Simulation of the magnetic field due to defects and verification using high-$T_c$ SQUID

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Abstract

The defect field due to a flaw in the conducting slab is studied numerically and experimentally in this report. It was found that the magnitude of the defect field exhibits a nearly exponential decrease with the increasing flaw depth, and the phase of the defect field shows a linear dependence on the flaw depth. The calculated defect field was compared with the results measured by using a high-$T_c$ SQUID. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

The theoretical analysis for the eddy-current problem is important for the quantitative non-destructive evaluation (NDE) with SQUIDs. The analytic formulas for the eddy-current distributions were studied for the unflawed samples excited by the sheet inducer [1,2] and by the circular coil exciter [3,4]. However, for the sample with a buried flaw, it is difficult to obtain the analytic formula for the eddy-current distribution because of complex boundary conditions imposed by the flaw. In this work, the eddy-current distribution in flawed conducting samples excited by the circular coil was investigated numerically by using a finite element method (FEM). The obtained defect field, which is the magnetic field due to the eddy current around defects, was compared with the results measured with a high-$T_c$ SQUID NDE system.

2. Techniques for calculations

For the calculation in this work, it was assumed that the circular coil generates the excitation field with the excitation frequency $< 1$ MHz. Thus, the magnetic field induced by the time-varying electric field is negligible. In addition, the sample was assumed to be non-magnetic with permeability $\mu = \mu_0$. For the eddy-current problem with a cylindrical symmetry, the analytic solution for the eddy-current distribution in the flawless sample was found to be [4]

$$J_{\text{eddy}}(r, z) = \frac{I_r}{2\pi} \int_0^\infty J_1(xr)J_1(xr) \times \exp\left(-\left(xz - z\sqrt{x^2 + j(2/\delta^2)}\right)\right) \times \frac{2x}{x + \sqrt{x^2 + j(2/\delta^2)}} \mathrm{d}x \quad (1)$$

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in which \( r_0 \) is the radius of the excitation coil, \( I \) is the excitation current, \( \lambda \) is the coil-to-sample distance, \( \omega \) is the angular excitation frequency, \( \delta = (2/\omega \sigma \mu_0)^{1/2} \) is the skin depth, \( \sigma \) is the conductivity and \( \mu_0 \) is the permeability of the sample. \( J_1(xr) \) is the Bessel function of first kind and one order. The coordinate \( z \) is the depth measured from the surface of the sample, and \( r \) is the radius measured from the \( z \)-axis.

A FEM is used to calculate the eddy-current distribution in both the flawed and unflawed samples. The geometry of the sample with a flaw was shown in Fig. 1. The conducting slab was divided into \( N \) rings with the cross-section \( c_k \times c_k \) and the radius \( r_k \) for the \( k \)th ring. The eddy current \( I_k \) was assumed to distribute uniformly in each ring. With a time dependent harmonic excitation current, \( I^\omega \), in the excitation coil, the interaction between the rings results in \( N \) coupled linear equations

\[
-j \omega M_{kl} I = (R_k + j \omega L_k) I_k + j \omega \sum_{k \neq k'} M_{k'k} I_{k'}
\]

in which \( M_{kl} \) is the mutual inductance between the excitation coil and the \( k \)th ring, \( M_{k'k} \) is the mutual inductance between rings, \( R_k \) is the resistance, and \( L_k \) is the self-inductance of the \( k \)th ring. \( R_k \) is calculated from \( R_k = c_k^2/2 \pi r_k \sigma \) while \( L_k \), and \( M_{k'k} \) were calculated as functions of \( c_k \), \( r_k \) and \( r_k' \) according to Ref. [5]. When all of the \( I_k's \) in Eq. (2) were found, the magnetic field measured by the SQUID was obtained by using the Biot–Savart law. The defect field was defined to be the difference in the magnetic field measured by the SQUID between the flawed sample and the unflawed sample,

\[
\frac{B \exp(-\delta r)}{B_{\text{flawed}} - B_{\text{unflawed}}}
\]

in which \( B \) on the left-hand side is the magnitude of the defect field, and \( \phi \) is the phase difference between the phase of the excitation current and the phase of the defect field.

3. Results and discussion

The distributions of the eddy-current density in the flawed and in the unflawed samples were calculated with the FEM as shown in Fig. 2. The geometry of the sample with a flaw used for calculating the eddy-current distribution was the same as that shown in Fig. 1. The sample was a thick aluminum disk with thickness of 1 cm and diameter of 4 cm, while the flaw in the sample was a disk-like hollow with thickness of 1 mm and diameter of 8 mm. The diameter of the circular coil exciter was 1.4 cm. With \( c_k = 1 \) mm, the sample was divided into 200 ring elements in order to calculate the eddy currents in the sample. It was found that the eddy-current distribution in the flawed sample shown in Fig. 2(b) is not significantly different from that for the unflawed sample shown in Fig. 2(a) except for zero current density in the hollow of the flaw. The maximum eddy-current density

![Fig. 1. Cross-section view of the sample with a buried flaw.](image1)

