SU(3) and nonet breaking effects in $K_L \to \gamma\gamma$ induced by $s \to d+2$ gluons due to an anomaly

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(Received 19 November 2002; published 27 May 2003)

In this paper we study the effects of $s \to d+2$ gluon on $K_L \to \gamma\gamma$ in the standard model. We find that this interaction can induce new sizable SU(3) and U(3) nonet breaking effects in $K_L \to \eta, \eta'$ transitions and therefore in $K_L \to \gamma\gamma$ due to large matrix elements of $\langle \eta' \eta' \rangle | \alpha_s G_{\mu\nu} G_{\mu\nu}^a | 0 \rangle$ from the QCD anomaly. These new effects play an important role in explaining the observed value. We also study the effects of this interaction on the $\Delta m_{K_L-K_S}$

DOI: 10.1103/PhysRevD.67.096005

It is well known that contributions from an intermediate hadronic state effect play an important role in many low energy processes. Some of the notable examples are $K_L \to \gamma\gamma$ [1,2] and $\Delta m_K = m_{K_L} - m_{K_S}^3$ [3–7]. For $K_L \to \gamma\gamma$, the direct contribution due to the quark level $s \to d \gamma\gamma$ alone accounts for only a small portion of the amplitude measured experimentally [2,7]. For $\Delta m_K$, the direct contribution due to the $\Delta S=2$ four-quark operator is again only a fraction of the experimental value depending on the value of the bag factor $B_K$ [3,4]. A simple method to estimate the contributions from intermediate hadronic states is the pole dominance approximation in which one assumes that a few low lying resonances saturate the contribution. The commonly identified resonances in the above two cases are $\pi^0, \eta, \eta'$ and $\eta''$. Combined with U(3) flavor symmetry, the $K_L \to \gamma\gamma$ amplitude can be estimated [2,6]. If a U(3) nonet is a good symmetry, the calculations are straightforward. However, not only the nonet but also SU(3) is known to be broken, so there are large uncertainties in these calculations. One should also study if there are some new contributions in the standard model (SM) which have not been examined so far. In this paper we show that indeed there is a new contribution to $K_L \to \gamma\gamma$ and $\Delta m_K$. This new contribution comes from the $s \to d+2$ gluon induced $K-\eta (\eta')$ transition, and the intermediate $\eta (\eta')$ subsequently decays into $\gamma\gamma$ or changes to another neutral kaon through the usual $\Delta S=1$ interaction. We find that, because of the large QCD anomaly hadronic matrix element $\langle \eta (\eta') | \alpha_s G_{\mu\nu}^a G_{\mu\nu}^a | 0 \rangle$ ($G_{\mu\nu}^a = \epsilon_{\mu\nu\rho\sigma} G_{\rho\sigma}^a$), the new contributions are sizable and also induce new sizable SU(3) and U(3) breaking effects.

The decay amplitude $A_{\text{dir}}$ of the direct contribution to $K_L \to \gamma\gamma$ from the quark level interaction $s \to d \gamma\gamma$ in the SM has been studied before [1,2,7]. Here we improve the calculations by including QCD corrections which also serve to set up our notation. In the SM, $s \to d \gamma\gamma$ can be generated at one-loop level by exchanging a W boson and quarks with two photons emitted from particles in the loop and particles in the external legs. The QCD corrected effective Hamiltonian for $s \to d \gamma\gamma$ is given by

$$H_{\text{eff}}(s \to d \gamma\gamma) = M_{\gamma\gamma}^{\gamma\gamma} + M_{\gamma\gamma}^{\gamma\eta},$$

where $M_{\gamma\gamma}^{\gamma\gamma}$ is the irreducible contribution with the two photons emitted from particles in the loop. $M_{\gamma\gamma}^{\gamma\eta}$ is the reducible contribution with at least one photon emitted from an external s or d quark.

