Trellis ternary line coding scheme for asynchronous optical CDMA systems

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Abstract

In this paper, we investigate the employment of a ternary line coding technique based on Ungerboeck’s trellis-coded method in asynchronous optical CDMA systems. The ternary coding we use is predicated upon the equal-weight orthogonal (EWO) scheme. Each user transmits two mutually orthogonal signature sequences to represent “+1” and “−1”, respectively, and nothing is transmitted for “0”. The receiver employs a maximum-likelihood soft-decoder to select the path with minimum Euclidean distance as the preferred path. This trellis ternary coding scheme applies set partitioning with partially overlapping subsets to increase the free Euclidean distance, which considerably improves system performance. Furthermore, due to line coding technique, such scheme comprises sufficient clock information, and thus benefits for baseband timing extraction (i.e. clock recovery). Numerical results reveal that the proposed trellis ternary coding scheme can significantly reduce the error floor and allow more active users to be accommodated in the network.

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1. Introduction

An optical code division multiple access (CDMA) technique has been proposed for high-speed fiber-optic networks \cite{1-11}. One of the most deteriorating factors in optical CDMA systems is multiple access interference (MAI), which may lead to an asymptotic error floor and thus limit the number of simultaneous active users. To mitigate the adverse impact of MAI and enhance the throughput of networks, many schemes and architectures have been discussed \cite{1-6}.

In commercial wireless spread spectrum communications, direct sequence spread spectrum (DSSS) and frequency hopping spread spectrum (FHSS) are the most promising techniques. Although hybrid systems (e.g. DSSS+FHSS) can further reduce MAI and strengthen network security, the system complexity and the enhancement of some core techniques (say, acquisition and tracking) are the inevitable costs. The data rate in optical CDMA systems is much higher than that in wireless spread spectrum communication systems; hence, this leads to stringent requirements of realizing optical CDMA systems for commercial applications.
applications. Therefore, the system complexity and some core techniques must be reasonable for implementation.

Conventionally, a sparse code is utilized as a signature sequence in direct-detection optical CDMA systems, for it can reduce the cross-correlation between different users. Optical orthogonal codes (OOCs) and prime sequences are often employed as signature sequences in asynchronous optical CDMA systems [1–3]. Such coding architectures have tunable encoders/decoders with fast reconfiguration time [8,9] and an effective interference reduction mechanism [5]; furthermore, the synchronization of signature sequence has also been discussed [12–14].

For “asynchronous” optical CDMA systems, each user transmits data asynchronously with each other; thus network synchronization is unnecessary [1–3]. However, the transmitter and intended receiver should be in exact synchronism; otherwise, timing error will lead to impairment of system performance. To eliminate timing error, a precise synchronization scheme is necessary at the receiver. Traditionally, the self-synchronization technique is often used for a synchronization subsystem in terms of benefits of bandwidth, transmitted power, and system complexity. Such a scheme acquires symbol synchronization (i.e. clock recovery) directly from the received signal, which must contain adequate timing information to allow for proper operation of clock recovery circuitry. Transmitted signals without sufficient level transitions (say, a long sequence with consecutive null symbols) will cause severe timing jitter or even transient loss of lock in baseband synchronization, and also foils adaptive equalization and echo cancellation [15,16]. In conventional optical CDMA systems [1–11], to overcome the problem mentioned above, scrambling and bit-stuffing techniques have been employed. Of course, the additional complexity is inevitable. Alternatively, the line coding technique is an effective way to overcome the problem described above. In the literature, some line coding techniques had also been discussed for applications in optical communication systems [17–19].