![Fig. 2. Eddy-current distribution in the flawed and unflawed aluminum slabs for the excitation frequency of 400 Hz. The induced eddy current is localized in the vicinity of the excitation coil. In addition, the eddy-current distribution in the flawed slab is not significantly different from that in the unflawed sample.](image2)
density occurs near the wire of the excitation coil, and the eddy-current density decreases rapidly for the positions away from the wire of the excitation coil. This is consistent with the fact that the excitation field is localized in the vicinity of the excitation coil. The localized excitation also implied that the eddy-current distribution in the finite slab should be similar to that in the semi-infinite slab because of the localization of the eddy-current distribution. Thus, in order to test the validity of the FEM, the eddy current in a finite slab calculated with the FEM was compared to that in a semi-infinite slab calculated with the analytic solution as it was shown in Fig. 3. For the excitation frequencies between 40 and 400 Hz, the magnitude of the eddy-current density at $r = 3.5$ mm was shown in Fig. 3(a), and the corresponding phase of the eddy current was shown in Fig. 3(b). The symbols in Fig. 3 are calculated by using the FEM described in Eq. (2), while the lines are calculated by using the analytic solution described in Eq. (1). It was found that the magnitude of the eddy-current density calculated with the FEM was in an excellent agreement with that calculated with the analytic solution at various depths. They both decreased exponentially with the increasing depth. In addition, the phase of the eddy currents calculated from Eq. (1) overlapped that calculated from Eq. (2) at various depths. The phases were linear functions of the depth. The agreement between the results calculated from Eqs. (1) and (2) was found to hold for various positions $(r, z)$ in the sample. Therefore, the FEM described in Eq. (2) was reliable in calculating the eddy-current distribution in the conducting sample.

The FEM was then utilized to calculate the defect field due to the flaw. The geometry of the sample was also the same as that shown in Fig. 1. The defect field calculated with Eqs. (2) and (3) was shown in Fig. 4. The magnitude of the defect field, which was the $B$ defined in Eq. (3), at 2 cm above the sample surface on the z-axis ($r = 0$) was shown in Fig. 4(a). The corresponding phase difference, $\phi$, at the same place was shown in Fig. 4(b). In Fig. 4(a), the magnitude the defect field is found to decrease about exponentially, which is quite similar to the depth dependence of the eddy-current density $J_{\text{eddy}}$ in the conducting sample. In addition, the phase difference $\phi$ exhibits the linear depth dependence, which also resembles the depth dependence of the phase of the eddy current shown in Fig. 3. The similarity in the depth dependence between the defect field and the eddy current was owing to the fact that the defect field is caused by the difference in the eddy-current distribution between the flawed and the unflawed sample. The defect field is generated by the current anti-dipole [4] at the position of the flaw. Hence, the depth dependence of the defect field must be related to the depth dependence of the eddy-current density.
Although the depth dependences of the eddy current and that of the defect field are similar, there are two major differences. Firstly, cross-overs are observed in the depth dependence of magnitude for the defect field with various excitation frequencies, but they were not observed in the depth dependence of magnitude for the eddy current. The cross-over means that there is a most effective excitation frequency for the defect field with a fixed flaw depth [6]. Secondly, although the phases are linear functions of the depth, the slope of the linear function for the defect field is greater than that for the eddy current by a factor of 2 with the same excitation frequency. Moreover, the difference in the slope does not disappear with the low excitation frequency. In other words, the phase of the eddy current at the depth of the flaw is not the same as the phase of defect field due to the flaw at the same depth, no matter how low the excitation frequency is. This difference suggests that the conducting material surrounding the flaw has induced additional eddy currents due to the disturbed eddy current near the flaw, which has a non-negligible contribution to the defect field. Therefore, for various excitation frequencies, it is inaccurate to calculate the phase of the defect field with the current anti-dipole [4] without the contribution from the conducting material surrounding the flaw.

To verify the calculated defect field experimentally, the induced defect field above a flawed aluminum sample was measured by using the SQUID-based NDE system reported in Ref. [6]. The excitation coil was the same as that shown in Fig. 1, and the dimensions of the sample and the flaw were the same as those reported in Ref. [6]. With the excitation frequency of 400 Hz, the measured magnitude and phase of the defect field are shown as the symbols in Fig. 5(a) and (b) respectively. The solid curve in Fig. 5(a) is the calculated magnitude the defect field for the flaw 8 mm in diameter and 1 mm in thickness. The measured magnitude was re-scaled to fit the calculated curve because of the differences in the flaw geometry. Although the flaw geometry is different, the depth dependence of the measured magnitude of the defect field was found to mock the calculated curve when the measured curve was re-scaled for the best fit as it was shown in Fig. 5(a). The lines in Fig. 5(b) are the calculated phase of the defect fields at 400 Hz for the flaws with different diameters. The solid line is for the flaw 1 mm in thickness and 8 mm in diameter, while the dashed lines are for the flaws with the same thickness but 4 and 12 mm in diameter. The calculated phase–depth curve was found to vary slightly for the flaw diameter from 4 to 12 mm. In addition, the measured phase of the defect field was in agreement with the calculated curves for the flaw depth <6 mm. For the deeper flaw depth, the greater deviation was observed between the measured phase and the calculated curve. The grown deviation...
may be due to the poor signal-to-noise ratio of the defect field for the deeper flaw. Anyway, both the measured and the calculated phases showed linear dependences on the flaw depth for the flaw depth <6 mm. Moreover, according to the simulation, the phase-depth relation almost remained the same for various flaw diameters. Therefore, it is possible to utilize the phase of the defect field for flaw depth evaluation. These facts support the method proposed in Refs. [6, 7] for the flaw depth evaluation by using the phase of the defect field.

4. Conclusion

For the conducting slab excited by a circular coil, the FEM is useful in calculating the defect field due to a flaw. The defect field exhibits the similar depth dependence for the flaws with different geometries, and the phase-depth relation of the defect field is linear. In addition, as the phase of the defect field varies slightly for various flaw diameters, it is possible to utilize the linear phase-depth relation to probe the depth of a flaw.

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References