The irreducible contribution $M_{\gamma\gamma}^{\gamma\gamma}$ is given by [2,7,8]

$$M_{\gamma\gamma}^{\gamma\gamma} = -\frac{16 \sqrt{2} \alpha_s m}{9 \pi} G_F N_a e^{\rho}(k_2) \frac{1}{2 k_1 \cdot k_2} \times \sum_{i=1}^{u, c, t} V_{id}^* V_{is} F(x, x_i) d y L R_{\mu \nu} e^{\star \mu}(k_1).$$

The reducible contribution $M_{\gamma\gamma}^{\gamma\eta}$ is given by [7,8]

$$M_{\gamma\gamma}^{\gamma\eta} = \frac{\sqrt{2} \alpha_s m}{6 \pi} \sum_{i=1}^{u, c, t} V_{id}^* V_{is} c_{12} \left( \frac{1}{p_{d} \cdot k_1} - \frac{1}{p_{s} \cdot k_2} \right) \times \sigma_{\mu \nu} \sigma_{\alpha \beta} \left( p_{d} \cdot k_1 \right)^2 + 2 i \left( \frac{p_{d} \mu}{p_{d} \cdot k_1} - \frac{p_{s} \mu}{p_{s} \cdot k_1} \right) \sigma_{\nu \beta} \left( k_2 \right)^2 + \left( k_1 \to k_2 \right) (\mu \to \nu, \nu \to \mu) \left(m_L + m_R \right) \times s e^{\star \mu}(k_1) e^{\star \nu}(k_2).$$

PACS number(s): 11.30.Hv, 13.25.Es, 14.40.Aq

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In the above $c_i$ are the Wilson coefficients defined in the following $\Delta S = -1$ effective Hamiltonian [9]:

$$H_{\text{eff}}(\Delta S = -1) = \frac{4G_F}{\sqrt{2}} \left[ V^{\ast}_{sd} V_{qs} (c_1 O_1 + c_2 O_2) - \sum_{k} \sum_{i=\nu,\tau} V^{\ast}_{id} V_{is} (c_i O_k) \right],$$

(5)

where the summation over $k$ is on all possible operators, four-quark operators, and quark-photon and quark-gluon operators, which are defined in Ref. [9]. The operators directly relevant to our calculations to the leading order are

$$O_1 = \bar{q} \gamma_{\mu} L q \bar{d} \gamma^\mu L s, \quad O_2 = \bar{d} \gamma_{\mu} L q \bar{q} \gamma^\mu L s,$$

$$O_{7\gamma} = \frac{e}{16\pi} \bar{d} \sigma_{\mu\nu} F_{\mu\nu} (m_L + m_s R) s,$$

$$O_{8G} = \frac{g_s}{16\pi} \bar{d} \sigma_{\mu\nu} T^a G_\mu^a \gamma^\nu (m_L + m_s R) s,$$

(6)

where $G_\mu^a$ and $F_{\mu\nu}$ are the gluon and photon field strengths. Here we have also written down the operator $O_{8G}$ which is needed for the study of $s \to d g g$.

To obtain the amplitude $A_{\text{dir}}$ for $K_L \to \gamma \gamma$ from the effective Hamiltonian $H_{\text{eff}}(s \to d \gamma \gamma)$, one needs to bind the $d$ and $s$ quarks to form a kaon, which involves long distance non-perturbative QCD effects. This effect cannot be calculated at present and is usually parametrized by a decay constant $f_K$ as

$$\langle 0 | \bar{d} \gamma^\mu \gamma_5 s | K^0 \rangle = -i f_K F_{\mu\nu}^K$$

with $f_K$ determined from data. We have

$$A_{\text{dir}}(K^0 \to \gamma \gamma) = \langle 0 | H_{\text{eff}}(O_{7\gamma}) | K^0 \rangle = \frac{2\sqrt{2} \alpha_{\text{em}} G_F}{9\pi} f_K (i N_{a_2} V_{ud}^\ast V_{us})$$

$$+ 3(\xi c_7^2 V_{td}^\ast V_{ts}) F_{\mu\nu}^K + 3(\xi c_7^2 V_{td}^\ast V_{ts} F_{\mu\nu}^K),$$

