Thermal–electrical–luminous model of multi-chip polychromatic LED luminaire

Bin-Juine Huang*, Chun-Wen Tang

New Energy Center, Department of Mechanical Engineering, National Taiwan University, Taipei 10617, Taiwan

Abstract

This paper proposed a thermal–electrical–luminous dynamic model of red–green–blue (RGB) light-emitting diode (LED) luminaire for lighting control. The thermal–electrical–luminous model consists of three parts, namely, electrical–thermal (E–T), electrical–luminous (E–L), and thermal–luminous (T–L) models. Using step response method, the electrical–thermal (E–T) model \( G(s) \) is derived as a first-order bi-proper system. The electrical–luminous (E–L) and thermal–luminous (T–L) models are zeroth order model with a constant gain since the luminous response to electric or thermal input is much faster. The thermal–electrical–luminous model shows that the luminous intensity is proportional to input power and inversely proportional to junction temperature. The dynamic response of luminous intensity is dominated by the electrical–thermal model \( G(s) \).

The whole thermal–electrical–luminous model can be further divided into a constant gain and a first-order bi-proper system. The constant gain causes the instantaneous response at power switch on; the first-order system represents the luminous variation due to junction temperature change which is mainly related to the heat sink design. The complete model can accurately describe luminous dynamic behavior and be used in control system design of RGB LED lighting luminaire.

1. Introduction

High brightness light-emitting diode (LED) is a promising technology for lighting. Due to the rapid technology improvement, the illuminating efficiency of LED has reached > 80 lm/W in commercial products and is proved energy saving as compared to traditional lightings, such as incandescent (<20 lm/W), fluorescent (<50 lm/W), and mercury (<70 lm/W) lamps. In addition, red–green–blue (RGB) LEDs are the only light source which can vary intensity and light color, is determined by power inputs of color LEDs. The light color is determined by luminous ratio of three color LEDs. However, the illuminations of LEDs vary with junction temperature variation due to self-heating of LEDs and variation of ambient temperature. Hence, the thermal effect will affect both illumination intensity and output color of LED. A lighting control of RGB LEDs is thus needed. In order to develop the control system, a system dynamics model of RGB LEDs taking into account the effect of thermal, electrical, and luminous properties must be derived first.

The RGB LED lighting can be controlled using feedback system design [3–7]. The derivation of a system dynamic model of LED luminaire is thus important for the feedback system design.

However, very few researchers investigate the system dynamics model of a LED luminaire. Masana [8] derived a RC thermal model for a general semiconductor package. Gu et al. [9] studied optical properties of LED at steady state. Muthu et al. [10] proposed a constant luminous model which ignores the thermal effect. Farkas et al. [11] developed a thermal model for luminous output and thermal resistance in monochromatic light-emitting unit. Huang et al. [12,13] derived a system dynamics model of a luminaire to relate the energy input to LED junction temperature. In fact, the junction temperature variation will affect the light output of LEDs. Little has been studied on this subject. The present paper intends to derive a thermal–electrical–luminous model of RGB LED luminaire for the control system design of RGB LED lighting.

2. Modelling of RGB LED luminaire

From the principle of solid-state lighting, the luminance of LED is induced from two physical mechanisms: energy effect and optoelectronic effect. Both effects are related to junction temperature. The thermal–electrical–luminous model of RGB LED luminaire thus consists of three major parts: electrical–thermal (E–T) model; electrical–luminous (E–L) model; thermal–luminous (T–L) model.

The RGB LED luminaire is a lighting fixture which is made of multiple RGB LED lamps with a heat sink. The schematic diagram of RGB LED luminaire is shown in Fig. 1. The LEDs are driven by electrical input power which will raise the LED junction temperature by self-heating [14]. The dynamic behavior of junction...
The thermal–electrical (E–T) model, $G(s)$, where $U$ is input power (W), $P_{LED}$ input power (W), $p$ pole of thermal dynamics, $s$ Laplace operator, $T$ temperature (°C), $T_{LED}$ junction temperature (°C)/$W$, $V_F$ averaged forward voltage (V), $V_{FD}$ initial forward voltage (V), $z$ zero of thermal dynamics.

