AN EXPANDED MODEL FOR THE VALUATION OF EMPLOYEE STOCK OPTIONS

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YUH-DAUH LYUU*

The unique characteristics of employee stock options make straightforward applications of traditional option pricing models questionable. This study extends the standard pricing model to account for the dilution effect, the employees’ exercise pattern, and the state-dependent employee forfeiture rate. It also performs comparative analysis of popular existing models and the proposed models. Finally, the impacts of the above-mentioned factors on the fair value of employee stock options are investigated. The results support the claim that our models reflect the reality better than existing models. © 2009 Wiley Periodicals, Inc. Jrl Fut Mark 29:713–735, 2009

INTRODUCTION

Under the Statement of Financial Accounting Standard Board (FASB, 1995, 2004) No. 123 “Share-Based Payment” (revised 2004), henceforth referred to as SFAS No.123(R) for the rest of the article, U.S. public companies are

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required to expense the cost of employee stock options at their fair values. The cost of the options is calculated by multiplying the option value given by a pricing model with those options’ expected life as the time to maturity and the number of granted options. The pricing of such options, however, is far from a simple exercise of applying textbook option pricing models. This is because employee stock options exhibit a number of characteristics that set them apart from exchange-traded options, which hinder the use of standard option pricing models. Among them, four stand out.

1. Employee stock options usually have a vesting period during which exercise is not permitted. The vesting period can range from one to several years.
2. If employees leave their company during the vesting period, their unvested options are forfeited.
3. If employees leave their company after the vesting period, the options will be exercised if they are in-the-money and forfeited if out-of-the-money.
4. Employees are not permitted to sell their employee stock options.

Because employee stock options are nontradable, if an employee has liquidity needs or wants diversity in the portfolio, he must exercise his vested and exercisable options and sell the stocks subsequently. Other factors such as the expected stock return, the employee’s risk aversion, investment opportunities, and wealth also affect the time when options are exercised. As a result, the options might be exercised suboptimally, that is, not exercised at the time, which maximizes the value of the options, due to these factors. According to SFAS No. 123(R), a tree model and a “closed-form model” both meet the criteria for obtaining a fair value of employee stock options. In particular, the fair value can be estimated either by the Black–Scholes (1973) model or the Cox, Ross, and Rubinstein (1979) binomial tree model. In addition, SFAS No. 123(R) requires that the two additional parameters be estimated:

1. The employee forfeiture rate during the vesting period, which is the probability that employees will leave the company each year during the vesting period. The rate is assumed to be constant over the vesting period.
2. The expected life of the options, which is the average time the vested options remain unexercised.

SFAS No. 123(R) states that the tree model has the advantage of being able to incorporate the peculiar characteristics of employee stock options properly. For example, it can accommodate employees’ exercise patterns, dividends, as well as what SFAS No. 123(R) terms “expected volatility” of the underlying stock over the options’ contractual terms. Thus, this article proposes a model
that enhances the Cox–Ross–Rubinstein (CRR) binomial tree model with the following considerations. First, exercising options results in an increase of the number of the company's shares. However, as the paid-in capital is lower than the prevailing stock price, the stock price should drop as a result. SFAS No. 123(R) states that if the market is reasonably efficient for the said company's shares, the dilution effect will be reflected in the sense that the stock price will fall in the amount based on the number of options exercised. However, to the best of our knowledge, no option valuation model in the literature accounts for this decrease in the stock price. To make the option value more faithful, the dilution effect will be incorporated.

Second, when calculating the option value, the expected life of the granted options is used as its time to maturity. The expected life of the options embodies employees’ exercise patterns, but the options’ total value is clearly not a linear function of their expected life. Thus, it is inappropriate to use the expected life as the options’ time to maturity and then multiply the resulting option value by the number of employee stock options as an estimate of the total cost of employee stock options. In contrast, our model will consider the employees’ exercise patterns explicitly. The option value will then be a weighted average value of those options that are exercised based on the exercise patterns.

Third, the assumption of a constant employee forfeiture rate over the vesting period of the options is obviously not realistic. When unvested employee stock options are in-the-money, employees who hold them will be less likely to leave the company voluntarily. This intuition implies that the employee forfeiture rate is negatively correlated with the stock price. A state-dependent employee forfeiture rate that links the employee forfeiture rate to the level of the stock price will be adopted in our model. Furthermore, the employees’ exercise pattern will also be considered explicitly.