The trellis-coded scheme for applications in baseband systems was first discussed by Wolf and Ungerboeck [20]. Applying Ungerboeck’s trellis-coded scheme [21], a ternary line coding technique is proposed in [22]. In [10,11], the OOC and prime sequence have been studied for an equal-weight orthogonal (EWO) signaling scheme. The EWO scheme requires two mutually orthogonal signature sequences to represent binary “+1” and “−1”. It has been found that the EWO signaling scheme has some superior merits over on-off keying (OOK). First of all, an OOK signaling scheme requires a dynamically adjustable receiving threshold. The optimal threshold for optical CDMA systems varies with the number of active users, and the use of a nonoptimal threshold may degrade the bit error rate (BER) by several orders of magnitude [10,11,23]. Receivers with dynamic thresholding schemes will increase the system complexity, and will even be intractable especially in high-speed transmission and bursty traffic situations. Secondly, when the lasers are power-limited, the EWO signaling scheme is preferable to the OOK signaling scheme because it utilizes twice the amount of average signal energy in transmission [10,24–27]. Besides, the OOK optical CDMA is more vulnerable than EWO in terms of network security [28]. In this paper, we utilize a ternary {+1, 0, −1} symbol set based on the EWO scheme [10,11], and also use the line coding method [22] to further improve system performance.

Avalanche photodiode (APD) is employed for photodetection. We apply Gaussian approximation [10] to analyze the bit error probability (BEP). Moreover, instead of OOC, the prime sequence is utilized as a signature sequence in our proposed scheme for two reasons. In the first place, for a large number of users and large received optical power, using a prime sequence can yield better performance than using OOC under the EWO scheme [10,11]. In addition, efficient and less complex encoders for prime codes have been proposed which have fast reconfiguration time [8,9]; thus it is suitable for ultrafast optical CDMA networking because of the all-optical architecture used.

This paper is organized as follows. In Section 2, systems using ternary codes and trellis-coding schemes for asynchronous optical CDMA systems are described. System performance analysis of the proposed scheme is given in Section 3. Section 4 presents some numerical results. Finally, conclusions are drawn in Section 5.

2. System description

In this paper, the prime sequence is employed as a signature sequence. The prime code can be constructed from the Galois field GF(p) = {0, 1, ..., p − 1}, where p is a prime number [1,2]. There are p codewords each with weight p, and length p^2. The cross-correlation value is no greater than 2, and the cardinality is p.

In [10,11], the EWO scheme was proposed that used two mutually orthogonal signature sequences a and a̅ to represent binary “+1” and “−1”, respectively. Here a̅ is generated from a shifted reverse sequence of a (or shifted version of a). In our ternary coding, “+1” and “−1” are transmitted via signature sequences a and a̅, respectively, and no signature sequence is transmitted for “0”.

Based on Ungerboeck’s trellis-coded modulation (TCM) scheme [21], a new ternary line coding technique is proposed in [22]. This trellis ternary line coding method can reduce decoding complexity and improve system performance compared with the conventional
Ungerboeck's trellis-coding approaches. In this paper, we will combine a ternary code based on [10,11] and the line coding technique [22] for application in asynchronous optical CDMA systems.

Fig. 1 illustrates our proposed asynchronous optical CDMA system using the trellis ternary coding (TTC) scheme. The binary sequence is encoded into a ternary sequence by the trellis ternary encoder; then it is spread into a signature sequence via a tunable optical sequence encoder. At the receiver, the correlators in the parallel arms correlate the received multiple access signal individually; then the outputs are converted into electrical signals by photodiodes. The two electrical signals are subtracted to produce a ternary sequence, which is followed by a soft-decision Viterbi decoder (maximum-likelihood soft-decoder). The Viterbi soft-decoder acting on the ternary sequence selects the path with minimum Euclidean distance as the preferred path. Finally, the ternary sequence of the preferred path is converted into a binary sequence.

In Fig. 2, we illustrate this approach with a code rate $R = 1/2$ as in [22]. In Fig. 2(a), the trellis ternary encoder based on set partitioning partitions these codewords into two-element subsets. Also, partially overlapping subsets (i.e., codewords $+ -$ and $- +$ are used twice) are employed to increase the free Euclidean distance [29]. Fig. 2(b) illustrates the resulting trellis structure. In Fig. 2(c), we can see that this four-state trellis ternary structure has squared free Euclidean distance $d_{\text{free}}^2 = 8$.