### Subscripts
- $a$ ambient air
- $B$ blue LEDs
- $G$ green LEDs
- $i$ notation of $R,G$ or $B$
- $j$ LED junction
- $P$ input power
- $R$ red LEDs
- $T$ junction temperature

### Greeks symbols
- $\Phi$ luminous intensity (cd)
- $\Phi_{LED}$ luminous intensity of RGB LED luminaire (cd)
- $\sim$ perturbation
- $-$ average

The electrical–thermal system dynamics can be treated as multi-variable system with three inputs (input power of red, green and blue LEDs – $P_R(s), P_G(s), P_B(s)$) and three outputs (junction temperature of red, green, and blue LEDs – $T_R(s), T_G(s), T_B(s)$). Neglecting the variation of ambient temperature, assuming the lumped condition for the temperature of LED chip, i.e. lumped assumption, and using linear perturbation concept, we obtain the thermal model $G(s)$ which is a 3 $\times$ 3 transfer function [15]:

$$G(s) = \begin{bmatrix} G_{RR}(s) & G_{RG}(s) & G_{RB}(s) \\ G_{GR}(s) & G_{GG}(s) & G_{GB}(s) \\ G_{BR}(s) & G_{BG}(s) & G_{BB}(s) \end{bmatrix} = \begin{bmatrix} \tilde{T}_R(s) \\ \tilde{T}_G(s) \\ \tilde{T}_B(s) \end{bmatrix} = \begin{bmatrix} \tilde{T}_R(s) \\ \tilde{T}_G(s) \\ \tilde{T}_B(s) \end{bmatrix} = G(s) \cdot \begin{bmatrix} \tilde{P}_R(s) \\ \tilde{P}_G(s) \\ \tilde{P}_B(s) \end{bmatrix} \quad \text{at } \tilde{T}_o \equiv 0. \tag{2}$$

The linearly-perturbed junction temperature can be derived as:

$$\tilde{T}_{LED}(s) = \begin{bmatrix} \tilde{T}_{R}(s) \\ \tilde{T}_{G}(s) \\ \tilde{T}_{B}(s) \end{bmatrix} = \begin{bmatrix} G_{RR}(s) & G_{RG}(s) & G_{RB}(s) \\ G_{GR}(s) & G_{GG}(s) & G_{GB}(s) \\ G_{BR}(s) & G_{BG}(s) & G_{BB}(s) \end{bmatrix} \cdot \begin{bmatrix} \tilde{P}_R(s) \\ \tilde{P}_G(s) \\ \tilde{P}_B(s) \end{bmatrix} \quad \text{at } \tilde{T}_o \equiv 0. \tag{3}$$

The block diagram of the E–T model is shown in Fig. 3.

### 2.2. Electrical–luminous (E–L) model, $E_P$

The electrical–luminous (E–L) model, $E_P$, relates the input power to the output luminous intensity. Since the light response to the input power is very fast, the E–L model of RGB LEDs is of

![Fig. 1. Schematic diagram of RGB LED luminaire.](image1)

![Fig. 2. The block diagram of thermal-electrical-luminous model of RGB LED luminaire.](image2)
on a at the same junction temperature, a single LED lamp is soldered core PCB (MCPCB). In order to measure the light spectrum of LED, the light engine is made of RGB LED lamps soldered on a metal mechanical parts: light engines, heat sink, shell, and circular baffle.

3.1. Experimental setup

An RGB LED luminaire was designed and built in the present study, as shown in Fig. 6. The lighting fixture consists of four lamps on £85 mm MCPCB which is attached to the light engine body. Each lamp has a thickness of 2.0 mm, total power dissipation of 25 W, and a weight of 900 g. The LED arrangement for the luminaire is a 2×2 configuration with a spacing of 20 mm. The light engine is a component that houses the LED lamps and is made of aluminum. The circular baffle material is black acrylonitrile–butadiene–styrene (ABS) and is used to isolate stray light.

3.2. Identification of thermal–electrical–luminous model of RGB LED luminaire

The mathematical model of RGB LED luminaire described above will be identified experimentally.

3.2.1. Experimental setup

An experimental apparatus was built for light measurement of RGB LED luminaire. The apparatus includes an integrating sphere, control system, photometric sensor, and a personal computer. The measuring system can provide color mixing and simulation.

