A new approach is presented for the load-frequency control of interconnected power systems using the theory of variable-structure systems and linear optimal control theory. A systematic procedure for the selection of the switching hyperplane, which is of vital importance in the design of variable-structure controllers, is developed by minimizing a performance index in the sliding mode operation. The proposed control scheme is illustrated by digital simulation of an interconnected power system consisting of a hydro power plant and a steam power plant.

Keywords: online control strategies, load flow, digital model

1. Introduction
Unpredictable changes in the load demand occur continuously in an interconnected power system. These changes in load always cause a mismatch between power generation and consumption, which adversely affects the quality of generated power in several ways. Among these, the frequency deviation, the time error (the time integral of deviations in frequency) and the deviation in the scheduled tie-line power, are the most important. Therefore, the objective of load frequency control (LFC) or automatic generation control (AGC) in interconnected power systems is twofold: minimizing the transient errors in the frequency and the scheduled tie-line power and ensuring zero steady-state errors of these two quantities.

The conventional and most widely used control by the industry is the classical proportional plus integral (PI) control\(^1\). Generally, the conventional approach using PI controllers results in relatively large overshoots in the transient frequency deviations. Furthermore, the settling time of the system frequency deviation is also relatively long.

A second approach based on the modern optimal control theory has been examined by many authors during the last decade\(^6\)-\(^12\). The main difficulties of the linear optimal control approach are summarized as follows.

- Because of the infinite final time specified in the performance index, the frequency deviation in the system cannot be effectively controlled.
- The controller designed by means of the linear optimal control approach depends on the parameters of the linear incremental model of the power system. The parameters of the system, in turn, depend on the operating condition of the power system. Therefore the linear optimal controller is sensitive to variations in the plant parameters or operating conditions of the power system.
- In practice, there is a maximum limit on the rate of change in the generating power of a steam power plant. This is restricted by the boiler characteristics and is in the range of 0.01-0.1 p.u./min\(^8\). It was found that the linear optimal controller yields unsatisfactory dynamic response in the presence of generation rate constraint\(^3\).

The purpose of this paper is to present a systematic approach for designing an optimal variable-structure controller (VSC) for the load frequency control of an interconnected power system. This approach makes use of the concept developed by Utkin and Yang\(^14\) and is an extension of the authors' previous works\(^13\),\(^15\). The main features of this work are as follows:

- It provides a systematic procedure for the selection of the switching vector or hyperplane of the VSC.
- The VSC so designed is optimal in the sense of minimizing a quadratic performance index specified in the sliding mode.
Other important features of the VSC are preserved, such as insensitivity to changes in the plant parameters, good transient performance, and relatively simple implementation.

II. Notation

\[ \Delta P_{gi} \]  
incremental change in generator output power, p.u.

\[ \Delta P_{ci} \]  
incremental change in speed changer position

\[ \Delta P_{dl} \]  
change of load, p.u.

\[ \Delta f_{id} \]  
incremental frequency deviation

\[ \Delta P_{tie} \]  
incremental change in tie-line power, p.u.

\[ K_r \]  
reheat coefficient of steam power plant

\[ T_r \]  
reheat time constant of steam power plant

\[ T_t \]  
turbine time constant of steam power plant

\[ T_g \]  
governor time constant of steam power plant

\[ T_{sw} \]  
nominal starting time of water in penstock

\[ u_i \]  
time constants of hydro plant

\[ z_i \]  
incremental change in speed changer position

\[ v_i \]  
change of load, p.u.

\[ \nu_i \]  
incremental frequency deviation

\[ \Delta P_{ci} \]  
incremental change in speed changer position

\[ \Delta P_{dl} \]  
change of load, p.u.

\[ \Delta f_{id} \]  
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nominal starting time of water in penstock

\[ u_i \]  
time constants of hydro plant

\[ z_i \]  
incremental change in speed changer position

\[ v_i \]  
change of load, p.u.

All symbols with subscript \( i \) refer to subsystem \( i \).

