TABLE IV

\( \rho \), Compared with \( \rho_0 \), Amplitude Distortion Only. \( \rho \) is the Image Correlation Predicted by the Theory. \( \rho_0 \) and \( \rho_0^* \) are the Measured Correlations when the Quantizer Design Was Optimized for Each Aperture Distribution, and for the Rayleigh Distribution, Respectively. The Values in the Bottom Row Are Obtained Based on Simulation Data Having a Rayleigh Distribution

<table>
<thead>
<tr>
<th>( H_s ) (bits)</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run</td>
<td>( \rho = K\rho_0 )</td>
<td>( \rho_0 )</td>
<td>( \rho_0^* )</td>
</tr>
<tr>
<td>301</td>
<td>0.70</td>
<td>0.70</td>
<td>0.69</td>
</tr>
<tr>
<td>303</td>
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<td>0.77</td>
<td>0.77</td>
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<tr>
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<td>0.89</td>
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<tr>
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<td>0.79</td>
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<tr>
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<td>0.81</td>
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<tr>
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</tr>
<tr>
<td>Rayleigh</td>
<td>0.95</td>
<td>0.94</td>
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</tr>
</tbody>
</table>

Spatial Pseudorandom Array Processing

CHENG-YUAN LIOU AND RUEY-MING LIOU

Abstract—A pseudorandom permuting procedure along with its array signal processing is introduced to resolve multiple coherent signal sources. Conventional adaptive beamforming algorithms fail to operate in such a situation or their performance will degrade. In addition, when applied to an irregularly spaced array or when background noise is colored, most of the existing adaptive algorithms are not capable of working. The important contribution of this work is that, by introducing a new procedure to the conventional processing algorithms, they can overcome the many difficulties which occur. We test this method with computer simulations, and their results are consistent with our prediction.

I. INTRODUCTION

In radar, sonar, and seismology, one is frequently interested in estimating the directions of arrival and the spectral densities of radiating sources from measurements provided by a passive array of sensors. The problem of simultaneous estimation of the directions of arrival and the spectral densities of the impinging sources can be regarded as a two-dimensional spectral estimation problem. Given spatial and temporal samples of the received signals, the problem is to determine the 2-D spectrum or the energy distribution in both the spatial and temporal domain. The spatial spectrum consists of point masses at different angles of arrival. The temporal spectrum may consist of point masses at different frequencies in the case of narrow-band sources or may be continuous in the case of wide-band sources.

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Since the number of samples (i.e., sensors) in the spatial domain is usually small, classical Fourier analysis yields low spatial resolution. As a result, alternative methods \cite{1} that provide high resolution have been developed. These methods are known to yield high resolution and asymptotically unbiased estimates. Although the details differ in various applications, the main assumptions and processing algorithms are the same. In particular, the key assumption in all the previous cited works is that the interfering signals are not correlated with the desired signal. Once correlated interferences happen to occur, they completely destroy the performance of adaptive array systems. Theoretically, these methods encounter difficulties only when the signals are perfectly correlated. In practice, however, significant difficulties arise even when the signals are highly correlated. The perfect correlation case (or coherence case) serves as a good model for the highly correlated signals.

In spite of its practical importance, the coherence case did not receive considerable attention until recently. The earliest pioneer works on this problem are that of Gabriel \cite{2} and Widrow \cite{3}; they described two similar approaches, both aimed at \textquoteleft\textquoteleft decorrelating\textquoteright\textquoteright the coherent signals. The scheme by Widrow et al., called \textquoteleft\textquoteleft spatial dither,'\textquoteright\textquoteright is based on mechanical movement of the array elements in some way. However, this technique does not provide a clear general procedure. Gabriel's scheme is based on \textquoteleft\textquoteleft Doppler smoothing,'\textquoteright\textquoteright he also mentioned that for the so-called \textquoteleft\textquoteleft single snapshot'\textquoteright case, a solution is sometimes possible via synthetic motion of a smaller sampling subaperture along the single snapshot data samples. Evans et al. \cite{4} presented an attractive solution to the problem for the case of a uniformly spaced linear array. Their solution is based on a preprocessing scheme referred to as \textquoteleft\textquoteleft spatial smoothing,'\textquoteright\textquoteright that essentially decorrelates the signals and thus eliminates the difficulties encountered with coherent signals. Shan et al. \cite{5} have done a complete analysis for the \textquoteleft\textquoteleft subaperture sampling' or \textquoteleft\textquoteleft spatial smoothing' preprocessing scheme. They provided an algorithm that can be applied to an on-line adaptive beamformer, and its performance is good even when coherent signals are presented. Su et al. \cite{6} modified the above method by a so-called \textquoteleft\textquoteleft parallel spatial smoothing' algorithm, using a parallel structure with a spatial averaging effect to overcome coherent jamming. Their conclusion is that a spatially smooth maximum likelihood estimate of the desired signal can be obtained when the adaptive beamformer converges.