2.3. Thermal–luminous (T–L) model, $ET$

The thermal–luminous (T–L) model, $ET$, relates the junction temperature to the output luminous intensity. Since the light response to the junction temperature variation is very fast, the T–L model of RGB LEDs is of zeroth order with a constant gain, defined as, in terms of linear perturbation concept:

$$ ET = \left[ E_{TR} \ E_{TC} \ E_{TB} \right] = \left[ \frac{\Phi_{LED}(s)}{T(s)} \ \frac{\Phi_{LED}(s)}{T(s)} \ \frac{\Phi_{LED}(s)}{T(s)} \right] $$

Fig. 5 is the block diagram of the thermal–luminous (T–L) model, $ET$.

2.4. Electrical–luminous (E–L) model, $EP$

The electrical–luminous (E–L) model, $EP$, relates the input power to the output luminous intensity. During the measurement of the light spectrum, the LED is driven by a constant-current with pulse-width modulation (DC-PWM) for minimal chromaticity shifts [16]. The input power is controlled by the duty cycle of DC-PWM. The junction temperature is measured by pulse method [13,18]. This single LED lamp acts as a sensor for measuring the LED junction temperature and is called “LED sensor”.

The circular baffle is used in front side to isolate the stray light during the measurement of the light spectrum. The specification of RGB LED lamp and luminaire are listed in Tables 1 and 2, respectively.

The LED is driven by a constant-current with pulse-width modulation (DC-PWM) for minimal chromaticity shifts [16]. The switching frequency is set at 120 Hz to avoid perceptible flicker [17]. The input power is controlled by the duty cycle of DC-PWM. The junction temperature is measured by pulse method [13,18].

A linear junction temperature–voltage relation at low current (1 mA) was first determined experimentally in a well-controlled environment [13]:

$$ T_j = a + bV_F. $$

The junction temperature can then be measured at the OFF-interval of DC-PWM using the LED sensor with 1 mA current.

An experimental apparatus was built for light measurement of RGB LED luminaire. The apparatus includes an integrating sphere, current meter with photopic detector, spectrometer, and a personal computer. The measuring system can provide color mixing.

Table 1

<table>
<thead>
<tr>
<th>Items</th>
<th>No. of chip</th>
<th>Peak wavelength (nm)</th>
<th>Photopic power (lm)</th>
<th>Power dissipation (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>1</td>
<td>621.2</td>
<td>33</td>
<td>1.21</td>
</tr>
<tr>
<td>Green</td>
<td>2</td>
<td>520.8</td>
<td>50</td>
<td>2.56</td>
</tr>
<tr>
<td>Blue</td>
<td>1</td>
<td>459.8</td>
<td>11</td>
<td>1.27</td>
</tr>
</tbody>
</table>

Table 2

<table>
<thead>
<tr>
<th>Items</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Light engine</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Four lamps on £85 mm MCPCB.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Lamps arrangement: 2×2.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Lamps spacing: 20 mm.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total power dissipation: 25 W</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Thickness of aluminum MCPCB: 2.0 mm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heat sink</td>
<td>Material: aluminum</td>
<td>Weight: 900 g.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shell</td>
<td>Material: aluminum</td>
<td>Surface area: 950 cm²</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Circular baffle</td>
<td>Material: black acrylonitrile–butadiene–styrene</td>
<td>Diameter: 125 mm</td>
<td>Length: 65 mm</td>
<td></td>
</tr>
</tbody>
</table>

Figure captions:

- Fig. 3: Input–output block diagram of the E–T model.
- Fig. 4: Input–output block diagram of the electrical–luminous (E–L) model, $EP$.
- Fig. 5: Input–output block diagram of the thermal–luminous (T–L) model, $ET$.
- Fig. 6: Schematic diagram of RGB LED luminaire.
data recording of luminous intensity and junction temperature. The experimental facility is shown in Fig. 7.

3.2. Electrical–thermal (E–T) model, \( G(s) \)

The electrical–thermal model \( G(s) \) can be identified using step response method. The E–T model \( G(s) \) is a \( 3 \times 3 \) MIMO model. All the elements can be identified experimentally using step test method. First, the step input power was applied to red LEDs while keeping all other inputs constant. The time responses of \( R–G–B \) junction temperatures then can be determined and analyzed to obtain \( G_{GR}(s), G_{GC}(s), G_{GB}(s) \), from Eq. (9):

\[
\begin{bmatrix}
\tilde{T}_R(s) \\
\tilde{T}_G(s) \\
\tilde{T}_B(s)
\end{bmatrix} = \begin{bmatrix}
G_{GR}(s) & G_{GC}(s) & G_{GB}(s) \\
G_{GR}(s) & G_{GC}(s) & G_{GB}(s) \\
G_{GR}(s) & G_{GC}(s) & G_{GB}(s)
\end{bmatrix} \begin{bmatrix}
\tilde{P}_R(s) \\
0 \\
0
\end{bmatrix}
\]

at \( \tilde{P}_C(s) = 0 \) and \( \tilde{P}_B(s) = 0 \),

\( (7) \)