Note that there are no consecutive zeros in this code; thus sufficient timing contents are ensured. For equally probable data symbols, the probability of transmitting ternary “0” is $1/4$. The free Euclidean distance is the minimum Euclidean distance between all pairs of distinct paths that diverge from a common state and remerge at another state some time later, which can be represented as

$$d_{\text{free}} = \min_{\{s_n\} \neq \{s'_n\}} \left\{ \sum_{n} d^2(s_n, s'_n) \right\}^{1/2}$$

The free Euclidean distance is the most important parameter that determines system performance of a TCM encoder. The free Euclidean distance of the trellis encoder in Fig. 2 can be calculated using a simple algorithm depicted in [22].

3. System performance analysis

At the receiver, the upper and lower correlators correlate the received multiple access signal individually, and then the correlated optical signals are incident upon APDs for photodetection. The received optical signal intensity over a chip interval $T_c$ can be modeled as a Poisson point process. The average number of absorbed photons is $\lambda_n T_c$, where $\lambda_n$ is the average arrival rate of
incident photons during a chip “1” (weight of signature sequence) transmission in the signature sequence, which can be represented as
\[ \lambda_s = \eta P_w / hf \]  
(2)
where \( P_w \) is the received optical power at optical correlator, \( \eta \) is the APD quantum efficiency, \( h \) is the Plank’s constant \((6.624 \times 10^{-34} \text{W s})\), and \( f \) is the optical carrier frequency.

Based on the EWO approach for analyzing our TTC scheme, the following system parameters are defined [10]:
\[ \gamma = \text{mean number of pulses per signature sequence period} \]  
(3)
where for prime sequence \( \gamma = p \).
\[ q = \gamma / F = 1 / p = \text{average duty cycle of code sequence} \]  
(4)
where \( F \) is the spreading factor.
\[ \mu_s = \lambda_s \gamma / T = \text{mean signal photoelectron count per ternary symbol} \]  
(5)
\[ \mu_b = \lambda_b T = K \lambda_s T / x = K \mu_s / xq = \text{mean background radiation photoelectron count per T seconds} \]  
(6)
where \( K \) is the number of active users and \( x \) is laser modulation–extinction power ratio.
\[ \mu_{bl} = \lambda_{bl} T = \text{mean APD bulk leakage current photoelectron count per T seconds} \]  
(7)
\[ \mu_0 = \lambda_0 T = \mu_b + \mu_{bl} = \text{mean total background photoelectron count per T seconds} \]  
(8)
\[ \mu_{sl} = \lambda_{sl} T = \text{mean APD surface leakage current photoelectron count per T seconds} \]  
(9)
\[ N_0 / 2 = 2KBT/e2RL = \text{two-sided power spectral density of thermal noise} \]  
(10)
\[ \gamma_{xy} / \gamma_s = \text{normalized average interference parameter for prime sequence, which can be found in [11]} \]  
(11)
where \( \gamma_{xy} \) is the average value of \( \gamma_{xy} \) for the reference user and \( \gamma_{xy} \) is an interference parameter with the reference user for sequences \( a^{(q)} \) and \( a^{(q)} \), which is derived in [10,11].

After parallel cancellation, the detection statistic \( Y \) can be approximated as a Gaussian random variable [10]. Based on the EWO method [10], if the transmitted signals are equally likely, the mean and variance of the detection statistic \( Y \) can be derived as
\[ \mu = \begin{cases} \beta G \mu_s & \text{for ternary } + 1 \\ -\beta G \mu_s & \text{for ternary } - 1 \\ 0 & \text{for ternary } 0 \end{cases} \]  
(12)
\[ \sigma^2 = \begin{cases} \beta F_s G^2 \mu_s q + 2q(\mu_s + \frac{N_0}{2}) + \sigma^2_{IK} & \text{for ternary } + 1 \text{ or } -1 \\ 2q(\mu_s + \frac{N_0}{2}) + \sigma^2_{IK} & \text{for ternary } 0 \end{cases} \]  
(13)
where
\[ \sigma^2_{IK} = 2F_s G^2 \mu_0 q_b + \beta^2 \left( \frac{3}{4} \right) (K - 1) G^2 \mu_0 \left[ \frac{2F_s}{\beta \mu_s} q + \frac{1}{3F} \gamma_{xy} \right] \]  
(14)
is the variance of “background radiation plus MAI” [10]. \( \beta = 1 - 1 / x \), and \( F_s = \text{APD excess noise factor} = K_{\text{eff}} G + (2 - 1 / G) (1 - K_{\text{eff}}) \).