(7)

where $H_{\text{eff}}(O_{7\gamma})$ indicates the term proportional to $O_{7\gamma}$ in the effective Hamiltonian of Eq. (5). $F_{\mu\nu}^K = (1/2) \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$. In obtaining the above result, we have used the fact that $F(x, x, c, t) \approx -1/2$ (large $x, x, c, t$) and $F(x, x, c) \approx 0$ (small $x, x, c$). We also neglected small contributions from $c_{7\gamma}$ which are proportional to $x_{u, c}$ [10], but have kept $c_7^2$ which is $-0.3$ in the SM.

The parameter $\xi$ is an average value of the quantity $\kappa = - (m_K^2/16) / (1/p_d' k_1 - 1/p_s' k_2 + 1/p_d k_1 - 1/p_s k_2)$. If one assumes that the $d$ and $s$ quarks share the kaon momentum equally, then $\xi = 1$ [2]. We have also estimated $\xi$ by calculating the quantity $\langle 0 | \bar{d} (1 + \gamma_5) s | K^0 \rangle$ using the perturbative QCD (PQCD) method and an appropriate distribution amplitude of quarks in the kaon [11]. This approach also obtains a value of order 1 for $\xi$. One should be aware that the applicability of PQCD may not be good here. However, we find that the contribution related to $\xi$ is not important as long as $\xi$ is of order 1. That is, the precise value of $\xi$ is not important here and we will set $\xi$ to be 1 in our later discussion.

To estimate the irreducible contribution, one needs to know the quantity $a_2 = c_1 + c_2 / N$. Without QCD corrections, $c_1 = 0$ and $c_2 = 1$. This gives a $a_2 = 1/3$. With QCD corrections the value for $a_2$ will be altered. The leading and next-to-leading order corrections to $c_1$ have been calculated [9]. The values of $c_i$ depend on the renormalization scale $\mu$. Since one does not know precisely where is the matching scale $\mu$, this causes uncertainty in $a_2$. For example, at the leading order, $a_2 = -0.27$ at $\mu \approx 1$ GeV, while at $\mu = 1.3$ GeV, $a_2 = -0.17$ with $\Lambda_{MS} = 325$ MeV. At the next-to-leading order the dependences on $\mu$ for each of the $c_i$ and $c_2$ are reduced, but leave $a_2$ still sensitive to $\mu$. For example, in the NDR scheme, for $\Lambda_{MS} = 325$ MeV, $a_2$ is $-0.08$ and $-0.1$ at $\mu = 1.0$ GeV and $\mu = 1.3$ GeV, respectively. Allowing the QCD parameter $\Lambda_{MS}$ to vary within the allowed range 215–435 MeV, $a_2$ can vary in the range $-0.1$ to $-0.35$ depending whether the NDR or HV scheme is used [9]. That is, the value of $a_2$ is not well determined even from next-to-leading order perturbative calculations. When all effects, perturbative and nonperturbative, are correctly treated, the final physical observables will not depend on the renormalization scale $\mu$. Unfortunately, such a calculation is not possible at present. The parameter $a_2$ behaves similarly to the one in hadronic $B$ and $D$ decays. In both $D$ and $B$ decays, the parameter $a_2$ determined from data ($|a_2| \sim 0.2–0.5$) is very different from the factorization value obtained by inserting $c_{12}$ at the relevant scale in the expression for $a_2$ [12]. One would expect a similar thing to happen in kaon decays although the details may be different. To take into account uncertainties in theoretical calculations of $a_2$, we will treat it as a free parameter and allow it to vary in the range of $-0.5$ to 0.5. One can also turn the argument around to obtain information about $a_2$ from $K_L \to \gamma \gamma$ data.