BACKGROUND

Traditionally, two accounting methods have been used to deal with employee stock options. The first is the intrinsic value-based method, which is based on the Accounting Principles Board Opinion No. 25 (1972). Under this method, the cost of an option is the excess of the market price on the grant date over the exercise price of the option, if any. The other method is the fair value-based method, which is based on SFAS No. 123 (1995). Under this method, the cost of an option is its fair value. Here, the fair value is estimated by an option pricing model. SFAS No. 123 recommends, but does not require, that companies adopt this method.

Most companies adopted the intrinsic value-based method. Because the exercise price is usually set to be the market price on the grant date, the cost is
reported zero on the income statement. In other words, the intrinsic value-based method does not decrease companies’ reported earnings. The resulting reported cost is clearly distorted as employee stock options do incur a cost to the issuing company (Guay, Kothari, & Sloan, 2003).

To address the distortion of treating employee stock options as zero cost on the income statement, the FASB issued SFAS No. 123 (R), which requires companies in the United States to adopt the fair value-based method. Now that employee stock option costs must be expensed under the new statement, how to price them has become an important issue for financial accounting reports.

In addition to the model proposed in SFAS No. 123(R), other popular models exist for valuing employee stock options. The popular binomial tree model that will be used throughout the article is one of them. Let $S_{i,j}$ and $C_{i,j}$ stand for the stock price and the value of the employee stock option at node $j$ of period $i$, respectively. Node $j$ of period $i$ is named $(i,j)$. $K$ is called the exercise price of the option, and $T$ is the maturity of the option in years. Suppose the binomial tree has $N$ time steps, each of length $\Delta t = T/N$. See Figure 1 for illustration. Define $r$ as the risk-free rate, $p$ as the risk-neutral probability that the stock price will move up in one period and $\sigma$ is the volatility of the underlying stock. During each period, the stock price at node $(i,j)$ may move up to price

![Diagram of binomial tree](image)

**FIGURE 1**
The structure of the binomial tree.
$S_{i,j}u$ at node $(i + 1, j)$ or move down to price $S_{i,j}d$ at node $(i + 1, j + 1)$. The following settings of the CRR binomial tree will be used:

\[
\begin{align*}
  u &= e^{\sigma \sqrt{\Delta t}} \\
  d &= 1/u \\
  p &= (e^{(r-D)\Delta t} - d) / (u - d)
\end{align*}
\]

where $D$ is the continuous dividend yield.

Let $q$ stand for the constant annualized employee forfeiture rate and $v$ the vesting period (in years). Depending on the context, which will always be clear, $q$ may refer to either the pre-vesting or the post-vesting employee forfeiture rate.

In the following, several popular models are reviewed. These are the SFAS 123 model, the Hull–White model, the utility maximization model, and the enhanced American model. As mentioned in the introduction section, as there are many factors affecting employees’ exercise pattern, the option may be exercised suboptimally, which means the resulting option value tends to be under-valued relative to the option value obtained from the CRR tree model.

**The SFAS 123 Model**

Under SFAS No. 123(R), either the Black–Scholes model or the CRR binomial tree model can be used to come up with the option value. In both cases, the expected life of the options must be used as the time to maturity. See Figure 2

```latex
\begin{align*}
EL_{i,j} &= 0, \quad j = 0,1,\ldots,N \\
\text{For } 1 \leq i \leq N-1, \quad 0 \leq j < i, \\
\{ \\
\quad &\text{If the option is vested and exercised, then } EL_{i,j} = 0 \\
\quad &\text{If the option is vested but held, then} \\
\quad &\quad EL_{i,j} = (1 - q\Delta t)[p \times EL_{i+1,j} + (1 - p) \times EL_{i+1,j+1} + \Delta t] \\
\quad &\quad \text{If the option is unvested, then} \\
\quad &\quad EL_{i,j} = p \times EL_{i+1,j} + (1 - p) \times EL_{i+1,j+1} + \Delta t \\
\} \\
\text{Return } EL_{0,0}
\end{align*}
```

**FIGURE 2**
The calculation of the expected life of employee stock options.
for the calculation of the expected life. The CRR binomial tree will be used as the SFAS model in numerical experiments because of its generality. Specifically,

\[ C_{\text{SFAS123}} = (1 - q)^v \times C_{T=\text{expected life}} \]

where \( C_{\text{SFAS123}} \) denotes the option value under the SFAS 123 model and \( C_{T=\text{expected life}} \) denotes the option value under the CRR binomial tree model with the expected life of the options substituting for the time to maturity. Note that \( C_{T=\text{expected life}} \) is multiplied by \((1 - q)^v\) to yield \( C_{\text{SFAS123}} \). This adjustment is made necessary by the possibility of employees leaving the company during the vesting period, which reduces the option value. The SFAS 123 model uses a constant \( q \) for the pre-vesting employee forfeiture rate; this assumption is simplistic. Furthermore, post-vesting employee forfeiture rate is not considered in the model; this omission also results in inaccuracies.


