In addition to these measurements at fixed substrate bias, the substrate gate can be used to modulate the channel at fixed drain/source bias $V_{ds}$ (see Fig. 5). This is a fundamentally different measurement to those previously described because in

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig3}
\caption{Measured lateral resonant tunnelling transistor showing nearly equal peak separations expected for harmonic oscillator potential}
\end{figure}

the former case (with fixed substrate bias) as the source/drain bias is swept, electrons are injected from the same source sub-band, so that the spectroscopy is indicative of the quantum well eigenvalues alone. However, under fixed $V_{ds}$, the occupation of the 2-D sub-bands in the source and drain, as well as the quantum well resonance states, are controllable by $V_{gs}$. Thus, the complicated mode mixing of quantum states in all three regions of the device is revealed in a rich spectrum of multiple negative transconductances. Although a quantitative explanation of the peak heights and spacings in the current curves of Fig. 5 is theoretically intractable at present, a consistent qualitative explanation can be given. As the substrate bias is increased, both the source sub-bands and the quantum well states are pulled below the Fermi level, but generally not at the same $V_{gs}$. Peaks in the current occur when unoccupied states in the well align with occupied source sub-bands; minima occur when a well state is pulled below the Fermi level. As the next unoccupied well state is pulled closer to the Fermi level, it becomes aligned with a source sub-band, and the current is once again increased. Clearly a more sophisticated theoretical treatment is needed to fully comprehend these measurements, however the first order effects are consistent with our equilibrium modelling of the structure.

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## CONVOLUTION-BASED DCT ALGORITHM

**Indexing terms:** Algorithms, Transforms, Convolution, Mathematical techniques, Information theory

Based on elements of number theory, a new convolution-based algorithm for computing the DCT (with power of two length) is proposed. In terms of computational counts, the proposed algorithm computes a length-$N$ DCT (with $N$ a power of two) using only $N$ multiplications.

**Introduction:** Since the discovery of the discrete cosine transform (DCT), many new algorithms for computing the DCT have been developed. These algorithms are either indirect computations using fast Fourier transforms or direct computations using fast convolution techniques.
Winograd convolution algorithm, or be implemented using the number theoretical transform (NTT) which needs only order N multiplications.

In this Letter, based on elements of number theory, a new convolution-based algorithm for computing the DCT (with power of two length) is proposed. In terms of computational counts, the proposed algorithm computes a length-N DCT using only N multiplications.

The proofs of theorems 1 and 2 can be found in Reference 6.

Theorem 1 implies that there is a one-to-one mapping between the following two subsets in Zp, (the integers modulo p) that is

\[(4n + 1) \pmod{p} = \{0, 1, \ldots, p-1\} \]

Theorem 2 states that if \( n > 2 \), then \( 2n^2 - 1 = 2n^2 \). The proofs of Theorems 1 and 2 can be found in Reference 6.

Theorem 3: For the matrix of index functions

\[ M = \{ [f(4n + 1) + 1 \pmod{4N}], n = 0, 1, \ldots, N-1 \} \]

there exist a circular convolution matrix C and two permutation matrices \( P_1 \) and \( P_2 \) such that

\[ M = P_1 C P_2 \]

Proof of Corollary 1: By Theorem 1

\[ 4n + 1 = 5n^1 \pmod{4N} \]

Therefore, we can reorder the rows and columns in M, i.e.

\[ C = \{ [f(5n^1 - 5n^0 \pmod{4N})], n = 0, 1, \ldots, N-1 \} \]

Thus, C is a circular convolution matrix, and the input and output reordering processes can be achieved by two permutation matrices \( P_1 \) and \( P_2 \), respectively.

According to Wang, there are four types of DCT definition, and the computation of the four types of DCT can be reduced to the computation of the type-IV DCT. Therefore, the fast algorithms for any type of DCT depend only on the computation of the type-IV DCT.