For $\xi$ of order 1, and $a_2$ in the range of $-0.5$ to 0.5, we find that the dominant direct $K^0 \to \gamma \gamma$ amplitude is from the irreducible contribution. We have

$$A_{\text{dir}}(K_L \to \gamma \gamma) = i A_{\text{dir}} \frac{1}{2} F_{\mu\nu} \tilde{F}_{\mu\nu},$$

$$A_{\text{dir}} = \frac{8 \alpha_{\text{em}} G_F}{9\pi} f_K N_{a_2} V_{ud}^\ast V_{us} \Re (V_{ud}^\ast V_{us}).$$

(8)

Using $V_{ud} = 0.9735$, $V_{us} = 0.2196$, and $f_K = 1.27 f_\pi$ [13], we obtain

$$A_{\text{dir}} = 2.54 \times 10^{-12} a_2 \text{ MeV}^{-1}.$$  

(9)

For $|a_2| = 0.5$, it is only about 35% of the experimental value of $3.5 \times 10^{-12}$ MeV$^{-1}$ [13]. Without QCD corrections $a_2 = 1/3$, $A_{\text{dir}}$ is about 24% of the total amplitude. There must be some other contributions to this process. These effects may come from contributions with intermediate hadronic...
states or even contributions from new physics beyond the SM. If one has a good understanding of all SM contributions, one can make a detailed study of new physics beyond the SM. It is probably too early to say that new physics is needed here due to large uncertainties in the possible hadronic intermediate contributions. Therefore we will work within the SM and see how contribution from hadronic intermediate states can affect the results.

Several analyses have been carried out using a pole model with $\pi^0$, $\eta$, and $\eta'$ to calculate the hadronic intermediate contribution. In this model, the amplitude $A_{\text{had}}$ from exchange of intermediate hadronic states is given by [6]

$$A(K_L\rightarrow gg) = \frac{1}{2N} \frac{2\alpha G_F}{\pi} f_K N_\alpha \text{Re}(V_{ud} V_{us}) \frac{1}{2} G^{\mu \nu}_a \tilde{G}^{\mu \nu}_a.$$  

The above interaction can induce large $K_L\eta, \eta'$ transitions and therefore contribution to $K_L\rightarrow \gamma \gamma$, because QCD can induce large matrix elements for $\langle \eta(\eta') | \alpha s G^{a}_{\mu \nu} \tilde{G}^{\mu \nu}_a | 0 \rangle$. The QCD anomaly implies that the divergence of the singlet current $a^*_\mu = \bar{u} \gamma'_\mu \gamma_5 u + \bar{d} \gamma'_\mu \gamma_5 d + \bar{s} \gamma'_\mu \gamma_5 s$ is not zero in the limit of zero quark masses, and is given by

$$\langle \eta(\eta') | \phi^0 a^*_\mu | 0 \rangle = \langle \eta(\eta') | 2i(m_u \bar{u} \gamma_5 u + m_d \bar{d} \gamma_5 d + m_s \bar{s} \gamma_5 s) | 0 \rangle - \langle \eta(\eta') | \frac{3\alpha_s}{4\pi} G^{a}_{\mu \nu} \tilde{G}^{\mu \nu}_a | 0 \rangle,$$

while for the octet current $a^8_\mu = \bar{u} \gamma'_\mu \gamma_5 u + \bar{d} \gamma'_\mu \gamma_5 d + \bar{s} \gamma'_\mu \gamma_5 s$ one obtains [14]

$$\langle \eta(\eta') | \phi^8 a^8_\mu | 0 \rangle = \langle \eta(\eta') | 2i(m_u \bar{u} \gamma_5 u + m_d \bar{d} \gamma_5 d + m_s \bar{s} \gamma_5 s) | 0 \rangle.$$  