In parameter estimation using eigenstructure techniques, the noise covariance must be known explicitly \cite{7}. A method based on some sort of translation or rotation of the array to overcome this problem has been developed in \cite{7}. It is our intent to propose a solution which does not need mechanical movements to the problem of estimation of direction of arrival for a broad class of unknown noise fields.

In Section II, some investigations of current adaptive arrays will be reviewed, and several difficulties with them will be pointed out, i.e., the signal cancellation phenomenon that arises when coherent interferences present. Current approaches that solve this problem will also be examined. In Section III, a random smoothing method will be introduced plus its rationale. A high-resolution technique for spectral estimation is reviewed for later use. In Section IV, we present computer simulations of our method. And the results of simulations are analyzed and compared to other methods. In Section V, this work is concluded with several remarks.

II. PROBLEM STATEMENTS

In this section, we will review the problem briefly. Then we will use a method by Su to point out the difference between his design and ours.

Consider a simple two-element Frost beamformer. Generally, the constraint of the Frost algorithm is to let the array have unit gain and zero phase shift over a certain frequency band in the looking direction, which can be preselected by time-delay steering of the array elements, while it eliminates all the off-look direction jammers by means of minimizing its own output power. Suppose the desired sinusoidal signal $S$ arrives from the looking direction and the jammer $J$ with the same frequency as the desired signal is arriving from an off-looking direction and thus keeps a fixed phase shift with the desired signal. Denote signal $S$ and jammer $J$ as $S = A \cdot e^{j\omega t}$ and $J = B \cdot e^{j\omega t} + B \cdot e^{j\omega t}$, where $A$ and $B$ are the corresponding amplitudes of $S$ and $J$, $\phi$ is the constant phase shift between desired signal $S$ and jammer $J$, and $\omega$ is the angular frequency. Suppose an array has two sensor elements which are placed in the plane parallel to the propagation direction of the waves. Element 1 and element 2 receive both the desired signal and jammer as

$$X_1 = A \cdot e^{j\omega t} + B \cdot e^{j\omega t}$$

and

$$X_2 = A \cdot e^{j\omega t} + B \cdot e^{j\omega t}$$

where $\Delta = d \cdot \sin \theta / c$, $d$: the interelement spacing, $c$: propagation speed of the waveform, $\theta$: the jammer's incidence angle from broadside. Let the received vector and the weight vector to be denoted as $X = [X_1, X_2]^T$, $W = [W_1, W_2]^T$; then the output of the beamformer is given by $y = X^T \cdot W = X_1 \cdot W_1 + X_2 \cdot W_2$. Note that all the mathematical representatives we used here are complex and the complex algorithm of linearly constrained adaptive beamformer by Su \cite{8} is used. Typically, the Frost constraint is set to cause the receiving array having a unit gain and zero phase shift in the looking direction. Thus, we have the following expressions:

$$\min_{\|w\|^2} \text{subject to } W_1 + W_2 = 1.$$
.. \( X_0(m) \) \( \mathbf{E} \): Let it be divided into \( K \) subarrays with each subarray having \( P \) sensors and adjacent subarrays having \( P - 1 \) overlapping sensors. Also suppose the desired signal \( E \) is impinging on the array; the desired signal is from the looking direction and the jammer is from an off-looking direction. Since the array is an equally spaced linear array, each element receives the desired signal from the direction where \( P = K - 1 \) for any \( m \). Then the overall system output is obtained as follows:

\[ y(m + K - 1) \approx A e^{j\omega t} W^\top \sum_{n=0}^n e_{j(n-1)\omega} \left[ \frac{1}{K} \sum_{k=1}^K e_{j(k-1)\omega} \right] \]

where

\[ a(m + K) = \sum_{n=1}^N W_0(m + K - 1) e^{j(n-1)\omega_0} \]

In the above equation, we see that the jammer can be modified by two factors. The first factor is a function of time and subject to the least mean square criterion and the linear constraint. The second term is given as \( 1/K \sum_{k=1}^K e_{j(k-1)\omega} \), which is the summation term results in a very small value. So when the adaptation process reaches steady state, the coherent jammer effect of the second term in resolving the directional spectrum by the signature (or envelope) part of the target's spectrum is much more important than the flat white noise part, can we reduce the spectral energy of those off-looking directions is whitened by the random permutation. In order to see this point and the resolution spectral analysis methods based on the \( R(i) \) function 

This is different from the conventional beamforming method which focuses on one frequency for all direction and obtains the directional spectrum for that frequency. In our method, we obtain the whole temporal power spectrum of all frequencies for one direction, and then steer to another direction.

The Rationales

The rationale for our algorithm is that the random permuting procedure can whiten the coherent signals coming from off-looking directions. With this kind of permutation, the correlation properties of the signal sources coming from all directions, except the steered one, are totally or partially destroyed. The spectral energy of those off-looking directions is whitened by the random permutation. In other words, the desired signal’s spectrum cannot be contaminated by coherence jammers from a different direction. The jammer’s signals will add a white-noise spectrum or thermal-noise spectrum to the baseline of the desired signal’s spectrum.