Proposed algorithm for computing type-IV DCTs: From Reference 3, the type-IV DCT can be rewritten as

\[ X(k) = \sum_{n=0}^{N-1} x(n) \cos \left[ \frac{2\pi \alpha}{8N} \right] \]

We prove that the work for computing N-point type-IV DCTs can be achieved by computing an N-point skew circular convolution and permutations using the following processes.

Step 1: Extend \( C \) (the notation defined in Reference 3) to \( C^{*} \) (notations for simplicity) as follows:

\[ Y(k) = \sum_{n=0}^{N-1} y(n) \cos \left[ \frac{2\pi \alpha}{8N} \right] \]

where

\[ y(n) = \begin{cases} \alpha(n) & 0 \leq n \leq N - 1 \\ 0 & N \leq n \leq 2N - 1 \end{cases} \]

and then

\[ X(k) = Y(k) \quad \text{for} \quad k = 0, 1, \ldots, N - 1 \]

(ii) Step 2: Reorder the input and output sequence. Similarly to the previous work, the above 2N-point transform can be rewritten as

\[ Y(k) = \sum_{n=0}^{2N-1} y(n) \cos \left[ \frac{2\pi \alpha}{8N} \right] \]

where

\[ y(n) = \begin{cases} \alpha(n) & n = 0, 1, \ldots, N - 1 \\ 0 & N \leq n \leq 2N - 1 \end{cases} \]

and then

\[ X(k) = Y(k) \quad \text{for} \quad k = 0, 1, \ldots, N - 1 \]

(iii) Step 3: The matrix representation of eqn. 2 is

\[ G_{2N} = \left[ \begin{array}{c} \cos \left[ \frac{2\pi \alpha}{8N} \right] \\ \vdots \\ \cos \left[ \frac{2\pi \alpha}{8N} \right] \end{array} \right] \]

From Corollary 1, it follows that the equation \( G_{2N} P_2 C_{2N} P_1 \) holds where \( P_1 \) and \( P_2 \) are two permutation matrices and \( C_{2N} \) is a 2N-point circular convolution matrix and can be represented as

\[ C_{2N} = \left[ \begin{array}{c} \cos \left[ \frac{2\pi \alpha}{8N} \right] \\ \vdots \\ \cos \left[ \frac{2\pi \alpha}{8N} \right] \end{array} \right] \]

Thus, the equation \( \alpha \) can be achieved by calculating an N-point skew circular convolution matrix.

By eqn. 4, it follows that the computation of \( C_{2N} \) can be achieved by calculating an N-point skew circular convolution and additional N additions/subtractions. Consider the following remarks:

(a) Remark 1: In step 1, we extend the input sequence with N zeros, therefore the N additions/subtractions in step 4 can be replaced by the 'sign change' operations.

(b) Remark 2: In step 1, we only need half of the output sequence. Therefore, the post-operations of eqn. 4 can be achieved by 'sign change' operations.

According to the above discussion, we can conclude that the computation of an N-point type-IV DCT can be achieved by an N-point skew circular convolution with some permutations and sign changes of input and output sequences.
Algorithms for discrete sinusoidal transforms: According to the previous works, the relations between some well known discrete sinusoidal transforms (DFT, DHT (discrete Hartley transform), DCT and DST (discrete Sine transform)) are very clear, and are listed as follows:

\[
\begin{align*}
\text{DFT}(N) & \Rightarrow \text{two DCT}^R(N/2) \\
\text{DHT}(N) & \Rightarrow \text{two DCT}^I(N/2) \\
\text{DCT}^R(N) & \Rightarrow \text{DCT}^R(N/2) \\
\text{DCT}^I(N) & \Rightarrow \text{DCT}^I(N/2) \\
\text{DST}^R(N) & \Rightarrow \text{two SCC}^R(N/4) \\
\text{DST}^I(N) & \Rightarrow \text{two SCC}^I(N/4)
\end{align*}
\]