Similarly, the step input powers were applied to green or blue LEDs, respectively, while keeping all other inputs constant. The time responses of \( R–G–B \) junction temperatures then can be determined and analyzed to obtain \( G_{GR}(s), G_{GC}(s), G_{GB}(s) \), from Eq. (8) and \( G_{GB}(s), G_{GC}(s), G_{GB}(s) \), from Eq. (9):

\[
\begin{bmatrix}
\tilde{T}_R(s) \\
\tilde{T}_G(s) \\
\tilde{T}_B(s)
\end{bmatrix} = \begin{bmatrix}
G_{GR}(s) & G_{GC}(s) & G_{GB}(s) \\
G_{GR}(s) & G_{GC}(s) & G_{GB}(s) \\
G_{GR}(s) & G_{GC}(s) & G_{GB}(s)
\end{bmatrix} \begin{bmatrix}
\tilde{P}_R(s) \\
0 \\
0
\end{bmatrix}
\]

at \( \tilde{P}_C(s) = 0 \) and \( \tilde{P}_B(s) = 0 \),

\( (8) \)

\[
\begin{bmatrix}
\tilde{T}_R(s) \\
\tilde{T}_G(s) \\
\tilde{T}_B(s)
\end{bmatrix} = \begin{bmatrix}
G_{GR}(s) & G_{GC}(s) & G_{GB}(s) \\
G_{GR}(s) & G_{GC}(s) & G_{GB}(s) \\
G_{GR}(s) & G_{GC}(s) & G_{GB}(s)
\end{bmatrix} \begin{bmatrix}
\tilde{P}_R(s) \\
0 \\
0
\end{bmatrix}
\]

at \( \tilde{P}_C(s) = 0 \) and \( \tilde{P}_B(s) = 0 \),

\( (9) \)

\( G(s) \) varied with the magnitude of input power. All the nine elements of \( G(s) \) were identified at different input power as first-order bi-proper system by using Rake’s analysis [13,19]. Table 3 shows the poles, zeros and gains of \( G(s) \) at different input power perturbation. An average model for E–T model \( G(s) \) is derived as, using the mean value of each parameter:

### Table 3

<table>
<thead>
<tr>
<th>No.</th>
<th>( G_{GR} )</th>
<th>( G_{GC} )</th>
<th>( G_{GB} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>( z )</td>
<td>( k )</td>
<td></td>
</tr>
</tbody>
</table>

#### Step \( P_1 (W) \)

- \( 1.8 \rightarrow 3.9 \): 13
- \( 3.9 \rightarrow 6.0 \): 14
- \( 6.0 \rightarrow 9.3 \): 15
- \( 9.3 \rightarrow 12.7 \): 16

#### Step \( P_2 (W) \)

- \( 1.9 \rightarrow 4.1 \): 25
- \( 4.1 \rightarrow 6.3 \): 26
- \( 6.3 \rightarrow 4.1 \): 27
- \( 4.1 \rightarrow 1.9 \): 28

This table demonstrates the poles, zeros, and gains of the thermal model under different input power perturbations.
The model parameters at each operating conditions are shown in Table 3. The comparison of the measured step response with the calculation from the average model, the element $G_{GR}(s)$ in $G_0(s)$, is shown in Figs. 8 and 9 shows the measured frequency response using 4 W step power input for each LEDs.

Fig. 8. The time response of $G_{GR}(s)$ using step power input of green LEDs from 8.3 W to 12.7 W.

$G_0(s) =$

\[
\begin{array}{ccc}
0.8132(s + 0.00192) & 0.2220(s + 0.00343) & 0.4190(s + 0.00301) \\
(3 + 0.00083) & (3 + 0.00083) & (3 + 0.00082) \\
0.9467(s + 0.00217) & 1.8164(s + 0.00137) & 1.7456(s + 0.00227) \\
(3 + 0.00081) & (3 + 0.00082) & (3 + 0.00081) \\
0.5934(s + 0.00111) & 0.3338(s + 0.00175) & 2.2140(s + 0.00122) \\
(3 + 0.00081) & (3 + 0.00082) & (3 + 0.00080)
\end{array}
\]

(10)

The model parameters at each operating conditions are shown in Table 3. The comparison of the measured step response with the calculation from the average model, the element $G_{GR}(s)$ in $G_0(s)$, is shown in Figs. 8 and 9 shows the measured frequency response using 4 W step power input for each LEDs.