The Hull–White model extends the SFAS 123 model by taking into account the possibility that employees may leave their company after the vesting period. Moreover, it explicitly incorporates employees’ early exercise strategy by assuming that all vested options are exercised early when the stock price is at least \( M \) times the exercise price, i.e., \( S_{i,j} \geq MK \) (Hull & White, 2002, 2003, 2004). The annualized pre-vesting and post-vesting employee forfeiture rates are assumed to be the same constant \( q \). See Figure 3 for an algorithmic description.
of the Hull–White model. Because the option is exercised as soon as the stock price reaches a certain multiple of the exercise price, the exercise strategy is sub-optimal. Suboptimality means the resulting option value is potentially undervalued compared with the option value obtained from backward induction through the tree.

THE UTILITY MAXIMIZATION MODEL

The utility maximization model assumes employees maximize their utilities rather than the expected return of the option. The vested option is exercised if the expected utility derived from exercising the option is greater than the utility from holding it. Marcus and Kulatilaka (1994) and Rubinstein (1995) propose the following utility function:

\[
U(W) = \begin{cases} 
W^{1-\gamma} & \text{if } \gamma > 0 \text{ and } \gamma \neq 1 \\
1 - \gamma & \text{if } \gamma = 1 \\
\ln W & \text{if } \gamma = 0 
\end{cases}
\]

where \( W \) is the total wealth of the employee, which consists of nonoption wealth and the employee stock options he holds, and \( \gamma \) is the coefficient of the employees’ risk aversion. The employee is said to be risk neutral if \( \gamma = 1 \), risk averse if \( \gamma < 1 \), and risk seeking if \( \gamma > 1 \). The model assumes that both nonoption wealth and cash realized from exercising the options are reinvested in the risk-free asset (Huddart, 2004).

In this model, the employee’s subjective expectation about the stock’s return will alter his exercise behavior. Consequently, the risk-free rate \( r \) is replaced by the subjective stock return \( \mu \) when computing the probability \( p \), i.e., \( p = [e^{(\mu-D)t - d}]/(u - d) \). See Figure 4 for an algorithmic description of the utility maximization model. The potential drawbacks of the utility maximization model include its dependence on a specific utility function and the need to estimate three parameters: the expected stock return \( \mu \), the initial wealth \( W_0 \), and the employee’s risk aversion \( \gamma \).

The Enhanced American Model

The enhanced American model is proposed by Ammann and Seiz (2004, 2005). The model explicitly incorporates early exercise by adjusting the exercise price of the option. The adjusted exercise price is used to determine the time to exercise the option but not the payoff of exercising the option. Denote the multiplicative adjustment factor by \( M^* \), which “triggers” or “delays” the exercise. The exercise criterion is \( \max(S_{i,j} - M^*K,0) > e^{-r\Delta t}[pC_{i+1,j} + (1-p)C_{i+1,j+1}] \) and \( S_{i,j} - K > 0 \).
Early exercise takes place when the exercise criterion is satisfied. If $M^* < 1$, then the exercise is said to be triggered, whereas if $M^* > 1$, then the exercise is said to be delayed. Figure 5 is an algorithmic description of the enhanced American model.

Under the enhanced American model, the option value with $M^*$ other than 1 is always lower than that with $M^* = 1$. The time when the option is exercised under the aforementioned criterion may not be the time that maximizes the option value. Hence, the option value under this exercise strategy is always lower than the option value with $M^* = 1$, which yields the maximum option value among all $M^*$. As a consequence, the strategy may not be optimal except for $M^* = 1$, and the resulting option value is likely to be undervalued compared with the option value obtained from backward induction through the tree.

THE ENHANCED EMPLOYEE STOCK OPTION VALUATION MODEL

This section introduces our three modifications to the CRR binomial tree model: the dilution factor, the exercise pattern of employees, and a state-dependent employee forfeiture rate.
The Dilution Model

Exercising employee stock options leads to issuance of new shares, which increases the number of outstanding shares. When employee stock options are exercised, the exercise price must be lower than the stock price if the employee is rational. Those newly issued shares with lower paid-in capital lead to dilution.

SFAS No. 123(R) states that if the market for the company’s shares is reasonably efficient, then the dilution effect from exercising employee stock options will be reflected right after the exercise. It concludes that no adjustment is needed to account for dilution when valuing employee stock options.