Based on the discussion of the proposed algorithm for computing type-IV DCTs, Section 2, we can compute the \( \text{DCT}^R(N) \) using \( N \)-point skew circular convolution (SCC(N)). Therefore, the following result may be derived from the recursive formulas of eqns. 5-8:

\[
\begin{align*}
\text{DFT}(N) & \Rightarrow \text{two SCC}^{R,N/4} \Rightarrow \text{two SCC}^{R,N/8} \Rightarrow \ldots \\
\text{DHT}(N) & \Rightarrow \text{two SCC}^{I,N/4} \Rightarrow \text{two SCC}^{I,N/8} \Rightarrow \ldots \\
\text{DCT}^R(N) & \Rightarrow \text{SCC}^{R,N/2} \Rightarrow \text{SCC}^{R,N/4} \Rightarrow \ldots \\
\text{DST}^R(N) & \Rightarrow \text{SCC}^{I,N/2} \Rightarrow \text{SCC}^{I,N/4} \Rightarrow \ldots
\end{align*}
\]

with some interblock additions and sign changes.

Although the DFT is defined in the complex number system, we can still derive an algorithm using only real SCCs.

**Conclusion:** We have developed an algorithm which transfers the problem of \( N \)-point type-IV DCT into the problem of \( N \)-point type-IV DCT, using \( \frac{N}{2} \)-point skew circular convolution. In theory, this algorithm can achieve the lower bound of the number of multiplications according to the minimum complexity polynomial algorithms. In practice, by means of the number theoretical transform, we can compute \( \text{DCT}^R(N) \) using only \( N \) multiplications, or we can use a filter-type structure that is very suitable for the VLSI implementation.

According to the relations between type-IV DCT and other famous transforms, we have mentioned that the other discrete sinusoidal transforms can be computed by means of the combination of some SCCs of smaller size, and possess the same advantages in both theoretical and practical as the type-IV DCT.

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Appendix: The proof of eqn. 3 is as follows:

\[
\begin{align*}
\cos \left( \frac{2 \pi}{N} \cdot 5^n \right) &= \cos \left( \frac{2 \pi \cdot 5^n}{8N} \right) \quad \text{(by theorem 3)} \\
&= \cos \left( \frac{5^n + 2 \pi}{8N} \right) \\
&= \cos \left( \frac{\pi + 2 \pi}{8N} \right) \\
&= -\cos \left( \frac{2 \pi \cdot 5^n}{8N} \right)
\end{align*}
\]

**TAPERED InP/InGaAsP WAVEGUIDE STRUCTURE FOR EFFICIENT FIBRE-CHIP COUPLING**

**Indexing terms:** Optoelectronics, Optical waveguides, Integrated optics, Optics

A novel passive InP/InGaAsP waveguide structure for low-loss coupling of monomode fibres to semiconductor devices having waveguides with small elliptical modes has been fabricated. The device consists of a fibre-matched waveguide, a tapered waveguide structure for the necessary mode transformation, and a small spot waveguide. The transformation of the fibre mode into a mode with a spot of 2-6µm lateral and 1-3-6µm vertical extension (FWHM) is demonstrated. Nearly linear dependence of transmission losses, as low as 6-9dB, are measured for uncoated devices with a 900µm long tapered section.

**Introduction:** To achieve large alignment tolerances and high efficiencies in the coupling of a monomode fibre to an optoelectronic semiconductor chip, the coupling unit is required to perform the following fundamental tasks: the large circular mode guided by the fibre to be transformed at low loss so that it matches the smaller and usually elliptic mode of the waveguide on the semiconductor chip.

Classical coupling techniques based on lenses1 or tapered fibres2 cannot fully meet these requirements as they conserve the circular form of the fibre mode. The modes associated with the mismatch in spot size are accepted as a tradeoff for the relative ease with which such rotationally symmetric devices may be designed and fabricated. Furthermore, the efficiencies attainable with these coupling units are limited by their aberrations. Finally, all these conventional techniques suffer from minute alignment tolerances representing a severe hazard to

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