Since $m_u, d, s$ are much smaller than $m_t$, one can neglect terms proportional to $m_{u,d}$. Then one obtains

$$\langle \eta(\eta') | \frac{3\alpha_s}{4\pi} G^{a}_{\mu \nu} \tilde{G}^{\mu \nu}_a | 0 \rangle = \frac{3}{2} \sqrt{\frac{\sqrt{2}}{f_1}} \cos \theta + f_8 \sin \theta p^2,$$

$$\langle \eta(p) | \frac{3\alpha_s}{4\pi} G^{a}_{\mu \nu} \tilde{G}^{\mu \nu}_a | 0 \rangle = \frac{3}{2} \sqrt{\frac{\sqrt{2}}{f_1}} \sin \theta + f_8 \cos \theta p^2,$$

where $f_{1,8}$ are the singlet and octet pseudoscalar decay constants.

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If there is no $\eta - \eta'$ mixing and all quark masses are equal, the $gg$ state, being a flavor singlet, can have only transitions to $\eta_1$. Because of the $\eta - \eta'$ mixing and the different quark masses, both $U(3)$ nonet and $SU(3)$ symmetries are broken.

The $K_L \rightarrow \eta, \eta'$ transitions induced by $s \rightarrow dgg$ will induce nonet and $SU(3)$ breaking in the total amplitude $A^{total}$. Normalizing the signs of each contribution to theoretical calculations, we finally obtain

$$A^{total} = A_{dir} + A (\pi^0 \rightarrow \gamma\gamma) \frac{\langle \pi^0 | H_W | K_L \rangle}{m_K^2 - m_\pi^2} \left[ 1 + \frac{m_K^2 - m_\pi^2}{m_K^2 - m_\eta^2} A (\eta \rightarrow \gamma\gamma) \left( 1 + \delta + \delta^{\eta\eta'} \cos \theta \right) \sqrt{3} \right] \left[ 1 + \frac{m_K^2 - m_{\eta'}^2}{m_K^2 - m_\eta^2} A (\pi^0 \rightarrow \gamma\gamma) \left( 1 + \delta + \delta^{\eta\eta'} \sin \theta - 2 \sqrt{\frac{2}{3}} (\rho + r^{\eta\eta'}) \cos \theta \right) \right].$$

where $\delta^{\eta\eta'}$ and $r^{\eta\eta'}$ are the $SU(3)$ and nonet breaking induced by the $s \rightarrow dgg$ interaction. They are given by

$$\delta^{\eta\eta'} = -\sqrt{2} f_K f_s m_K^2 G_F \Re \langle V^\ast_{ud} V_{us} \rangle \frac{\langle \pi^0 | H_W | K_L \rangle}{m_K^2 - m_\pi^2} a_2,$$

$$r^{\eta\eta'} = -\frac{f_1}{2 f_s} \delta^{\eta\eta'}.$$

We find

$$\delta^{\eta\eta'} = 0.96 \frac{f_s}{f_K} a_2,$$

$$r^{\eta\eta'} = -0.48 \frac{f_1}{f_K} a_2.$$  

We see that the corrections can be sizable and cannot be neglected.

We now provide some details for numerical calculations. There are several parameters involved in $A_{dir}$, the mixing angle $\theta$, the decay constants $f_1, f_8$, the $SU(3)$ and $U(3)$ nonet breaking parameters $\delta$ and $\rho$, and the parameter $a_2$. Chiral perturbation calculations and fitting data not involving $K_L \rightarrow \gamma\gamma$ have obtained $\theta = -20^\circ$, $\delta = 0.17$, $f_8 = 1.28 f_\pi$, and $f_1 = 1.10 f_\pi$ [15]. We will use these values for these parameters in the calculation of $K_L \rightarrow \gamma\gamma$. There is not a reliable estimate for the parameter $\rho$. Since we are interested to see how the new $s \rightarrow dgg$ interaction induces $U(3)$ nonet breaking effect, we will take $\rho = 1$ and attribute nonet breaking solely to $r^{\eta\eta'}$. As has been discussed, $s \rightarrow dgg$ also induces $SU(3)$ breaking effect. This effect was not included in other fittings. We therefore should include this new $SU(3)$ breaking effect also.