Fig. 9. The frequency response of $G_{GR}(s)$ model identified using step power input of green LEDs from 8.3 W to 12.7 W.

Fig. 10. The junction temperature responses of all nine elements in $G(s)$ using 4 W step power input for each LEDs.
and the calculation from the average model. Fig. 10 presents the junction temperature responses of all nine elements in the thermal model $G(s)$ for 4 W step input power to each LEDs which are calculated using the above identified model.

3.3. Electrical–luminous (E–L) model, $E_p$

The optoelectric response of LEDs is much faster than the thermal behavior of luminaire. The electrical–luminous (E–L) model, $E_p$, can thus be treated as zeroth order system. Moreover, input power and junction temperature will influence linearly the luminous intensity of LEDs [14]. The reception of eyes is also linear due to linearity laws [17]. Therefore, $E_p$ is constant.

The luminous response of different color LEDs due to power input can be determined separately using isolating method. That is, applying step input power to red LED, while keeping the power input of other LEDs at constant, and measuring the luminous intensity of LEDs, we can determine the elements of $E_p$:

\[ \Phi_p(s) = E_p \cdot \tilde{P}_{LED}(s) = [E_{PR} \ E_{PC} \ E_{PB}] \cdot \begin{bmatrix} \tilde{P}_R(s) \\ 0 \\ 0 \end{bmatrix} = E_{PR} \cdot \tilde{P}_R(s) \quad \text{at} \quad \tilde{P}_C(s) = 0 \quad \text{and} \quad \tilde{P}_B(s) = 0. \]  

\[ \Phi_p(s) = E_p \cdot \tilde{P}_{LED}(s) = [E_{PR} \ E_{PC} \ E_{PB}] \cdot \begin{bmatrix} 0 \\ \tilde{P}_C(s) \\ 0 \end{bmatrix} = E_{PC} \cdot \tilde{P}_C(s) \quad \text{at} \quad \tilde{P}_R(s) = 0 \quad \text{and} \quad \tilde{P}_B(s) = 0. \]  

\[ \Phi_p(s) = E_p \cdot \tilde{P}_{LED}(s) = [E_{PR} \ E_{PC} \ E_{PB}] \cdot \begin{bmatrix} 0 \\ 0 \\ \tilde{P}_B(s) \end{bmatrix} = E_{PB} \cdot \tilde{P}_B(s) \quad \text{at} \quad \tilde{P}_R(s) = 0 \quad \text{and} \quad \tilde{P}_C(s) = 0. \]  

A procedure was designed to obtain the step input power and luminous intensity. Input power of one color LEDs firstly was applied at a fixed value and others set to 0 W. The junction temperature of powered LEDs will rise by self-heating. The luminous intensities were recorded at junction temperature 60 °C. In this study, three input power levels were used for each color LEDs (red LEDs: 5.45 W, 4.24 W and 3.03 W; green LEDs: 4.48 W, 3.20 W and 1.92 W; blue LEDs: 2.22 W, 1.33 W and 0.44 W). The results are shown in Fig. 11. $E_p$ is then determined:

\[ E_p = [E_{TR} \ E_{TC} \ E_{TB}] = [279.52 \ 542.87 \ 161.39]. \]  

3.4. Thermal–luminous (T–L) model, $E_T$

The thermal–luminous (T–L) model, $E_T$, relates the junction temperature to the output luminous intensity. Since the light response to the junction temperature variation is very fast, the T–L model of RGB LEDs is of zeroth order with a constant gain, defined in Eq. (5).