However, the information on the total number of exercised employee stock options and the times that the options are exercised as well as their amounts is not available to the public in a timely fashion. In many countries, timely disclosure of such information is not even required and details about the employee stock options that have been exercised are usually disclosed in the annual or quarterly financial statements. Hence, there will be a time lag between the time the options are exercised and the time such information is disclosed. Thus, it is almost impossible for the market to reflect the dilution effect efficiently. Assuming the market is efficient enough to reflect the exercise of employee stock options fully is therefore not realistic.

Although stock price decreases when dilution occurs, to the best of our knowledge, no valuation model takes the dilution effect into account. To make

\[
C_{N,j} = \max(S_{N,j} - K, 0), \quad j = 0, 1, \ldots, N
\]

For \(1 \leq i \leq N - 1\), \(0 \leq j \leq i\),

\{

If the option is vested and the exercise criterion holds, then

\[C_{i,j} = S_{i,j} - K\]

If the option is vested but the exercise criterion does not hold, then

\[C_{i,j} = q\Delta t \times \max(S_{i,j} - K, 0) + (1 - q\Delta t)e^{-r\Delta t}[pC_{i+1,j} + (1 - p)C_{i+1,j+1}]\]

If the option is unvested, then

\[C_{i,j} = (1 - q\Delta t)e^{-r\Delta t}[pC_{i+1,j} + (1 - p)C_{i+1,j+1}]\]

\}

Return \(C_{N,0}\)

FIGURE 5
Algorithmic description of the enhanced American model.
the option value more reasonable, the dilution effect will be taken into account as follows. Denote the number of outstanding shares before the exercise of employee stock options by $\omega$ and the number of granted employee stock options by $\theta$. Assume one employee stock option entitles one to buy one share of the underlying stock for $K$ dollars. Assume either none of the options is exercised or all of the options are exercised at the same time. The diluted stock price after exercising $\theta$ employee stock options is defined as

$$\frac{S_{i,j}\omega + K\theta}{\omega + \theta}$$

at node $Q_{i,j}$ (Hull & White, 2004). The stock price at each node is still computed in the same way as in the CRR tree model; in other words, the same $u$, $d$, and $p$ are used. The only modification is that the payoff from exercising the options is based on the diluted stock price. Figure 6 is a precise algorithmic description of the dilution model.

\begin{verbatim}
C_{i,j} = \max \left( \frac{S_{i,j}\omega + K\theta}{\omega + \theta} - K, 0 \right), \quad j = 0, 1, \ldots, N
\end{verbatim}

For $1 \leq i \leq N-1$, $0 \leq j \leq i$,

{ \begin{align*}
\text{If the option is vested and} & \quad \frac{S_{i,j}\omega + K\theta}{\omega + \theta} - K > e^{-r\Delta t}[pC_{i+1,j} + (1-p)C_{i,j+1}], \text{then} \\
C_{i,j} &= \frac{S_{i,j}\omega + K\theta}{\omega + \theta} - K
\end{align*} } \begin{align*}
\text{If the option is vested and} & \quad \frac{S_{i,j}\omega + K\theta}{\omega + \theta} - K \leq e^{-r\Delta t}[pC_{i+1,j} + (1-p)C_{i,j+1}], \text{then} \\
C_{i,j} &= q\Delta t \times \max \left( \frac{S_{i,j}\omega + K\theta}{\omega + \theta} - K, 0 \right) \\
&\quad + (1-q\Delta t)e^{-r\Delta t}[pC_{i+1,j} + (1-p)C_{i,j+1}]
\end{align*} 

If the option is unvested, then

$$C_{i,j} = (1-q\Delta t)e^{-r\Delta t}[pC_{i+1,j} + (1-p)C_{i,j+1}]$$

Return $C_{0,0}$
\end{verbatim}

FIGURE 6
Algorithmic description of the dilution model.
The Early Exercise Model

SFAS No. 123(R) requires that the time to maturity of employee stock options be replaced by its expected life in their evaluation. As the expected life of the options is the average time that the options remain unexercised, it embodies information about the employees' exercise policies. However, it is clearly not satisfactory to use a single “representative” option value with the expected life as its time to maturity multiplied by the total number of employee stock options as the estimate of the total cost of the options. Instead, it is more accurate to calculate the average of the options’ values using each option’s life.