Without the $s \rightarrow dgg$ effect, we find that the amplitude $A^{total}$ is equal to $5.5(1+0.46a_2) \times 10^{-12}$ MeV$^{-1}$, which is considerably larger than the experimental value $3.5 \times 10^{-12}$ MeV$^{-1}$ [13] for $a_2 < 0.5$. With the new effect, we find

$$A^{total} = 5.5(1 + 2.14a_2) \times 10^{-12} \text{ MeV}^{-1}. \quad (19)$$

To reproduce the central experimental value, $a_2$ is required to be $-0.17$, which is a reasonable value to have.

The detailed numerical results depend on several parameters. Even with other parameters fixed, one can also introduce a phase in $a_2$. To fit the $K_L \rightarrow \gamma\gamma$ data, the values for the magnitude and phase of $a_2$ can vary. We would like to emphasize, however, that the new effect discussed can play an important role in $K_L \rightarrow \gamma\gamma$ independent of the details.

The new contributions for $K_L \rightarrow \eta\eta'$ transitions also induce a new hadronic intermediate state effect in the $K_L$ and $K_S$ mass difference parameter $\Re (M_{12})$ in the pole dominance approximation. We find [6]

$$2m_K \Re (M_{12}) = \frac{|\langle \pi^0 | H_W | K^0 \rangle|^2}{m_K^2 - m_\pi^2} \left[ 1 + \frac{m_K^2 - m_\pi^2}{m_K^2 - m_\eta^2} \frac{1 + \delta + \delta^{\eta\eta'} \cos \theta + 2 \sqrt{\frac{2}{3}} (\rho + r^{\eta\eta'}) \sin \theta}{\sqrt{3}} \right] \left[ 1 + \frac{m_K^2 - m_{\eta'}^2}{m_K^2 - m_\eta^2} \frac{1 + \delta + \delta^{\eta\eta'} \sin \theta - 2 \sqrt{\frac{2}{3}} (\rho + r^{\eta\eta'}) \cos \theta}{\sqrt{3}} \right]. \quad (20)$$
Without the new effects, the above would lead to $\Delta m_K = -0.5 \times 10^{-12}$ MeV, which is a non-negligible portion of the experimental value of $3.5 \times 10^{-12}$ MeV. With the new effects and $a_s = -0.17$ as determined from $K_L \rightarrow \gamma \gamma$, the contribution to $\Delta m_K$ is $-0.9 \times 10^{-12}$ MeV, and again it cannot be neglected. The new effect in $K_L \rightarrow \pi^0, \eta, \eta'$ transitions can have a sizable contribution to $\Delta m_K$.

The $s \rightarrow d\bar{g}g$ process can also induce $K_L$-glueball mixing, which would also affect $K_L \rightarrow \gamma \gamma$ and $\Delta m_{S-L}$, as pointed out in Ref. [7], where a light glueball mass 1.4 GeV was used. Recent lattice calculations indicate that the pseudoscalar glueball mass is about 2.3 GeV [16]. With such a large mass the glueball–$\eta$ ($\eta'$) mixing contribution should be small and therefore the effects are smaller than the effects discussed earlier.

In conclusion we have evaluated additional contributions to $K_L \rightarrow \eta(\eta')$ transitions from $s \rightarrow d\bar{g}g$ in the standard model. These transitions induce new sizable SU(3) and U(3) breaking effects and have significant effects on contributions to $K_L \rightarrow \gamma \gamma$ and $\Delta m_K$.

The work of X.G.H. was supported in part by the National Science Council under Grant NSC 91-2112-M-002-42, and in part by the Ministry of Education Academic Excellence Project 89-N-FA01-1-4-3. The work of C.S.H. and X.Q.L. is supported in part by the National Natural Science Foundation. X.G.H. would like to acknowledge the hospitality of the Institute for Theoretical Physics in Beijing where part of this work was carried out. He would also like to thank Hai-Yang Cheng for useful discussions.