The luminous response of different color LEDs can be calculated as:

\[ \Phi_T(s) = E_T \cdot \tilde{T}_{LED}(s) = [E_{TR} \ E_{TC} \ E_{TB}] \cdot \begin{bmatrix} \tilde{T}_R(s) \\ 0 \\ 0 \end{bmatrix} = E_{TR} \cdot \tilde{T}_R(s) \quad \text{at} \quad \tilde{T}_C(s) = 0 \quad \text{and} \quad \tilde{T}_B(s) = 0, \]  

\[ \Phi_T(s) = E_T \cdot \tilde{T}_{LED}(s) = [E_{TR} \ E_{TC} \ E_{TB}] \cdot \begin{bmatrix} 0 \\ \tilde{T}_C(s) \\ 0 \end{bmatrix} = E_{TC} \cdot \tilde{T}_C(s) \quad \text{at} \quad \tilde{T}_R(s) = 0 \quad \text{and} \quad \tilde{T}_B(s) = 0, \]  

\[ \Phi_T(s) = E_T \cdot \tilde{T}_{LED}(s) = [E_{TR} \ E_{TC} \ E_{TB}] \cdot \begin{bmatrix} 0 \\ 0 \\ \tilde{T}_B(s) \end{bmatrix} = E_{TB} \cdot \tilde{T}_B(s) \quad \text{at} \quad \tilde{T}_R(s) = 0 \quad \text{and} \quad \tilde{T}_C(s) = 0. \]  

Input power was applied at fixed level in one color LEDs, and set other LEDs at 0 W. The curve of luminous intensity and junction temperature were recorded continuously. In this study, input power level were 4.84 W for red, 3.33 W for green and 0.83 W for blue LEDs. The results are linear as shown in Fig. 12. $E_T$ can be obtained by using linear regression method as

\[ E_T = [E_{TR} \ E_{TC} \ E_{TB}] = [-12.76 \ -4.32 \ -0.69]. \]  

3.5. Thermal–electrical–luminous model of RGB LED luminaire

Combining the above results, the thermal–electrical–luminous model is described as:

\[ H_{LED}(S) = E_p + E_T \cdot G(S) \]

\[ = \begin{bmatrix} 260.97 - 0.0371 \\
534.69 - 0.0176 \\
156.04 - 0.0134 
\end{bmatrix} \]
The complete thermal–electrical–luminous model of RGB LED luminaire $H_{LED}(s)$ needs experimental verification. To simulate the general lighting application, the operating condition was set at luminous intensity 3800 cd and color temperature 4500 K. The light appears as normal warm white color. Under this condition, the conversion of illuminance at 2 m distance is 950 lux which is suitable for indoor lighting application [20]. A step test procedure with four input power perturbations was employed. The perturbed inputs are listed in Table 4. The first step input is big and causes large junction temperature rise. The luminous intensity response can be calculated using the electrical–thermal model $G_T(s)$ and the thermal–luminous (T–L) model, $E_T$. For small perturbation input to individual color LEDs with small temperature variation, the luminous response can be calculated using the electrical–luminous (E–L) model $G_e$. Since the light color is determined by luminous intensity, the conversion of illuminance at 2 m distance is 950 lux which is suitable for indoor lighting application. The complete model can describe the luminous dynamic behavior and can be used in control system design of RGB LED lighting luminaire.

### Table 4

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Perturbation of input power (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_T$</td>
</tr>
<tr>
<td>0</td>
<td>4.84</td>
</tr>
<tr>
<td>13,000</td>
<td>0</td>
</tr>
<tr>
<td>19,000</td>
<td>0</td>
</tr>
<tr>
<td>25,000</td>
<td>0.3</td>
</tr>
</tbody>
</table>

3.6. Verification of thermal–electrical–luminous model of RGB LED luminaire

The complete thermal–electrical–luminous model of RGB LED luminaire $H_{LED}(s)$ needs experimental verification. To simulate the general lighting application, the operating condition was set at luminous intensity 3800 cd and color temperature 4500 K. The light appears as normal warm white color. Under this condition, the conversion of illuminance at 2 m distance is 950 lux which is suitable for indoor lighting application [20]. A step test procedure with four input power perturbations was employed. The perturbed inputs are listed in Table 4. The first step input is big and causes large junction temperature rise. The luminous intensity response can be calculated using the electrical–thermal model $G_T(s)$ and the thermal–luminous (T–L) model, $E_T$. For small perturbation input to individual color LEDs with small temperature variation, the luminous response can be calculated using the electrical–luminous (E–L) model $G_e$. Since the light color is determined by luminous intensity, the conversion of illuminance at 2 m distance is 950 lux which is suitable for indoor lighting application.

The complete model can describe the luminous dynamic behavior and can be used in control system design of RGB LED lighting luminaire.

