With $\mu \to e\gamma$ decay forbidden by multiplicative lepton number conservation, we study muonium-antimuonium transitions induced by neutral scalar bosons. Pseudoscalars do not induce conversion for triplet muonium, while, for singlet muonium, pseudoscalar and scalar contributions add constructively. This is in contrast with the usual case of doubly charged scalar exchange, where the conversion rate is the same for both singlet and triplet muonium. Complementary to muonium conversion studies, high energy $\mu^+e^- \to \mu^-e^+$ and $e^-e^- \to \mu^-\mu^-$ collisions could reveal spectacular resonance peaks for the cases of neutral and doubly charged scalars, respectively.

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I. INTRODUCTION

The interest in muonium-antimuonium ($M-\bar{M}$) conversion dates back to a suggestion by Pontecorvo [1], which pointed out the similarity between the $M-\bar{M}$ and $K^0-K^0$ systems. Feinberg and Weinberg [2] noted further that $M-\bar{M}$ conversion is allowed by conservation of multiplicative muon number, muon parity, but forbidden by the more traditional additive muon number. It thus provides a sensitive test of the underlying conservation law for lepton number(s) and probes physics beyond-the standard model. One advantage of studying $M-\bar{M}$ conversion is that, once the effective four-fermion Hamiltonian is given, everything is readily calculable since it involves just atomic physics. The experiment is quite challenging, however, while on the theoretical front, it has attracted less attention from model builders compared to decay modes such as $\mu \to e\gamma$ which are in fact forbidden by the multiplicative law.

The effective Hamiltonian is traditionally taken to be of $(V-A)(V-A)$ form,

$$\mathcal{H}_{M\bar{M}} = \frac{G_{M\bar{M}}}{\sqrt{2}} \bar{\mu}\gamma_5(1-\gamma_5)e\bar{\nu}\gamma_5(1-\gamma_5)e + \text{H.c.},$$

(1)

and experimental results are given [3] as upper limits on $R_g \equiv G_{M\bar{M}}/G_F$, where $G_F$ is the Fermi constant. The present limit is $R_g < 0.16$ [4]. The limit has just been improved to the $10^{-2}$ level [5] by an ongoing experiment [6] at PSI, with the ultimate goal of reaching down to the $10^{-3}$ level.

Explicit models that lead to effective interactions of Eq. (1) were slow in coming. In 1982, Halprin [7, 8] pointed out that in left-right symmetric (LRS) models with Higgs triplets, doubly charged scalars $\Delta^{++}$ can mediate $M-\bar{M}$ transitions at the tree level in the $t$ channel [Fig. 1(a)]. The effective interaction, after Fierz rearrangement, can be put in the $(V \pm A)(V \pm A)$ form of Eq. (1). This not only encouraged experimental interests [3], it also stimulated theoretical work [9]. In particular, Chang and Keung [10] give the conditions for a generic model. Hence, the doubly charged scalar boson is well established as a leading candidate for inducing $M-\bar{M}$ transitions. However, in a recent model [11] for radiatively generating lepton masses from multiple Higgs doublets, it was pointed out in passing that the flavor-changing neutral Higgs bosons responsible for mass generation could also mediate $M-\bar{M}$ conversion. A remnant $Z_2$ symmetry serves the function analogous [10] to Feinberg-Weinberg's muon parity that forbids $\mu \to e\gamma$ transitions, while the effective four-fermion operators responsible for $M-\bar{M}$ transitions are not of the form of Eq. (1). In this paper we explore neutral scalar induced $M-\bar{M}$ oscillations [12–14] in the general case. Constraints from $g-2$ and $e^+e^- \to \mu^+\mu^-$ scattering data are studied. We point out that, complementary to muonium studies, high energy $\mu^+e^- \to \mu^-e^+$ and $e^-e^- \to \mu^-\mu^-$ collisions could clearly distinguish between (flavor-changing) neutral and doubly charged scalar bosons. Comments on several specific models are also given.

