakage at the piston is hardly estimated accurately and is also ignored.

**SYSTEM PERFORMANCE ANALYSIS AND IMPLEMENTATION**

**System Performance Analysis**

A PC-based computer program is developed and packaged (called "STCS 2.0") for the system performance analysis of split-type Stirling refrigerator. A typical PV diagram is shown in Figure 3.

The frequency response of $G_d(s)$ and $G_p(s)$ are shown in Figure 4 and 5. The net cooling capacity at various loss coefficient $C_d$ and cold-end temperature $T_L$ is shown in Figure 6. $Q_{net}$ at various operating frequency $f$ and $C_d$ is shown in Figure 7. The above results is consistent with the previous studies [3 – 5]. However, it takes only several seconds for the computation on PC 486.

It is also found that $G_p(j\omega)$ has a peak gain (Figure 5). This indicates that an optimum operating frequency exists at the peak gain since $Q_{max}$ is proportional to the gain $a_{ep}(=|G_p(j\omega)|)$. The performance at this peak gain corresponds to the optimum value of $Q_{net}$ in Figure 7 for a given $C_d$.

**Testing Results and Application of "STCS 2.0"**

A split-type Stirling refrigerator is designed and built in the laboratory for experiments. The lowest temperature obtained is around 100 K, with 0.25W at 150 K. The loss coefficient $C_d$ can be estimated from the package STCS 2.0 by using the test results. An empirical correlation for $C_d$ is further obtained: $C_d = 224.9 + 1.026 T_L$, $\text{N s/m}$, as shown in Figure 8.

It was found experimentally that there is gas leakage between the displacer and the cylinder due to seal design problem and manufacturing defects. $C_d$ obtained from the experiments thus includes the effect of gas leakage in addition to the frictional loss.
CONCLUSION

A linear network model is developed for the system performance analysis of split-type Stirling refrigerators. It is shown that the analytical results are consistent with the conventional heat transfer analysis. However, the computational speed is increased tremendously and numerical problems is completely avoided. A PC-based software package “STCS 2.0” was also developed according to the linear network model. STCS 2.0 can be further used to derive an empirical correlation of $C_d$ by using the test results. The system design of Stirling refrigerator can then be simplified a great deal.

REFERENCES


Acknowledgment — The present study was supported by the National Science Council, Taiwan, R.O.C., through Grant No. NSC81-0401-E002-587. The authors are also grateful to Prof. Guo Fangzhong at Huazhong University of Science and Technology, Wuhan, China for his valuable discussions.
Expansion Space Temperature: 60 K
Compression Space Temperature: 380 K
Warm Space Temperature: 330 K
Displacer Stroke: 9.96 mm
Displacer Phase Angle: 13.25 deg
Expansion Space Pressure Phase Angle: -29.51 deg
Net Cooling Capacity: 2.615 W

Figure 3. PV diagram

Figure 4. Frequency response of $G_{dp}(s)$

Figure 5. Frequency response of $G_{dp}(s)$

Figure 6. $Q_{net}$ at various $C_d$ and $T_L$.

Figure 7. $Q_{net}$ at various $C_d$ and $f$.

Figure 8. Experimental results of $C_d$. 