III. Mathematical model

To simplify the presentation let us consider an interconnected power system which consists of a hydro plant and a steam plant with reheat turbines. The block diagram of the system is shown in Figure 1, where subsystem 1 is a hydro plant and subsystem 2 is a steam plant.

From the block diagram and the associated transfer functions, we can write the state equation of the system as

\[ x = Ax + Bu + Fz, \quad x(0) = 0 \]  

where \( x \) is the state vector, \( u \) is the control vector and \( z \) is the disturbance vector, and the system matrices \( A, B \) and \( F \) are given by:

![Block diagram of interconnected hydrothermal power system](image-url)
The state equation, equation (1), is usually written in a more convenient form

\[ \dot{x} = Ax + Bu, \quad x(0) = -x_{ss} \]

where \( x \) is the new state vector which equals the old state vector minus its steady state value \( x_{ss} \).

The blocks designated by controller \( i, \ i = 1, 2, \) play a central role in system control. For comparison purposes we shall examine three types of controllers.

III.1 Conventional controller
For the conventional controller, the transfer functions are simply the integrator gain, \( -K_ii \), because the conventional controller is a proportional plus integral control.

III.2 Linear optimal controller
According to linear optimal control theory the optimal control law for the system with state equation, equation (2), and a quadratic performance index

\[ J = \frac{1}{2} \int_0^\infty (x^TQx + u^TRu) \, dt \]

is given by

\[ u = -Kx \]

where

\[ K = R^{-1}B^TP \]

and the matrix \( P \) is the solution of the algebraic matrix Riccati equation

\[ PA + A^TP - PBR^{-1}B^TP + Q = 0 \]

III.3 Optimal VSC
The fundamental concepts of variable-structure systems theory may be found in Itkis\(^1\)\(^6\), with brief introductions elsewhere\(^15\)\(^,\)\(^18\). A block diagram of the optimal VSC is shown in Figure 2, where the control law is a linear state feedback law whose coefficients are piecewise constant functions.

Figure 2. Block diagram of the VSC controller

The optimal VSC has two important characteristics:

- The switching hyperplanes are specified by the equations

\[ o_i(x) = c_i^Tx = 0, \quad i = 1, \ldots, m \]

or

\[ o(x) = c^Tx = 0 \]

where \( c_i \) are the switching vectors which are determined by minimizing a quadratic performance index.

- The optimal control laws are given by

\[ u_i = -\psi_i^Tx = -\sum_{j=1}^{n} \psi_{ij} x_j, \quad i = 1, \ldots, m \]

where

\[ \psi_{ij} = \begin{cases} \alpha_{ij}, & \text{if } x_j \sigma_i > 0, \quad i = 1, \ldots, m, \\ \beta_{ij}, & \text{if } x_j \sigma_i < 0, \quad j = 1, \ldots, n \end{cases} \]

The criteria for the selection of the constant vectors \( c \), \( \alpha \) and \( \beta \) will be given in section V.

IV. Problem statement
The optimal generation control problem in interconnected power systems is to design controllers which generate a best actuating signal to control the frequency deviation in each subsystem and the tie-line power resulting from sudden changes in the load. By a best actuating signal we mean that the signal is generated according to a well-defined optimality criterion.

As stated in a previous paper\(^13\), for disturbance conditions it is desirable for the system to have a proper transient frequency deviation, zero static frequency deviation and reasonably small tie-line power exchange. In order to meet these requirements for control performance and to make use of the important advantage of variable-structure systems (the sliding mode), we shall choose a quadratic
performance index in terms of the state vector in the sliding mode for the interconnected power system under consideration.