A more elaborate scheme that explicitly incorporates the exercise pattern of the options throughout their life will be proposed here. The exercise ratio at each period and the value of the option that expires at that period will be multiplied. (The exercise ratio is the proportion of the number of options exercised at that period relative to the total number of options granted.) Then these products are summed to arrive at a weighted average option value, our model option value.

The next question is about what the exercise ratios look like over time. According to the empirical findings of Bajaj, Mazumdar, Surana, and Unni (2006), there is a spurt of early exercise immediately after initial vesting. Later, the early exercise slows down to a relatively stable rate for the bulk of the remaining vested life until close to the expiration date when it increases sharply. See Figure 7 for illustration. This pattern will be approximated next with the help of the $\chi^2$ distribution.

The main reason for using the $\chi^2$ distribution to capture qualitatively the aggregate exercise behavior of employees is the convenience and ease with which it can fit the observed exercise pattern of employees. We choose not to use the empirical results in the article for the employees’ exercise pattern for a few reasons. A single set of historical data describes a specific employee stock option plan and a particular group of employees, but the terms of employee stock options may differ from company to company and from plan to plan. Once the terms change, employees’ exercise behavior will follow suit. Even if the exercise pattern of employees is derived from a database composed of a lot of employee stock option plans from different companies, the exercise pattern may not be representative of any particular plan. Moreover, historical data cannot match employees’ future exercise patterns because different groups of employees may have different exercise patterns. It is also hard to predict the time the employee will exercise the options; many factors affect an employee’s exercise behavior, and these factors may also change with time. Thus, even with historical data, employees’ exercise pattern remains elusive. Furthermore, some companies may not have data for their own employees’ past exercise patterns.
This study therefore decides to employ the $\chi^2$ distribution to demonstrate the idea of weighted average option value.

Recall the probability density function of the $\chi^2$ distribution

Figure 8 depicts the probability density function of the $\chi^2$ distribution under various $k$ with the parameter $x$ standing for the time in years. Now turn our objective curve in Figure 7 upside down to obtain a curve $C$. Observe that the curves on the right panel of Figure 8 fit $C$ better. Hence, the parameter $k$ is set to be the vesting period (in years) plus 4 and $x$ is set to start from the length of the vesting period in years. For example, if the vesting period is one year, then $x$ starts from 1. The setting of the parameter $k$ and $x$ is for better fitness of the observed exercise pattern.

As mentioned above, we are looking for a curve that resembles the one described in Bajaj et al. (2006), i.e., Figure 7. Toward that end, the $\chi^2$ distribution’s probability density function is turned upside down. Moreover, to make sure that the resulting curve is a valid probability density function, we have to

$$f(x) = \begin{cases} \frac{(1/2)^{k/2}}{\Gamma(k/2)} x^{k/2-1} e^{-x/2} & \text{if } x \geq 0 \\ 0, & \text{if } x < 0. \end{cases}$$
find the specific height, $y$, which makes the area between $f(x)$ and $y$ (the shaded area in Figure 9) for $x$ in the post-vesting period equal 1. This can be done by solving the equation $y = 1 + \int_v^T f(x)dx/(T - v)$, where $v$ is the vesting date and $T$ is the maturity date, both measured in years. After obtaining the height $y$ numerically, subtract $f(x)$ from it at each period. Hence, $g(x) \equiv y - f(x)$ becomes the proposed exercise pattern. Figure 10 depicts the exercise patterns of employee stock options for $k = 5 \sim 8$ that look qualitatively similar to the goal in Figure 7. Of course, a company may use any function $g(x)$ to serve as the exercise pattern if it possesses inside information or above-average predicting power.

In the following, $s$ stands for the number of periods in the vesting period, $n$ is the number of periods after the vesting period, and the total number of periods from the vesting date to the maturity date $T$ of the options is $h$. Thus, $s + h = N, 1 \leq n \leq h$ (see Figure 11). The exercise ratio is formally defined as

$$R_n = \frac{g(x_n)}{\sum_{m=1}^{h} g(x_m)}, 1 \leq n \leq h.$$
Finally, the weighted average option value is

\[ C_{AVG} = \sum_{n=1}^{h} (1 - q)^{(s+n)\Delta t} \times R_n \times C_n \]

where \( C_n \) is the European option value with the time to maturity equal to \((s + n)\Delta t\).