According to our scheme, the signals coming from the off-looking direction are steered by the random permuting procedure and their \( Y_i(m), m = 1, 2, \ldots, M \) should be close to random noise or thermal noise. And their entropy should be increased by this random permutation, too. In order to see this point and the resolution ability in the direction spectrum, we employ the entropy rate function \( H \) to show this effect. The definition of \( h \) for a Gaussian-distributed random variables of zero mean is given by

\[ h = \lim_{L \to \infty} \frac{1}{L} \ln \det (R) \]

where \( R = \frac{1}{L} \ln \det (R) \) and \( L \) is the order of correlation matrix. Theoretically, when a signal is a pure tone, its \( R \) is one eigenvector dominant and \( h \) tends to \(-\infty \). When a signal is close to thermal noise (or white noise), it has a very large value of \( h \). We test this idea by simulation of a very narrow-band signal \( s(t) \) which is impinging on an array from the normal direction of the array. Applying our algorithm shown in Fig. 1 and using (1)-(4), we can estimate the \( R \) matrix for each steered direction. Then we substitute this \( R \) in (6). The result is plotted in Fig. 2. It is clear that this
algorithm will preserve the information content of the signal only when the array is steered toward the signal. When the direction steered is not consistent with the arrival direction of the signal, the \( R \) is close to white noise, that is, its \( h \) is increased by the permutation. Instead of deriving the theoretical approach to the resolution bandwidth reached by taking the expectation values of correlations between adjacent directions, we give the following simple intuitive explanation to the solution of resolution bandwidth. Since the whiteness (or randomness) of unwanted signals, which have no correlation property with \( R(i) = 0 \) for \( i \neq 0 \) after the random permutation, designates the independence between the desired (steered) signal, which has some correlation properties, and unwanted signals, according to the definition of resolution, the ability of resolution in the direction spectrum can be properly indicated by the width of the notch in Fig. 2. And the widths of the notches can be predetermined by computer simulations for a given array. In the above analyses, we do not assume that the sensors are equally spaced. When the array is composed of irregularly spaced sensor elements, the autocorrelation functions of all permuted time sequences will still preserve the autocorrelation function properties of the signals from the steered direction except \( R(0) \).

IV. SIMULATION RESULTS

This section provides several computer simulations. The results of simulations support our prediction. The example we considered had ten (\( Q = 10 \)) planar wavefronts at directions of arrival \( -90 + 18 \cdot (q - 1) \) degree, \( q = 1, 2, \cdots, 10 \). All of the ten signal sources are perfectly coherent with the same amplitude, namely, \( S_q(t) = \sin 0.5 \pi t \). In the first case, the array is linear and uniformly spaced with ten sensor elements. Each element is assumed to be omnidirectional, and the interelement spacing is one-half wavelength. The ambient white noise is assumed to be negligible. Two-hundred snapshots (\( M = 200 \)) for a steered direction are thus obtained. We then apply the algorithm shown in Fig. 1 with the

Fig. 3. (a) Spatiotemporal spectrum by the random smoothing method with a uniformly spaced linear array. (b) Profile of Fig. 3(a) at \( f = 0.25 \) (or \( \omega = 0.5 \pi \)).

Fig. 4. (a) Spatiotemporal spectrum by the delay-and-summed beamformer with the same array as in Fig. 3. (b) Profile of Fig. 4(a) at \( f = 0.25 \) (or \( \omega = 0.5 \pi \)).
substitution of (1)-(5) in proper places and obtain the power spectrum for that direction. Then we steer to another direction, from -90° to 90°, degree by degree. The result is shown in Fig. 3(a). The profile of Fig. 3(a) at frequency 0.25 is presented in Fig. 3(b). The ten peaks corresponding to the ten signal sources are clearly seen in these figures. The resolution bandwidth along the axis of the direction spectrum is close to the width of the notch of Fig. 2. Fig. 4(a) and (b) shows the results of simulation using conventional delay-and-summed algorithm in the same environment. In this figure, the ten coherent signals are totally lost by the coherence.

In the second case, suppose three of the ten sensors, namely, sensor number 2, 5, and 9, are damaged. Thus, the array is linear but irregularly spaced. The results, obtained from 200 "snapshots" using our algorithm are presented in Fig. 5(a). The profile of Fig. 5(a) at frequency 0.25 is presented in Fig. 5(b). We still can identify the ten peaks corresponding to the ten signal sources. There is no other method that can resolve this case efficiently as far as we know.

V. CONCLUSIONS

A "random smoothing" algorithm for array signal processing is proposed to overcome signal cancellation effects in correlated jamming environment. Our method is able to handle the particular situation when the array is irregularly spaced, especially when some of the sensors have been damaged and when the background noise is colored. The effectiveness of our method has been verified by many simulations. Due to the estimation of the ensemble average $R(i)$, our algorithm requires additional computations. Our scheme is easily extended to a multidimensional irregularly spaced array for broad-band signals and it has been verified by enormous simulations.