Next, we will derive the upper bound of system performance for our proposed scheme. Let \( Z \) be the transmitted ternary sequence and \( Z' \) be the ternary sequence that diverges from \( Z \) at a common state and remerge at another state some time later. Also denote \( \xi \) as the received sequence. Consider that the desired signal is perturbed by MAI and noise with variance \( \sigma^2 / \mu_s^2 \); this makes Euclidean distances between sequences normalized.

Applying the union-bounding technique, the error event probability \( P_e \) of ternary codes can be obtained by summing the error probability over all possible incorrect paths which remerge with all possible correct paths [30]. Thus, \( P_e \) can be upper bounded as
\[ P_e \leq \sum_{d=d_{\text{min}}}^{\infty} A_d P_d \]  
(15)
where \( d \) represents the Euclidean distance between signal sequences, \( A_d \) is the average number of code sequences at distance \( d \) from a specific code sequence with the average taken over all code sequences in the code, and \( P_d \) is the pairwise error probability.

The pairwise error probability can be expressed as
\[ P_d = P(Z \rightarrow Z') = P(||Z - \xi|| \geq ||Z' - \xi||) \]
\[ = P(||Z - \xi|| - ||Z' - \xi|| \geq 0) \]
\[ = P(m \geq \frac{||Z - Z'||}{2 \sigma / \mu}) = Q \left( \frac{m \mu}{2 \sigma} \right) \]  
(16)
where \( m \) is a Gaussian random variable and \( Q(x) = (1 / \sqrt{2\pi}) \int_x^\infty e^{-y^2 / 2} dy \).
To evaluate the error event probability in (14), an efficient computational algorithm will be required for searching \( d \) and \( A_d \) (i.e. distance spectrum) [30]. An alternative approach to find the upper bound of the error event probability is based on the transfer function method without knowing \( d \) and \( A_d \) [31]. It is represented
as the closed-form equivalent of the infinite summation in the right-hand side of (14). The modified generating function \( T(W, L, I) \) of a trellis code can be calculated as a transfer function of a generalized state diagram [22,30]. Let \( T(W, L, I) \) be the modified generating function of a trellis code given by

\[
T(W, L, I) = \sum_{d, l, i} A_{d, l, i} W^d L^i I^i
\]

where \( A_{d, l, i} \) is the number of error events that have squared Euclidean distance \( d \), length \( l \), and \( i \) information bit errors.

A closed-form upper bound of error event probability using the transfer function method, based on the inequality \( Q(\sqrt{x + y}) \leq Q(\sqrt{x}) e^{-y/2} \), can be obtained as [31]

\[
P_e \leq Q\left(\frac{\mu d_{\text{free}}}{2\sigma} \right) e^{d_{\text{free}}^2 / 8\sigma^2} T(W, L, I)
\]

Also, the BEP can be expressed as

\[
P_b \leq \frac{1}{n} Q\left(\frac{\mu d_{\text{free}}}{2\sigma} \right) e^{d_{\text{free}}^2 / 8\sigma^2} \frac{\partial T(W, L, I)}{\partial I}
\]

where \( n \) is the number of data bits per codeword, and in our case (i.e. the TTC scheme with code rate \( R = 1/2 \) [22]) \( n = 1 \). Also, \( \mu/\sigma \) represents the ratio of mean and square root of variance for detected ternary +1 [22]; \( W = e^{-d^2 / 8\sigma^2}, L = 2^{-n} \), and \( I = 1 \) [22,31].

The modified generating function of the trellis ternary code with code rate \( R = 1/2 \) we use here can be found in [22], which is

\[
T(W, L, I) = 4L^4 W^8 I^2 + 4L^3 W^{10} I^2 + 4L^2 + L^5 W^{12} I^2 + \cdots
\]

From the transfer function, we can see that there are four pairs of paths with squared free Euclidean distance \( d_{\text{free}}^2 = 8 \) (also see Fig. 2). The following numerical calculations are all based on the transfer function method.