### The Employee Forfeiture Rate Model

Recall that after computing the option value with backward induction, the option value is multiplied by \((1 - q)^{v}\) to arrive at \(C^{FAS123}\) for SFAS No. 123(R). However, the assumption of a constant employee forfeiture rate is clearly not realistic. When unvested employee stock options are in-the-money, employees who hold them will be less likely to leave the company voluntarily. It stands to reason that the employee forfeiture rate is negatively correlated with the stock price. As the extent to which the option is in-the-money can be measured by moneyness, \( S_{i,j}/K \), and the employee forfeiture rate is negatively correlated with the stock price, the reciprocal of moneyness, namely \( K/S_{i,j} \), is used to revise the pre-vesting employee forfeiture rate from \( q \) to \( qK/S_{i,j} \).

After the vesting period, the exercise ratio \( R_n \) will be used to model the early exercise behavior. If the option is vested and

\[ 0 < S_{i,j} - K \leq e^{-r\Delta t}[pC_{i+1,j} + (1 - p)C_{i+1,j+1}] \]

the employee forfeiture rate is adjusted to be \((q + R_n)\Delta t\). Here is the reasoning. As the exercise value is positive, if for any reason the employee must exercise the options, he will exercise them. Thus, the employee forfeiture rate is adjusted upward by the amount of \( R_n \). On the other hand, if the option is vested but the exercise value is zero or negative, the employee forfeiture rate is adjusted to be \((q - R_n)\Delta t\). This is because a rational employee will never exercise the options under the circumstances. Hence, the employee forfeiture rate is adjusted downward by the amount of \( R_n \). The rate \((q \pm R_n)\Delta t\) may be called the early exercise ratio. This is because once the options are vested, they may be exercised based on various reasons, including employees’ severance, the level of the stock, and so on. Hence, the rate \((q \pm R_n)\Delta t\) incorporates many factors that
may affect early exercise. Figure 12 is a precise algorithmic description of the employee forfeiture rate model.

The Complete Model

Now combine the three extensions into a single, coherent model. The definitions of $s$, $n$, $h$, and $R_n$ are the same as in the section “The Early Exercise Model.” The precise algorithmic description of the complete model appears in Figure 13.

### NUMERICAL RESULTS AND COMPARATIVE ANALYSIS

This section compares the models proposed in the section “The Enhanced Employee Stock Option Valuation Model” and some popular models.
According to Ammann and Seiz (2004), as long as the expected lives of the options in the Hull–White model, the utility maximization model, and the enhanced American model are the same, these three models generate almost the same value. Thus, only the SFAS 123 model and the Hull–White model are compared with the models proposed in the section “The Enhanced Employee Stock Option Valuation Model.” The sample option parameters used in numerical experiments are shown in Table I. In the section “The Dilution Effect on the Value of Employee Stock Options,” the length of the vesting period is set to be three years, whereas for the rest of the section, the option values under different models will be compared by varying the length of the vesting period.

The Dilution Effect on the Value of Employee Stock Options

Define the dilution rate as the number of employee stock options exercised over the total number of outstanding shares of the company. It is assumed that

\[ C_{n,j} = \max \left\{ \frac{S_{n,j} \theta + K \theta}{\theta + \theta} - K, 0 \right\}, \quad m = s + n, \quad n = 1,2,\ldots,h \quad \text{and} \quad j = 0,1,\ldots,m \]

For \( 1 \leq i \leq n - 1, \quad 0 \leq j \leq i \),

\[
\begin{align*}
&\text{If the option is vested and } S_{i,j} - K > 0, \text{ then } \\
&C_{i,j} = [1-(q + R_n)\Delta t] \times e^{-\alpha \Delta t} \left[ pC_{i+1,j} + (1-p)C_{i+1,j+1} \right] \\
&\text{If the option is vested and } S_{i,j} - K \leq 0, \text{ then } \\
&C_{i,j} = [1-(q - R_n)\Delta t] \times e^{-\alpha \Delta t} \left[ pC_{i+1,j} + (1-p)C_{i+1,j+1} \right] \\
&\text{If the option is unvested, then } \\
&C_{i,j} = \begin{cases} \\
1 - \frac{K}{S_{i,j}} q \Delta t \times e^{-q \Delta t} \left[ pC_{i+1,j} + (1-p)C_{i+1,j+1} \right], & \text{if } \frac{K}{S_{i,j}} q \Delta t < 1 \\
0, & \text{if } \frac{K}{S_{i,j}} q \Delta t \geq 1 \\
\end{cases}
\end{align*}
\]

The weighted average option value is \( C_{AVG} = \sum_{n=0}^{s} R_n \times C_n^* \), where \( C_n^* \) is the option value derived above with the time to maturity equal to \((s+n)\Delta t\).