II. NEUTRAL-SCALAR-INDUCED $M-\bar{M}$ CONVERSION

Consider neutral scalar and pseudoscalar bosons $H$ and $A$, with the interaction

$$-\mathcal{L}_Y = \frac{f_H}{\sqrt{2}} \bar{\mu} e H + i \frac{f_A}{\sqrt{2}} \bar{\nu}\gamma_5 e A + \text{H.c.}$$

(2)

Imposing a discrete symmetry $P_e$ [10] such that the electron as well as $H, A$ fields are odd while the muon field is even, produces odd in number of electrons (plus positrons) such as $\mu \to e\gamma$ and $\mu \to ee\bar{\nu}$ are forbidden. Namely, scalar bosons may not possess flavor diagonal and nondiagonal couplings at the same time. $P_e$ is nothing but a variation of the multiplicative muon number of Feinberg and Weinberg [2]. The interaction of Eq. (2) induces [Figs. 1(b) and 1(c)] the effective Hamiltonian

$$\mathcal{H}_{S,P} = \frac{f_H^2}{2m_H^2} \bar{\mu} e \bar{\mu} e - \frac{f_A^2}{2m_A^2} \bar{\nu}\gamma_5 e \bar{\nu}\gamma_5 e$$

(3)

at low energy that is relevant for mediating $M-\bar{M}$ conversion. The conversion matrix elements for $S^2$ and $P^2$
where $F$ is the muonium total angular momentum, while $a$ is its Bohr radius. Thus, only scalars induce muonium conversion in the spin triplet state, while for singlet muonium, the effects of scalar and pseudoscalar channels add constructively. Note that for $(V + A)^2$ interactions of Eq. (1), we always get $8G_{M\bar{M}}/\pi a^2$ for both singlet and triplet muonium [2]. One clearly sees that separate measurements of singlet versus triplet $M - \bar{M}$ conversion probabilities can distinguish between neutral scalar, pseudoscalar, and doubly charged Higgs boson induced interactions.

In practice, $M$ is formed as a mixture of triplet and singlet states. It is crucial whether the (anti)muon decays in the presence of magnetic fields. Any sizable field strength lifts the degeneracy of $M - \bar{M}$ for $F = 1$, $\mu = \pm 1$ states, and hence effectively "quenches" [2] the $M - \bar{M}$ conversion. This is normally the case under realistic conditions, but experiments correct for this and report $G_{M\bar{M}}$ (or $R_g$) for zero $B$ field. It is important to note, however, that in so doing, one inadvertently ignores the possible differences in the neutral (pseudo)scalar case.

Let us take the example of the ongoing PSI experiment [6]. Muonium is formed and stays in the presence of 1 kG magnetic field. In this case, muonium states are populated as 32%, 35%, 18%, and 15%, respectively, for $(F, \tau) = (0, 0), (1, +1), (1, 0), \text{and} (1, -1)$. Only the $\tau = 0$ modes are active for muonium conversion; hence, the effective triplet probability comes only from $|c_1,0|^2 = 18\%$, down from 68%. For $(V + A)^2$ interactions, one simply corrects for a factor of 1/2 reduction.

For our case of neutral-scalar-induced interactions, the experimental limit on $G_{M\bar{M}}$ relates to scalar couplings as

$$\frac{G_{M\bar{M}}^{\text{expt}}}{\sqrt{2}} = \frac{1}{8} \sqrt{|c_{0,0}|^2 \left( \frac{f_H^2}{m_H^2} + \frac{f_A^2}{m_A^2} \right)^2 + |c_{1,0}|^2 \left( \frac{f_H^2}{m_H^2} \right)^2}.$$  

Several cases are of interest: (a) $f_A = 0$; (b) the "U(1) limit" of $m_A = m_H$ ($H$ and $A$ form a complex neutral scalar), with $f_A = f_H$; (c) $f_H = 0$ (pseudoscalar only). For case (a), the result is rather similar to Eq. (1). For case (b), constructive interference strongly enhances the effect in singlet channel. For case (c), only the singlet $(0, 0)$ part is active. With the newly attained [5] limit of $R_g < 10^{-2}$, we have the bounds

$$f^2/m^2 \lesssim (0.9, 0.4, 0.6) \times 10^{-6} \text{ GeV}^{-2},$$  

respectively, for the three cases, where $f/m$ stands for $f_H/m_H$ except for case (c).