V. Synthesis of the optimal VSC

Suppose that the system considered is described by the state equation, equation (2), where the dimension of the control vector \( u \) is as usual less than that of the state vector \( x \). We shall follow the concepts developed by Utkin and Yang \(^{14} \) to summarize the design procedure for the optimal VSC with some modifications as follows:

- **Step 1. Find the equation for the system motion in the sliding mode.**

  First, we define the coordinate transformation
  \[
  y = Mx
  \]
  such that
  \[
  MB = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}
  \]
  where \( M \) is a nonsingular \( n \times n \) matrix and \( B_2 \) is a nonsingular \( m \times m \) matrix. Differentiating equation (10) with respect to time and then substituting for \( \dot{x} \) from equation (2), we have
  \[
  \dot{y} = MAM^{-1}\dot{y} + MBu
  \]
  Now writing equation (12) in the form
  \[
  \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} u
  \]
  where \( A_{11} \), \( A_{12} \), \( A_{21} \) and \( A_{22} \) are respectively \((n-m) \times (n-m), (n-m) \times m, m \times (n-m) \) and \( m \times m \) submatrices.

  Second, let the equations of the switching hyperplane of the VSC be specified by equation (7) with switching vector \( c_1 \) to be determined. In terms of \( y_1 \) and \( y_2 \) we write equation (7) in the form
  \[
  o(y) = c_{11}y_1 + c_{12}y_2 = 0
  \]
  where \( c_{11} \) and \( c_{12} \) are respectively \( m \times (n-m) \) and \( m \times m \) submatrices satisfying the relation
  \[
  [c_{11} c_{12}] = C^TM^{-1}
  \]
  Thus the first equation of equation (13), that is
  \[
  \dot{y}_1 = A_{11}y_1 + A_{12}y_2
  \]
  together with equation (14) uniquely determine the motion of the system in the sliding mode.

- **Step 2. Specify the optimal sliding mode.**

  The problem of designing an optimal VSC in the sliding mode can be treated as a linear optimal state regulator problem. Without loss of generality, we shall assume that the submatrix \( c_{12} \) in equation (14) is an identity matrix and that the performance index in the sliding mode be given by
  \[
  J = \frac{1}{2} \int_{t_s}^{t_f} \dot{y}^T Qy \, dt
  \]
  where \( t_s \) is the time instant at which the sliding mode begins and \( Q \) is a real, symmetric, positive semidefinite matrix. Generally \( t_s \) depends on the initial state \( y(0) \) and the location of the switching hyperplanes of the VSC. In terms of \( y_1 \) and \( y_2 \) the performance index in equation (17) can be written as
  \[
  J = \frac{1}{2} \int_{t_s}^{t_f} \left( y_1^T Q_{11}y_1 + 2y_1^T Q_{12}y_2 + y_2^T Q_{22}y_2 \right) \, dt
  \]
  where \( Q_{11} \), \( Q_{12} \) and \( Q_{22} \) are known submatrices of \( Q \) and \( y_2 \) is regarded as control vector.

  Utkin and Yang \(^{14} \) have shown that if (a) \( Q_{22} > 0 \), (b) the pair \((A, B)\) is controllable and (c) the pair \((A_{11} - A_{12}Q_{21}Q_{T}^{-1}Q_{11}, D)\) is observable, where
  \[
  D^TM = Q_{11} - Q_{12}Q_{22}^TQ_{12}^T
  \]
  then the optimal control law is given by
  \[
  y_2 = -(Q_{12}^T A_{12}^TP + Q_{22}^T Q_{11}^{-1})y_1
  \]
  where \( P \) is the solution of the algebraic matrix Riccati equation
  \[
  PA^T + (A')^T P - PB'(R')^{-1}(B')^T P + Q' = 0
  \]
  and
  \[
  A' = A_{11} - A_{12}Q_{11}^{-1}Q_{12}^T
  \]
  \[
  B' = A_{12}
  \]
  \[
  R' = Q_{22}
  \]
  \[
  Q' = D^TD = Q_{11} - Q_{12}Q_{22}^TQ_{12}
  \]