### 4. Discussion and conclusions

The present paper has derived a thermal–electrical–luminous dynamic model of red–green–blue (RGB) light-emitting diode (LED) luminaire for lighting control. The thermal–electrical–luminous model consists of three parts, namely, electrical–thermal (E–T), electrical–luminous (E–L), and thermal–luminous (T–L) models. Using step response method, the electrical–thermal (E–T) model $G_T(s)$ is identified experimentally as a first-order bi-proper system. The electrical–luminous (E–L) and thermal–luminous (T–L) models are zeroth model with a constant gain since the luminous response to electric or thermal input is much fast.

The thermal–electrical–luminous model shows that the luminous intensity is proportional to input power and inversely proportionally to junction temperature. The dynamics of luminous intensity is dominated by the electrical–thermal model $G_T(s)$. The complete model can describe the luminous dynamic behavior and can be used in control system design of RGB LED lighting luminaire.

The electrical–thermal (E–T) model $G_T(s)$, Eq. (10), can be separated into a constant model $G_c(s)$ and a first-order model $G_b(s)$, as shown in Eq. (20). The constant model $G_c(s)$ represents the instantaneous heating of LED chip due to poor cooling in chip package. The first-order model $G_b(s)$ is resulted from slow dynamic behavior of the heat sink. It is seen that all the nine poles of $G_b(s)$ are very close to each other, within ± 2%. The time response of optoelectric effect is at nano-second level and much faster than thermal effect. The electrical–luminous (E–L) model $E_L$ is thus a zero-order system. The luminous intensity is proportional to input power. However, $E_{PC}$ is greater than others because there are twice numbers of green chips in each LED lamp [17].
The thermal–luminous (T–L) model $E_T$ describes the luminous response of junction temperature change. It is a zero-order and constant system. The luminous intensity is inversely proportional to junction temperature. The luminous decay rate by thermal effect is thus determined by electrical–luminous (E–L) model $E_T$ and the thermal–luminous (T–L) model $E_T$. Furthermore, the spectrum shift and decay of red LEDs due to junction temperature effect is much greater than other LEDs [9] since $E_{T,R}$ for red LEDs is greater than other LEDs. The prediction of E–T and T–L model coincides very well with experiment as shown in Fig. 13a.

Since the input power increase will cause a junction temperature rise and reduce luminous intensity. Therefore, both E–T and T–L model must be tested at the same time. For the verification of the E–L model, the input power step is small in order to reduce junction temperature rise. The large gain of the E–L model helps in validating the model using small step. By examining the response at the instant of applied step, the jump of predicted luminous response coincides with experimental results at 13,000 s, 19,000 s, and 25,000 s.

The present thermal–electrical–luminous model for a RGB LED luminaire is able to predict the luminous response. However, the slight deviation occurs in the last two step responses. This deviation is caused by the disturbance of ambient temperature since the effect of ambient temperature is ignored in the present model. According to the E–T model of Eq. (2), the sequential increasing input power steps in Table 4 will raise junction temperatures while the ambient temperature keeps constant. However, at the duration of the last step test from 23,000 s in Fig. 13b, the ambient temperature is larger than input power step change. The junction temperature decrease since the effect of decreasing ambient temperature is larger than input power step change. The lower junction temperature thus induces a higher luminous intensity (Fig. 13a) according to T–L model. This explains why the luminous intensity responses in Fig. 13a shows opposite trend between experimental results and the thermal–electrical–luminous model during the last step transient.

The whole thermal–electrical–luminous model can be further divided into two parts as:

$$H_{LED}(s) = \left[ E_T + E_T \cdot G(s) \right] = \left[ E_T + E_T \cdot G(s) \right] + \left[ E_T \cdot G(s) \right] = H_T(s)$$

where

$$H_T(s) = \left[ \frac{260.97}{s + 0.0371} \right] - \left[ \frac{534.69}{s + 0.000816} \right] = \left[ \frac{156.04}{s + 0.0176} \right] - \left[ \frac{0.0114}{s + 0.000816} \right]$$

$H_T$ is constant matrix which can be treated as the instantaneous response at switch on. $H_T$ represents the luminous variation due to junction temperature change which is mainly related to the heat sink.

The thermal–electrical–luminous model can be used in the control system design for RGB LED lighting.

**Acknowledgement**

The present study was supported by National Science Council, Taiwan, under Grant No. NSC95-2221-E-002-244-MY2.

**References**