Return \( C_{AVG} \)

FIGURE 13
Algorithmic description of the complete model.
TABLE I
Sample Option Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contractual life (T)</td>
<td>10 years</td>
</tr>
<tr>
<td>Stock price (S)</td>
<td>$50</td>
</tr>
<tr>
<td>Exercise price (K)</td>
<td>$50</td>
</tr>
<tr>
<td>Risk-free rate (r)</td>
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</tr>
<tr>
<td>Expected volatility (σ)</td>
<td>30%</td>
</tr>
<tr>
<td>Expected dividend yield (D)</td>
<td>1%</td>
</tr>
<tr>
<td>Employee forfeiture rate (q)</td>
<td>3%</td>
</tr>
</tbody>
</table>

there is no change in the company’s equity except for the exercise of those employee stock options under this specific employee stock option plan. Table II shows the option values computed with the sample parameters under the SFAS 123 model, the Hull–White model, and the dilution model. From Table II, it can be observed that when the dilution rate is relatively large, the impact of dilution on the option value becomes significant. A study shows that by the late 1990s, the outstanding employee stock options of large companies exceed 6.9% of their total outstanding shares on average (Core & Guay, 2001). Under the sample parameters with a 7% dilution rate, the option value is $13.3540. The difference between the option value with and without the dilution rate is nearly $1. The difference is nonnegligible as it is almost 7% of the option value without considering the dilution effect.

Table II is plotted in Figure 14, which compares the option values under the SFAS 123, the Hull—White, and the dilution models under various dilution rates. It demonstrates clearly that, as the dilution rate increases, the option value under the dilution model decreases. In contrast, the option values under the SFAS 123 model and the Hull–White model remain the same regardless of the dilution rate because these two models do not consider the dilution effect.
Increasing numbers of companies grant stock options to their employees as rewards. When the number of granted options is substantial, the difference between the costs of employee stock options with and without considering the dilution effect can be large in dollar terms. Thus, the dilution factor should not be ignored when valuing employee stock options.

### The Approximation of Early Exercise Patterns of Employees by the $\chi^2$ Distribution

Table III compares the option values under the SFAS 123 model, the Hull–White model, and the early exercise model based on the parameters in

### TABLE II

The SFAS 123 Model, the Hull–White model, and the Dilution Model Compared

<table>
<thead>
<tr>
<th>SFAS 123 Model</th>
<th>Hull–White Model</th>
<th>Dilution Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Option Value</td>
<td>Option Value</td>
<td>Dilution Rate (%)</td>
</tr>
<tr>
<td>14.2881</td>
<td>11.2307 (with $M = 1$)</td>
<td>0.00</td>
</tr>
<tr>
<td>0.01</td>
<td>14.2874</td>
<td>0.05</td>
</tr>
<tr>
<td>0.10</td>
<td>14.2746</td>
<td>0.50</td>
</tr>
<tr>
<td>1</td>
<td>14.1474</td>
<td>5</td>
</tr>
<tr>
<td>10</td>
<td>12.9898</td>
<td>10</td>
</tr>
<tr>
<td>20</td>
<td>10.9914</td>
<td>30</td>
</tr>
<tr>
<td>40</td>
<td>10.2063</td>
<td>50</td>
</tr>
<tr>
<td>50</td>
<td>9.5259</td>
<td>60</td>
</tr>
<tr>
<td>70</td>
<td>7.9382</td>
<td>80</td>
</tr>
<tr>
<td>90</td>
<td>7.4444</td>
<td>100</td>
</tr>
</tbody>
</table>

### TABLE III

The SFAS 123 Model, the Hull–White Model, and the Early Exercise Model Compared

<table>
<thead>
<tr>
<th>Vesting Period $v$ (in Years)</th>
<th>SFAS 123 Model</th>
<th>Hull–White Model</th>
<th>Early Exercise Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>12.8771</td>
<td>9.2836</td>
<td>14.4958</td>
</tr>
<tr>
<td>3</td>
<td>14.2881</td>
<td>11.2307</td>
<td>14.8353</td>
</tr>
<tr>
<td>4</td>
<td>15.2195</td>
<td>12.6952</td>
<td>15.1829</td>
</tr>
</tbody>
</table>
Table I. Table III is plotted in Figure 15, which compares the option values under the SFAS 123, the Hull–White, and the early exercise models with different vesting period $v$. From Figure 15, it is apparent that the option values under the SFAS 123 model and the Hull–White model change a lot more than the early exercise model as the length of the vesting period, $v$, increases. In other words, the option value under the early exercise model has low sensitivity to the length of the vesting period.