  - **Step 3. Determine the optimal switching hyperplane.**

    Since \( c_{12} = I \) by assumption, it follows from equation (14) that
    \[
    y_2 = -c_{11}^T c_{11} y_1 = -c_{11}^T y_1
    \]
    On comparing equation (19) with equation (21), we find
    \[
    c_{11} = Q_{12}^T A_{12}^TP + Q_{22}^T Q_{11}^{-1}Q_{12}^T
    \]
    From equation (15) we obtain
    \[
    c^T = [c_{11} I] M
    \]
    Therefore the switching hyperplanes in equation (7) or equation (14) are completely determined by equation (23).
Table 1 System parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>3 Hz/p.u.MW</td>
</tr>
<tr>
<td>$v_1$</td>
<td>0.383 p.u.MW</td>
</tr>
<tr>
<td>$T_1$</td>
<td>0.6 s</td>
</tr>
<tr>
<td>$T_2$</td>
<td>5 s</td>
</tr>
<tr>
<td>$T_3$</td>
<td>32 s</td>
</tr>
<tr>
<td>$T_w$</td>
<td>1 s</td>
</tr>
<tr>
<td>$K_{p1}$</td>
<td>20 Hz/p.u.MW</td>
</tr>
<tr>
<td>$T_{p1}$</td>
<td>3.76 s</td>
</tr>
<tr>
<td>$2\pi T_{12}$</td>
<td>0.545 p.u.MW</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_2$</td>
<td>2.4 Hz/p.u.MW</td>
</tr>
<tr>
<td>$v_2$</td>
<td>0.425 p.u.MW</td>
</tr>
<tr>
<td>$T_2'$</td>
<td>0.08 s</td>
</tr>
<tr>
<td>$T_2'$</td>
<td>10 s</td>
</tr>
<tr>
<td>$K_{p2}$</td>
<td>120 Hz/p.u.MW</td>
</tr>
<tr>
<td>$T_{p2}$</td>
<td>20 s</td>
</tr>
<tr>
<td>$\Delta P_{21}$ max</td>
<td>0.1 p.u/min</td>
</tr>
</tbody>
</table>

VI. Example

To illustrate the effectiveness of the optimal VSC, let us consider the interconnected power system in Figure 1, where a hydro plant is interconnected with a steam plant via a tie-line. The values of the system parameters are given in Table 1.

Using the parameter values of Table 1, we find the following system matrices:

\[
A = \begin{bmatrix}
0 & 0.383 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
\]

\[
B = \begin{bmatrix}
0 & 0
\end{bmatrix},
\]

We investigate the system responses for two cases using digital simulation. In the first case, there are no constraints on the state variables of the system. In the second case, there are constraints on the maximum power generation rate of the steam plant.

VI.1 System response without generation rate constraints

We examine the effectiveness of three types of controllers: the conventional controller, the linear optimal controller, and the optimal VSC for the case of no generation rate constraints being present in the system.

Case 1. The conventional controller

The control laws for the conventional control used in the simulation study are

\[
\begin{align*}
    u_1 &= -0.04x_1 \\
    u_2 &= -0.01x_7
\end{align*}
\]

Case 2. The linear optimal controller

The linear optimal control laws are given by

\[
\begin{align*}
    u_1 &= K_1 x \\
    u_2 &= K_2 x
\end{align*}
\]

where:

\[
\begin{align*}
    K_1 &= [-2.46, 0.503, -2.8, -19.9, 0.68, -4.7, 1.99, -0.899, -0.746, -0.24, 0.0667] \\
    K_2 &= [-2, -0.814, -3.17, -8.75, -0.0221, -1.25, -2.45, -3.91, -4.72, -3.63, 0.934]
\end{align*}
\]

The matrices $Q$ and $R$ are chosen with elements

\[
q_{11} = q_{22} = q_{66} = q_{77} = q_{88} = 10
\]

\[
q_{33} = q_{44} = q_{55} = q_{99} = q_{10,10} = q_{11,11} = 1
\]

All other elements of $Q$ are zero. $R = I$.

Case 3. The optimal VSC

For optimal VSC, we select the matrix $M$ with elements

\[
M_{11} = M_{22} = M_{33} = M_{44} = M_{5,10} = M_{6,6} = M_{77} = M_{88} = M_{99}
\]

\[
M_{10,5} = M_{10,10} = M_{11,11} = 1
\]

\[
M_{5,11} = M_{10,11} = -0.5, M_{34} = 2, M_{45} = -0.1562
\]

All other elements of $M$ are zero.