## III. OTHER CONSTRAINTS

Some other constraints on $H, S, P$, such as the anomalous magnetic moments of the electron and muon, should be considered. Defining $a \equiv (g - 2)/2$, we find that

$$\delta a_e \simeq \frac{f^2}{16\pi^2} m_e \left( \frac{m_e}{3m^2} + \frac{3m_H}{2m^2} + \frac{m_{1/2}}{m^2} \right),$$  

where $\mp$ is for $H$ or $A$ contribution, respectively, while for $a_\mu$ one interchanges $e \leftrightarrow \mu$. Comparing experimental measurements [3] with QED prediction, we find $\delta a_e^{\text{expt}} = (146 \pm 46) \times 10^{-10}$ and $\delta a_\mu^{\text{expt}} = (27 \pm 69) \times 10^{-10}$. The effective bound from $\delta a_e^{\text{expt}}$ on $f^2/m^2$ is of order $G_F$, except for the U(1) limit case. In the latter case, cancellations between $H$ and $A$ lead to a much weaker limit. However, for muon $g - 2$ the leading term (proportional to $m_{1/2}$) comes from the first term of Eq. (8) which does not suffer from $H - A$ cancellation. Hence, it gives a bound of order $10G_F$ for all cases. In any rate, these limits are considerably weaker than Eq. (7).

An interesting constraint comes from high energy $e^+e^- \rightarrow \mu^+\mu^-$ scattering cross sections, which probe the interference effects between the contact terms of Eq. (2) [Fig. 1(b) in the $t$ channel] and standard diagrams. For case (b), the effective contact interaction can be put in standard form [16] for a compositeness search:

$$\mathcal{H}_{e\mu\mu} = \frac{f^2}{2m^2} (\bar{\mu}e \bar{\mu} - \bar{\mu}e \mu \tau e \bar{\mu}e),$$  

where $A \equiv A_{LR}^+$. Setting $g^2/(4\pi) = 1$, the combined
limit gives \( \Lambda(ee\mu) > 2.6 \text{ TeV} \) [15], which translates to \( f_\mu^2/m_\mu^2 < 1.9 \times 10^{-6} \text{ GeV}^{-2} \). This can be converted to a limit on \( M-M \) conversion by assuming Eq. (6),

\[
C_{M-M} < 0.06 G_F,
\]

which is better than the existing [4] \( M-M \) conversion bound of \( R_\mu < 0.16 \), but is somewhat weaker than the new bound of order \( 10^{-2} \) that has just been reported [5]. We shall return to an important consequence of the bound on \( \beta M-M \) conversion by assuming Eq. (7), on models of the type of Ref. [11] later.

IV. HIGH ENERGY MUON-ELECTRON COLLISIONS

If \( M-M \) conversion is observed, one would certainly have to make separate measurements in singlet versus triplet states to distinguish between the possible sources. Complementary to this, one could explore signals at high energies. It was pointed out a long time ago by Glashow [16] that studies of \( e^+e^- \rightarrow \mu^+\mu^- \) collisions and \( M-M \) conversion are related to each other. Indeed, shortly after the first \( M-M \) experiment [17], studies of \( e^+e^- \) collisions at SLAC improved the limit on \( G_{M-M} \) by a factor of 10 [18]. Although such efforts have not been repeated, it has been stressed recently by Frampton [19] in the context of diepton gauge bosons [20]. It is clear that if \( \Delta^- \) exists it would appear as a resonance peak in energetic \( e^+e^- \rightarrow \mu^+\mu^- \) collisions.

In contrast, it has rarely been mentioned [8] that \( \mu^+e^- \rightarrow \mu^-e^+ \) collisions may also be of great interest. Even for \( \Delta^- \) bosons, the cross section can be sizable for \( \sqrt{s} \sim m_\Delta \). However, if neutral scalars that mediate \( M-M \) conversion exist and the masses are of order TeV or below, one would have spectacular \( s \)-channel resonances in \( \mu^+e^- \) collisions. Even the nonobservation of \( M-M \) conversion does not preclude this possibility. Let us take the recent PSI bound [5] on \( M-M \) conversion at the \( 10^{-2} \) level; that is, \( f_\mu^2/m_\mu^2 \) is bound by Eq. (7). Assuming just a single scalar boson \( H \) [case (a)] that saturates such a bound and that \( H \rightarrow \mu^+\mu^- \) only, we plot in Fig. 2 the cross section \( \sigma(\mu^+e^- \rightarrow \mu^-e^+) \) vs \( \sqrt{s} \) for \( m_H = 0.25, 0.5, 1, \) and 2 TeV. The result for \( \Delta^- \) constrained by \( G_{M-M} \lesssim 10^{-2} \) is also shown in Fig. 2 as dashed lines for similar masses. Note that for \( f = 0.1-2 \), which is the plausible range for Yukawa couplings advocated in Ref. [11], Eq. (7) implies that the lower bound for \( m_H \) ranges between 100 GeV and 2 TeV. For \( e^-e^- \rightarrow \mu^-\mu^- \) collisions, the curves are rather similar, with the role of \( H \) and \( \Delta^- \) interchanged. It is clear that \( \mu^+e^- \) or \( e^-e^- \) colliders in the few hundred GeV to TeV range have the potential of observing huge cross sections, and could clearly distinguish between \( H \) and \( \Delta^- \).