As more corporations grant employee stock options as bonus, the cost of the options can rise sharply. In the SFAS 123 model and the Hull–White model, the option costs can be easily controlled by manipulating the length of the vesting period. And if the option cost is manipulated, the reported cost of the options will surely be distorted. When this happens, the income statement cannot represent the true cost of the options, and the goal of SFAS No. 123(R) will be left unfulfilled. The early exercise model greatly reduces the possibility that the option cost is manipulated by corporations.

**State-Dependent Employee Forfeiture Rates**

Table IV tabulates the option values as calculated under the SFAS 123, the Hull—White, and the employee forfeiture rate model under different vesting periods with the parameters in Table I. Table IV is then plotted in Figure 16. It shows the discrepancy between the SFAS 123 model and the employee forfeiture rate model becomes greater as the length of vesting period lengthens, but the difference is not significant. The reason may be that the expected life of the options is short, and it is shorter than the length of the vesting period plus two years (i.e., $v + 2$) regardless of $v$ under the sample parameters. It can be imputed to the fact that the duration under which the state-dependent employee forfeiture rate is in effect is short. Moreover, the
magnitudes of adjustments to pre-vesting and post-vesting employee forfeiture rates, namely \((K/S_{ij} - 1)q\) and \(R_n\), respectively, are small. Thus, the incorporation of a state-dependent employee forfeiture rate does not result in big changes to the option value. However, as stated in the section “The Dilution Effect on the Value of Employee Stock Options,” when the number of employee stock options is large, the dollar amount can still be huge.

Incorporating a state-dependent employee forfeiture rate impacts the option value in a complex manner as the parameters are varied. Figure 17 depicts how the (relative) price difference behaves as the volatility, the risk-free rate, the continuous dividend yield, and the employee forfeiture rate \(q\) change. Price difference in Figure 17 means the percentage difference between the option values under the employee forfeiture rate model and the SFAS 123 model divided by the value under the employee forfeiture rate model. Again, the calculations are based on the parameters in Table I except for the parameter that is varied in each panel. This figure shows that, although the discrepancy between the SFAS 123 model and the employee forfeiture rate model seems minor in Figure 16, the discrepancy can be

<table>
<thead>
<tr>
<th>Vesting Period (v) (in Years)</th>
<th>SFAS 123 Model</th>
<th>Hull–White Model</th>
<th>Employee Forfeiture Rate Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10.5376</td>
<td>6.5163</td>
<td>10.6338</td>
</tr>
<tr>
<td>2</td>
<td>12.8771</td>
<td>9.2836</td>
<td>13.0132</td>
</tr>
<tr>
<td>3</td>
<td>14.2881</td>
<td>11.2307</td>
<td>14.5588</td>
</tr>
<tr>
<td>4</td>
<td>15.2195</td>
<td>12.6952</td>
<td>15.6928</td>
</tr>
</tbody>
</table>

**FIGURE 16**
Option valuation models with different vesting periods.
significant as the vesting period increases. Thus, even a small change in the employee forfeiture rate may yield a noticeable drop in the option value (Jennergren & Näslund, 1993).

In conclusion, the state-dependent employee forfeiture rate does accommodate the changes in stock price and the employees’ exercise pattern to some extent.

The Combined Effect

Table V presents the overall effect of the three extensions and compares it with the SFAS 123 and Hull–White models based on the parameters in Table I as the vesting period varies. Table V is plotted in Figure 18. From Figure 18, it is clear that the option value under the complete model is less steep and always higher than the other two models. This is mainly owing to the incorporation of the employees’ exercise pattern as approximated with the help of the distribution. The incorporation of dilution and the state-dependent employee forfeiture rate improves the accuracy further.

To sum up, after the three modifications added to the model, the option $\chi^2$ value is expected to be more realistic.
CONCLUDING REMARKS

This article addresses three key issues while valuing employee stock options: the dilution effect, the employees’ exercise pattern, and the employee forfeiture rate. This article also presents a comparative analysis of some popular models and our models.

For the dilution factor, the decrease in the option value is roughly proportional to the increase in the dilution rate. Dilution can only be ignored under the case that the cost of the options is small relative to the other dollar amounts on the income statement. To make the option value more precise, dilution should be taken into account. With the employees’ exercise pattern approximated by the $x^2$ distribution, the difference between option values with different vesting periods becomes much smaller. Hence, this addition avoids such drawbacks as the underestimation of option values and manipulation of the options’ cost. Although adding the state-dependent employee forfeiture rate does not affect the option value much, it still improves the option value’s accuracy and cannot be ignored when the dollar amount is high.

BIBLIOGRAPHY