The equivalent weighting matrices $Q'$ and $R'$ in equation (20) are given by

\[
q'_{11} = q'_{22} = q'_{66} = q'_{77} = q'_{88} = 100, q'_{33} = q'_{44} = q'_{55} = q'_{99} = 1
\]

\[
R' = I
\]

and all other elements are zero. By virtue of equations (20) and (22) we obtain from equation (23) the matrix

\[
e = \begin{bmatrix}
8.52 & 2.53 & 21.8 & 49.7 & 2.0341 & 28.7 & -5.24 \\
5.23 & 3.28 & 6.66 & 13.4 & -0.00414 & 8.88 & 8.52
\end{bmatrix}
\]

\[
\times \begin{bmatrix}
11.4 & 7.26 & 1.99 & 0.00673
\end{bmatrix}
\]

\[
= \begin{bmatrix}
8.52 & 2.53 & 21.8 & 49.7 & 2.0341 & 28.7 & -5.24 \\
5.23 & 3.28 & 6.66 & 13.4 & -0.00414 & 8.88 & 8.52
\end{bmatrix}
\times \begin{bmatrix}
11.4 & 7.26 & 1.99 & 0.00673
\end{bmatrix}
\]

The controller gains used for the simulation study are

\[
\alpha_{11} = \alpha_{12} = -\beta_{11} = -\beta_{12} = 1
\]

\[
\alpha_{27} = \alpha_{26} = -\beta_{27} = -\beta_{28} = 1
\]

and all other gains are zero.

For a comparison study the interconnected power system shown in Figure 1 is investigated for using the above three types of controllers. The dynamic responses of the system for a step change of 0.01 p.u. in the load $\Delta P_{21}$ are obtained by digital simulation and shown in Figure 3. In order to illustrate the switching actions of the optimal VSC, the controller gains $\Psi_{11}$ and $\Psi_{21}$ are also included in
Figure 3. System responses without generation rate constraints (\(\Delta P_{r1} = 0.01 \) p.u.): -- -- -- conventional controller (case 1); -- -- -- linear optimal controller (case 2); -- -- VSC controller (case 3): (a) frequency deviation in area 1, \(\Delta f_1\); (b) frequency deviation in area 2, \(\Delta f_2\); (c) tie-line power deviation, \(\Delta P_{tie}\); (d) generated power deviation in area 1, \(\Delta P_{g1}\); (e) generated power deviation in area 2, \(\Delta P_{g2}\); (f) controller gain \(\Psi_{11}\); (g) controller gain \(\Psi_{21}\)
the figure. The system responses of Figure 3 indicate that the optimal VSC is more effective than the other two in the sense of having smaller overshoots and settling times.

VI.2 System response with generation rate constraints
In practice, there exists a limit on the rate of change in the generating power of a steam plant. This restriction is normally in the range of $0.01 \sim 0.1$ p.u./min due to practical operating limits of the boiler characteristics. For simulation studies we shall assume the generation rate constant to be 0.1 p.u./min.

Simulation results obtained by using the same controllers as in section 6.1 are shown in Figure 4. However, in this case the VSS controller gains are reduced to 0.01. It can be seen from Figure 4 that when there are generation rate constraints on the steam plant, the dynamic responses of the system experience larger overshoots and settling times, compared to the case without generation rate constants. But the optimal VSC still yields better performance than the conventional controller.

It should be noted that, in the presence of generation rate constraint, the linear optimal controller will result in oscillatory responses with increasing amplitudes and will not settle down within a period of 2 min. As a result, the dynamic responses obtained by using the linear optimal controller are not shown in Figure 4.