The development of \( \mu^+\mu^- \) colliders has received some attention recently [21]. Perhaps one could also consider the \( \mu^+e^- \) collider option, especially if one could utilize existing facilities. As muons are collected via \( \pi \rightarrow \mu \) decay, existing accelerator complexes that have both electron and proton facilities, such as CERN or the DESY ep collider HERA, are preferred. Since \( \mu^+ \) is easier to collect and cool, while \( e^- \) requires no special effort, \( \mu^+e^- \) collisions should be easier to perform. For example, take \( E_e \) to be the CERN \( e^+e^- \) collider LEP II beam energy of 90 GeV; if intense 200 GeV to 7 TeV \( \mu^+ \) beams could be produced, one could attain \( \sqrt{s} \simeq 100 \) GeV to 1.1 TeV. Compared with problems like \( \mu^- \) decay before collision for \( \mu^+\mu^- \) colliders [21], \( \mu^-e^- \) events in \( \mu^+e^- \) collisions have practically no background. Future linear colliders should be able to span an even wider energy range, perhaps performing \( e^-e^- \), \( \mu^+e^- \), \( \mu^-e^- \) as well as \( e^+e^- \) collisions.

V. COMMENTS ON SPECIFIC MODELS

So far we have been rather general in our treatment. We now turn to some specific models and check the utility of the recent PSI bound on these models.

In the model of Ref. [11], scalar interactions of the type of Eq. (2) were used to generate charged lepton masses iteratively order by order, via effective one-loop diagrams with lepton seed masses from one generation higher. To be as general as possible, we are not concerned with the generation of \( m_\mu \) from \( m_\tau \) here. However, in analogy to the softly broken \( Z_\theta \) symmetry of Ref. [11], some discrete symmetry can be invoked to forbid electron mass at the tree level but allow it to be generated by \( m_\mu \) via one-loop diagrams as shown in Fig. 3. Since \( m_{H,A} \gg m_\mu \), we have

\[
\frac{m_e}{m_\mu} \approx \frac{f^2}{32\pi^2} \ln \left( \frac{m_H^2}{m_A^2} \right).
\]

Note that \( f_H = f_A = f \) is necessary for divergence cancellation; hence, in the \( U(1) \) limit [11] of \( m_A = m_H \) the mass generation mechanism is ineffective. We see that,
because the factor of $1/32\pi^2 \sim 1/300$ is already of order $m_e/m_\mu$, if $m_A \neq m_H$ but are of similar order of magnitude, in general we would have $f \sim 1$. This looks attractive for scalar masses far above the weak scale since one could have large Yukawa couplings but at the same time evade the bound of Eq. (7). However, in the more ambitious model of Ref. [11], radiative mass generation mechanism is pinned to the weak scale; namely, Higgs boson masses cannot be far above the TeV scale for the sake of naturalness. In this case, although Eq. (11) still looks attractive and is a simplified version of the more detailed results of Ref. [11], with $f \sim 1$ and $m_H, m_A \lesssim \text{TeV}$, the bound of Eq. (7) cannot be satisfied. We thus conclude that the bound of Eq. (7), derived from the new bound on $M\bar{M}$ conversion [5] from PSI, rules out the possibility of radiatively generating $m_\tau$ solely from $m_\mu$ via one-loop diagrams involving lepton-number-changing neutral scalar bosons that have weak scale mass. A model where $m_\tau$ dominantly comes from $m_\mu$ at the one-loop level, with a minor contribution from $m_\mu$, will be presented elsewhere.