### 4. Numerical results

In this section, the typical APD parameters used here are the same as those in [10] and listed in Table 1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_{\text{eff}} )</td>
<td>APD effective ionization ratio</td>
<td>0.01</td>
</tr>
<tr>
<td>( R_b )</td>
<td>APD load resistance</td>
<td>1030 ( \Omega )</td>
</tr>
<tr>
<td>( T_e )</td>
<td>Preamp equivalent noise temperature</td>
<td>1100 K</td>
</tr>
<tr>
<td>( f )</td>
<td>Optical carrier frequency</td>
<td>3.63 \times 10^{14} \text{ Hz}</td>
</tr>
<tr>
<td>( N_0/2 )</td>
<td>Thermal noise two-sided PSD</td>
<td>1.15 \times 10^{-12} \text{ W/Hz}</td>
</tr>
<tr>
<td>( \lambda_b )</td>
<td>APD bulk leakage current</td>
<td>6.12 \times 10^9 \text{ photons/s}</td>
</tr>
<tr>
<td>( \lambda_s )</td>
<td>APD surface leakage current</td>
<td>7.46 \times 10^9 \text{ photons/s}</td>
</tr>
<tr>
<td>( G )</td>
<td>APD mean gain</td>
<td>250</td>
</tr>
<tr>
<td>( \eta )</td>
<td>APD quantum efficiency</td>
<td>0.6</td>
</tr>
<tr>
<td>( R_b )</td>
<td>Bit rate</td>
<td>10 Mbps</td>
</tr>
<tr>
<td>( z )</td>
<td>Laser modulation–extinction power ratio</td>
<td>( \infty )</td>
</tr>
</tbody>
</table>

Fig. 3. Bit error probability comparisons with \( p = 11 \) and 13, respectively, in the case of number of active users \( K = 10 \) and bit rate \( R_b = 10 \text{ Mbps} \).

The optimum APD gain depends on a large number of receiver parameters, such as dark current, effective ionization ratio, received optical power, etc. [24–27]. Fig. 7 shows the BEP versus mean gain of APD with number of active users \( K = 10 \), received optical power improved when using the TTC scheme. Furthermore, a system using a longer signature sequence has much better performance, because increasing the length of the signature can reduce the impact of multiple access interference [1,10].

Figs. 5 and 6 illustrate BEP comparisons with \( p = 11, 13, 17 \), and 19, under a different number of active users \( K \) in the case of received optical power \( P_w = -70 \text{ dBm} \) and bit rate \( R_b = 10 \text{ Mbps} \). As the number of active users \( K \) becomes large (i.e. MAI increased), system performance deteriorates in all cases. However, the TTC scheme can yield better performance than the EWO scheme with several orders.

The optimum APD gain depends on a large number of receiver parameters, such as dark current, effective ionization ratio, received optical power, etc. [24–27]. Fig. 7 shows the BEP versus mean gain of APD with number of active users \( K = 10 \), received optical power
5. Concluding remarks

In this paper, we apply the TTC scheme for asynchronous optical CDMA systems, which is based on the EWO scheme [10,11] and line coding technique [22]. Such arrangements not only can considerably improve system performance but also allow for proper operation of timing extraction circuitry because sufficient clock information is embedded in the transmitted signal.

The prime sequence is used as signature sequence, because for a large number of users and large received optical power $P_w$, the optimum APD gain can achieve the lowest BEP; for $p = [11, 21, 25, 31]$ the optimum APD gain is $G_{opt} = [181, 187, 196, 200]$, respectively. In Fig. 8, we present the optimum mean gain of APD under different received optical powers $P_w$ for number of active users $K = 10$ and bit rate $R_b = 10$ Mbps.
optical power systems, using a prime sequence can yield better performance than those using OOC under the EWO scheme [10,11]. For prime codes, increasing the prime number \( p \) enlarges the cardinality, and also reduces the adverse impact of multiple access interference. To achieve full network functionality, reconfiguration to address different network users requires tunable encoders. All-optical tunable encoders for prime codes with fast reconfiguration time are available [8,9]. From the practical standpoints of system performance and component feasibility, our proposed scheme is suitable for high-speed optical networking.

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