It is interesting to examine whether the VSC is more suitable for the steam plant than the hydro plant. In order to investigate this situation we consider the following two combinations:

Case 4 The hydro plant using a conventional controller and the steam plant using an optimal VSC. In this case we use the following control laws in the simulation:

For the hydro plant:

$$u_1 = -0.04X_1$$

For the steam plant:

$$u_2 = - \sum_{i=1}^{11} \psi_i x_i$$

$$\psi_i = \begin{cases} \alpha_i, & x_i > 0 \\ \beta_i, & x_i < 0 \end{cases}$$

Figure 4. System responses with generation rate constraints ($\Delta P_{g1} = 0.01$ p.u.); --- conventional controller (case 1); --- VSC controller (case 3); (a) frequency deviation in area 1, $\Delta f_1$; (b) frequency deviation in area 2, $\Delta f_2$; (c) tie-line power deviation, $\Delta P_{tie}$; (d) generated power deviation in area 1, $\Delta P_{g1}$; (e) generated power deviation in area 2, $\Delta P_{g2}$.
We select the matrix
\[
M = \begin{bmatrix}
1 & 0 & -0.5 \\
0 & 1 & 1
\end{bmatrix}
\] (28)

The equivalent weighting matrices \(Q'\) and \(R'\) are chosen to be
\[
q_{11} = q_{22} = q_{66} = q_{77} = q_{88} = 100 \\
q_{33} = q_{44} = q_{55} = q_{99} = q_{10,10} = 1
\]

\(R' = 9\)

and all other elements of \(Q'\) are zero.

We find the switching vector
\[
c = [2.68 \ 3.71 \ 30.8 \ -221 \ 41.9 \ 48.2 \ 30 \ 42.3 \\
37.5 \ 17.7 \ -7.85]^T
\] (29)

The variable-structure controller gains are chosen as
\[
\alpha_7 = \alpha_8 = -\beta_7 = -\beta_8 = 0.01
\]
and all other gains are zero.

Case 5. The steam plant using a conventional controller and the hydro plant using an optimal VSC. In this case we use the following control laws in the simulation:
\[
u_2 = -0.01 x_7
\]

The matrix \(M\) is selected to have elements
\[
M_{11} = M_{22} = M_{33} = M_{44} = M_{5,11} = M_{66} = M_{77} = M_{88} = M_{99} = M_{10,10} = M_{11,5} = 1, \ M_{34} = 2, \ M_{45} = -0.1562
\]
and all other elements are zero.

The equivalent weighting matrices \(Q'\) and \(R'\) are the same.
as that of case 4. Then the switching vector is found as
\[
\mathbf{c} = [30 16 82 218 -7.36 96.4 -1.52 11.8 \\
8.85 18.6 -8.38]^T
\]  
(30)

The VSC gains are chosen to be
\[
\alpha_1 = \alpha_2 = -\beta_1 = -\beta_2 = 0.01
\]
and all other gains are zero.

Simulation results for cases 3–5 are shown in Figure 5. Because of the generation rate constraint on the steam plant, in the first several (about 8–10) seconds cases (3) and (4) yield almost exactly the same response. However, case (5) yields improved response. Based on these observations from Figure 5 it appears that using a conventional controller for the steam plant and an optimal VSC for the hydro plant is a better combination for automatic generation control.

VII. Conclusions

It has been shown in this paper that it is possible to design an optimal VSC for the automatic generation control of interconnected hydrothermal power systems. This VSC is optimal in the sense that the performance of the system in the slide mode is specified by a quadratic performance index which is to be minimized.

If there are no generation rate constraints in the steam power plant, it has been found that the optimal VSC yields better system performance, which is achieved at the cost of higher generation rate. On the other hand, if severe generation rate constraints are present in the system such as in case 5, it might be better to use a conventional controller in the steam plant.

It should be noted that the procedure for designing the switching vector of the optimal VSC is a systematic one, while the problem of how to select the VSC gains remains to be solved. In this paper, the VSC gains have been obtained by trying several values and then selecting those which yield good dynamic responses.

VIII. Acknowledgement

The authors wish to acknowledge the financial assistance received from the Natural Sciences and Engineering Research Council of Canada under Grant A-5127 and the National Science Council of the Republic of China.

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