The model of Ref. [12] enforces an $e \leftrightarrow \mu$ permutation symmetry, in contrast to the $Z_2$ (softly broken down to $Z_2$) type symmetry of Ref. [11]. In the elaborate form of extending to include three generations of quarks, the Derman model is ruled out by the observation of hadronic $b$ decays. However, the model still stands when, analogous to the attitude taken here in the present work, it is restricted to the lepton sector only. The chief phenomenological distinction between the permutation symmetry model and that of the scenario of Ref. [11] is in the Yukawa coupling strength. The permutation symmetry dictates that the heaviest flavor determines the scale of all Yukawa couplings. Thus, restricting oneself to the $e, \mu$ sector only, one would get rather weak couplings, in contrast to the general Yukawa couplings of the present paper or Yukawa couplings that are of order 1, which is advocated in Ref. [11]. Such effects may not be so easy to distinguish via $M\bar{M}$ conversion experiments, but should be easily distinguishable in high energy collisions.

In supersymmetric theories containing $R$-parity-violating terms [14], $s$-channel $\tilde{\tau}$ (tau neutrinos, a kind of neutral scalar) exchange could also induce $M\bar{M}$ conversion, resulting in $(\bar{S}-\bar{F})(S+F)$ operators. It is interesting to note that this effect evades $\mu \to e\gamma$ bound not as a result of some multiplicative lepton number, but because the product of $R$-parity-violating couplings responsible for $M\bar{M}$ conversion does not enter into processes such as $\mu \to e\gamma$ at the one-loop level. It would certainly be remarkable if neutral scalar bosons produced via energetic $\mu^+\epsilon^-$ collisions turn out to be sneutrino partners of the $\tau$ lepton. Since $|G_{MM\bar{M}}| \lesssim 2 \times 10^{-2} G_F [100 \text{ GeV}/m(\tilde{\tau})]^2$ [14], if $m(\tilde{\tau}) \sim \text{few hundred GeV}$, it could evade $M\bar{M}$ conversion experiments [5], but may readily show up in $\mu^+\epsilon^-$ collisions. The actual dominant decay channels would depend on details of the model.

VI. DISCUSSION AND CONCLUSION

We make some brief remarks before closing. Neutral scalars with flavor-changing couplings may appear to be exotic [22]. However, with multiplicative lepton number, one evades the bounds from $\mu \to e\gamma$ decay and the like. In this light, we note that any model with more than one Higgs doublet in general would give rise to flavor-changing neutral scalars. Second, the couplings of Eq. (2) demand that $H$ and $A$ carry weak isospin; hence, they must have charged partners. These charged scalars can induce the so-called "wrong neutrino" decay $\mu^- \to \nu_e \bar{\nu}_\mu$ [3]. Third, the conversion matrix elements for $(S \pm P)^2$ part of Eq. (3) can be Fierz related to the $e\mu$ sector only, while the more widely allowed $(S \mp P) (S \mp P)$ parts are related to $(V \pm A)(V \mp A)$ operators, which were considered by Fujii et al. [23] in the context of dilepton gauge bosons. In general, $M\bar{M}$ conversion may have four different kinds of sources: doubly charged scalar or vector bosons in the $t$ channel or neutral scalar or vector bosons in the $s$ or $t$ channel. Dilepton gauge boson models are therefore of the second type. Neutral vector bosons would come from horizontal gauge symmetries, but models are somewhat difficult to construct [24]. Detailed measurements of singlet versus triplet $M\bar{M}$ conversion, as well as high energy $\mu^+\epsilon^- \to \mu^+\epsilon^\pm$ and $\epsilon^-e^- \to \mu^-\mu^-$ collisions, should be able to identify the actual agent for these lepton-number-violating interactions.

Let us summarize the novel features of this paper. We have emphasized that neutral (pseudo)scalars may well induce muonium-antimuonium transitions. All one needs to invoke multiplicative lepton number rather than adhering to the traditional but more restrictive additive lepton number conservation. In this way, stringent limits from $\mu \to e\gamma$ decay, etc., are evaded. The induced operators differ from the usual $(V-A)(V-A)$ form, and care has to be taken when one interprets experimental limits. In particular, measuring $M\bar{M}$ conversion strength in both singlet and triplet muonium can distinguish between different interactions. A limit of $G_{MM\bar{M}} < 10^{-2} G_F$, just reported by an experiment at PSI, rules out the possibility of radiatively generating $m_\tau$ solely from $m_\mu$ at one-loop order via neutral scalar bosons with weak scale mass. Complementary to $M\bar{M}$ studies, high energy $\mu^+\epsilon^- \to \mu^+\epsilon^\pm$ collisions may reveal resonance peaks for flavor-changing neutral scalars, while the more widely known doubly charged scalar would appear as resonances in $\epsilon^-e^- \to \mu^-\mu^-$ collisions.

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